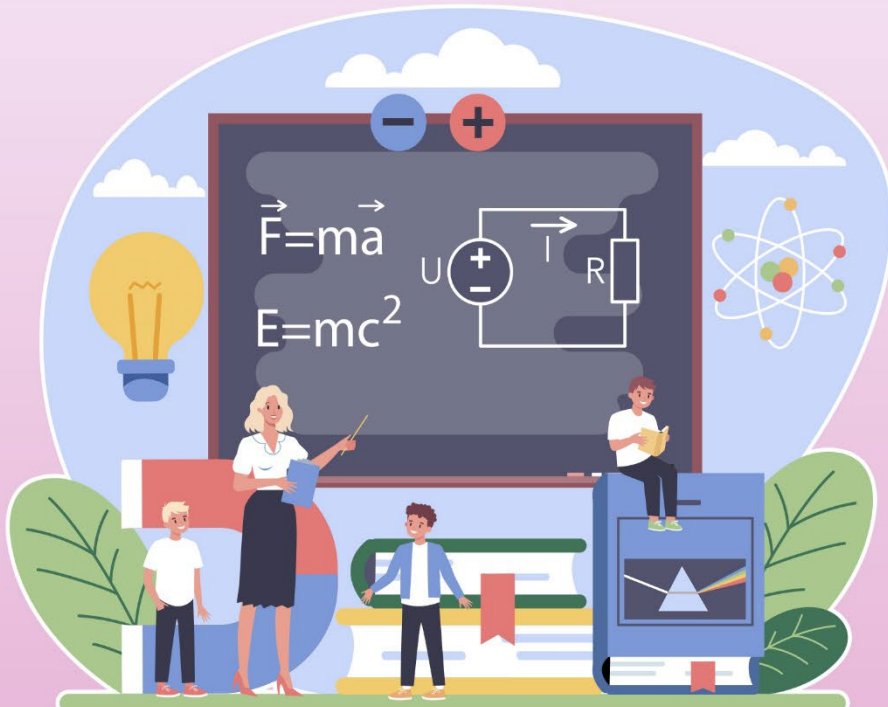




भौतिकी Physics

कक्षा / Class XI
2026-27

विद्यार्थी सहायक सामग्री
Student Support Material



केन्द्रीय विद्यालय संगठन ~ Kendriya Vidyalaya Sangathan

संदेश

शिक्षा केवल पुस्तकों तक सीमित ज्ञान नहीं है; यह स्वयं को पहचानने, अपने विचारों को विस्तार देने और जीवन की चुनौतियों का आत्मविश्वास से सामना करने की सतत् यात्रा है। केन्द्रीय विद्यालय संगठन का सदैव यह प्रयास रहा है कि प्रत्येक विद्यार्थी के लिए ऐसा शिक्षण वातावरण हो, जहाँ जिज्ञासा को प्रोत्साहन मिले, सीखना आनंददायी बने और प्रत्येक बालक अपनी अंतर्निहित क्षमताओं को पहचानकर उन्हें विकसित कर सके।

राष्ट्रीय शिक्षा नीति 2020 ने शिक्षा को सिर्फ रटने से समझने, सूचना से ज्ञान और ज्ञान से जीवनोपयोगी दक्षताओं की ओर अग्रसर किया है। आज का समय कृत्रिम बुद्धिमत्ता (Artificial Intelligence – AI), डिजिटल नवाचार और निरंतर बदलती तकनीकों का है। ऐसे युग में केवल तथ्यों का ज्ञान पर्याप्त नहीं है; आवश्यक है कि विद्यार्थी मनन- चिंतन करना सीखें, प्रश्न पूछें, समस्याओं का समाधान खोजें और नए विचारों के साथ आगे बढ़ें। यही क्षमताएँ उन्हें भविष्य के लिए तैयार करेंगी।

इन्हीं उद्देश्यों को ध्यान में रखते हुए केन्द्रीय विद्यालय संगठन के पाँचों आंचलिक शिक्षा एवं प्रशिक्षण संस्थानों (ZIETs) के माध्यम से, समर्पित एवं अनुभवी शिक्षकों द्वारा कक्षा 9 से 12 के विद्यार्थियों के लिए यह विद्यार्थी सहायक सामग्री तैयार की गई है। यह केवल परीक्षा की तैयारी की संकलन सामग्री नहीं है, बल्कि विषयों को सरलता से समझने, अवधारणाओं को स्पष्ट करने, नियमित अभ्यास करने और आत्मविश्वास के साथ आगे बढ़ने का एक विश्वसनीय साथी है।

इस सामग्री में विषय-वस्तु को विद्यार्थियों की आवश्यकताओं के अनुरूप सरल, व्यवस्थित एवं दक्षता-आधारित दृष्टिकोण से प्रस्तुत किया गया है। मुझे प्रसन्नता है कि इसे हमारे शिक्षकों के अनुभव, नवाचार और विद्यार्थियों के प्रति उनकी प्रतिबद्धता ने समृद्ध बनाया है। मुझे विश्वास है कि यह सामग्री विद्यार्थियों की सीखने की यात्रा को अधिक सहज, रोचक और प्रभावी बनाएगी तथा उन्हें बोर्ड परीक्षाओं के साथ-साथ जीवन की व्यापक चुनौतियों के लिए भी तैयार करेगी।

मेरा विश्वास है कि जब एक जिज्ञासु विद्यार्थी, प्रेरित शिक्षक और सार्थक शिक्षण संसाधन एक साथ आते हैं, तब शिक्षा केवल उपलब्धि का माध्यम नहीं रहती, बल्कि व्यक्तित्व निर्माण की एक सशक्त प्रक्रिया बन जाती है। यही भावना इस विद्यार्थी सहायक सामग्री के केंद्र में है।

मेरी हार्दिक शुभकामनाएँ हैं कि आप सभी सीखने की इस यात्रा में निरंतर आगे बढ़ें, अपने सपनों को नए आयाम दें और ज्ञान, संवेदनशीलता तथा नवाचार के साथ राष्ट्र निर्माण में अपना महत्वपूर्ण योगदान दें।

शुभकामनाओं सहित।


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About the Student Support Material

The Student Support Material (SSM) for Class XI Physics has been developed for the academic session 2026–27 under the guidance of Kendriya Vidyalaya Sangathan (KVS), Headquarters, New Delhi. The responsibility for preparing this material was entrusted to the KVS Zonal Institute of Education and Training (ZIET), Mysuru.

To ensure quality, accuracy, and student-friendly presentation, ZIET Mysuru collaborated with experienced Physics teachers from various Kendriya Vidyalayas across different regions. Their collective expertise has helped in developing a comprehensive, well-structured, and examination-oriented resource for learners.

The SSM has been prepared in accordance with the latest CBSE curriculum and assessment framework, the National Education Policy (NEP) 2020, Competency-Based Education (CBE) principles, and NCERT learning outcomes. It aims to strengthen conceptual understanding, scientific reasoning, problem-solving skills, and the application of Physics in real-life situations.

The material has been organized systematically to make learning engaging and effective. Each chapter generally includes:

- Infographics
- Important Definitions, Laws, and Formulae
- Solved Numerical Problems
- Multiple Choice Questions (MCQs)
- Assertion-Reason and Case-Based Questions
- Competency-Based Questions
- Short and Long Answer Questions
- Practice Exercises and Revision Material

Special Features

- ✓ Simple and student-friendly language
- ✓ Emphasis on conceptual clarity and scientific thinking
- ✓ Step-by-step solutions to numerical problems
- ✓ Real-life applications of Physics concepts
- ✓ Competency-based and examination-oriented questions
- ✓ Logical and systematic presentation of content

This Student Support Material serves as both a study guide and a learning companion. It encourages inquiry, critical thinking, and independent learning while helping students prepare effectively for classroom assessments and examinations. Regular use of this material will help learners develop confidence, strengthen their conceptual foundations, and achieve academic success in Physics.

We hope that this collaborative effort by KVS ZIET Mysuru and participating Physics teachers will significantly benefit students and contribute to their academic growth and scientific temper.

How to Use this Student Support Material (SSM)

Welcome to the Student Support Material for Class XI Physics. This resource has been carefully designed to help you understand fundamental concepts, strengthen problem-solving skills, and prepare effectively for school examinations and future competitive pursuits.

Best Practices for Effective Learning

To make the most of this material, follow the steps given below:

- **Build Conceptual Understanding:** Begin by reading the concept notes thoroughly and focus on understanding the physical principles rather than memorizing facts and formulae.
- **Learn Key Definitions, Laws, and Formulae:** Familiarize yourself with important terms, laws, principles, and mathematical relationships presented in each chapter.
- **Practice Numerical Problems:** Solve the worked-out examples and numerical questions regularly to develop accuracy, logical thinking, and confidence in problem-solving.
- **Strengthen Analytical Skills:** Attempt competency-based, application-oriented, and case-study questions to connect Physics concepts with real-life situations.
- **Revise Systematically:** Use concept maps, infographics, and chapter summaries for quick revision and reinforcement of learning.
- **Evaluate Your Understanding:** Solve MCQs, short-answer, and long-answer questions to assess your preparation and identify areas requiring further attention.
- **Discuss and Explore:** Engage in discussions with teachers and classmates to clarify doubts and deepen your understanding of concepts.

Versatile Usage

This SSM can be effectively used for:

- Daily Classroom Learning and Homework
- Unit Tests and Periodic Assessments
- Half-Yearly and Annual Examinations
- Revision and Self-Assessment
- Strengthening Foundations for Higher Studies

Regular and sincere use of this material will help you develop conceptual clarity, scientific reasoning, problem-solving abilities, and confidence in Physics. Most importantly, it will encourage you to view Physics not merely as a subject but as a way of understanding the world around you.

Happy Learning and Best Wishes for Your Success!

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CBSE Class XI Physics

Chapter 1: Units & Dimensions

Measure Correctly • Think Scientifically • Learn Visually



1 NEED FOR MEASUREMENT

Measurement is important in Physics for comparing an unknown quantity with a standard unit. It allows us to express physical properties with precision.

Real-life examples



School lab visuals and temperature



Temperature, time and mass

Sporting examples



Length



Mass



QUICK RECALL

Physics becomes meaningful only when quantities are measured accurately.

2 SYSTEMS OF UNITS

	Length	Mass	Time
CGS	centimetre (cm)	gram (g)	second (s)
FPS	foot (ft)	pound (lb)	second (s)
MKS	metre (m)	kilogram (kg)	second (s)
SI	metre (m)	kilogram (kg)	second (s)

✓ SI SYSTEM = Internationally Accepted System



3 SI FUNDAMENTAL QUANTITIES & UNITS

Length
metre (m)

Memory Trick:
'Length is standard'

Mass
kilogram (kg)

Memory Trick:
'Mass is quantity'

Time
second (s)

Memory Trick:
'Time is duration'

Electric
Current
ampere
(A)

Temperature
kelvin (K)

Memory Trick:
'Kelvin is temperature'

**Amount of
Substance**
mole (mol)

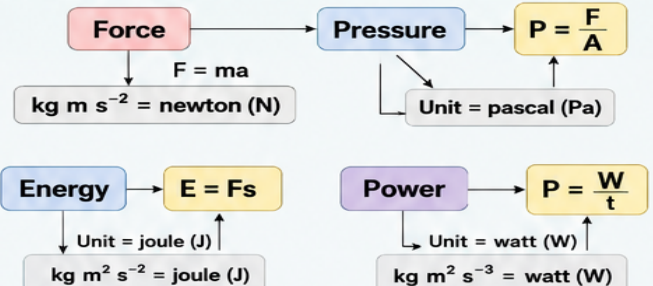
Memory Trick:
'Mole is quantity'

**Luminous
Intensity**
candela (cd)

Memory Trick:
'Candela is brightness'

Note: Electric current (A) is not a derived unit.

4 DERIVED UNITS



5 SIGNIFICANT FIGURES

RULES BOX

- Non-zero digits are significant.
- Zeros between non-zero digits are significant.
- Leading zeros are NOT significant.
- Trailing zeros after decimal are significant.

Solved examples: 0.0032 → 2 significant figures
2.500 → 4 significant figures
100 → 1 significant figure



6 ERROR & UNCERTAINTY IN MEASUREMENT

Types of Errors

- Absolute Error** (Δa) = $|a_m - a_t|$
where a_m = measured value,
 a_t = true value

- Mean Absolute Error**

$$\Delta a_{\text{mean}} = \frac{\sum |\Delta a_i|}{n}$$

- Relative Error** = $\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$

- Percentage Error** = $\left(\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \right) \times 100$



Mean absolute error:
closeness of repeated
measurements.

Percentage error:
accuracy of measurement.

MOST IMPORTANT: Experimental accuracy depends on least error.

7 DIMENSIONS OF PHYSICAL QUANTITIES

Dimensional formula as $[M^a L^b T^c]$ (Mass, Length, Time)

Velocity

$[LT^{-1}]$

Acceleration

$[LT^{-2}]$

Force

$[MLT^{-2}]$

Work (Energy)

$[ML^2T^{-2}]$

Pressure

$[ML^{-1}T^{-2}]$

Memory trick: **Exponent highlighting**
(Match exponents with the basic dimensions)

9 QUICK FORMULA REVISION ZONE

Length = metre (m)

Mass = kilogram (kg)

Time = second (s)

Electric Current = ampere (A)

Temperature = kelvin (K)

Pressure = pascal (Pa)

Force = newton (N)

Luminous Intensity = candela (cd)

Quick Recall

8 DIMENSIONAL ANALYSIS & APPLICATIONS

- ✓ Checking correctness of equations

Checking equation:
Check dimensional validity of equations.

Example: $v = u + at$
 $[LT^{-1}] = [LT^{-1}] + [LT^{-2}][T] = [LT^{-1}]$ ✓

- ✓ Converting units

Converting units:
Use dimensional relations.

Example: $1 \text{ km/h} = \frac{1000 \text{ m}}{3600 \text{ s}}$
 $= 0.2778 \text{ m s}^{-1}$

- ✓ Deriving relations

Deriving relation:
Use dimensions to get valid relations.

Example: $T = 2\pi \sqrt{\frac{l}{g}}$
(Simple pendulum)

Basis: DIMENSIONAL HOMOGENEITY PRINCIPLE
(Only dimensionally consistent equations are physically meaningful.)

10 PRACTICAL APPLICATIONS & PROBLEM SOLVING

$$\Delta a_i = |a_i - a_{\text{mean}}|$$

$$\Delta a_{\text{mean}} = \frac{\sum |\Delta a_i|}{n}$$

$$\Delta a_i = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$$

$$\text{Relative Error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$$

$$\text{Percentage Error} = \left(\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \right) \times 100$$



Accurate Measurements Lead to Accurate Physics!



EXAM SUCCESS

MULTIPLE CHOICE QUESTIONS

1. One of the combinations from the fundamental physical constants is hc/G . The unit of this expression is

- (a) kg^2 (b) m^3 (c) s^{-1} (d) m

2. Which of the followings is the proper representation of a 10 cm long scale?

- (a) The scale of the length is 10 cm. (b) This is a 10 cm. long scale.
(c) This a 10 c.m. long scale. (d) The scale of the length is 10 c.m.

3. The unit of work is derived from which fundamental quantities?

- (a). Mass & length (b). Force & distance (c). Mass, length, and time (d). Power & time

4. Number of significant figures in 0.0024500 is

- (a) 3 (b) 5 (c) 8 (d) 7

5. Which number has 3 significant figures?

- (a) 0.070 (b) 700 (c) 7.00×10^2 (d) 0.007

6. The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is

- (a) 663.821 (b) 664 (c) 663.8 (d) 663.82

7. Which of the following quantities is dimensionless?

- (a) Force (b) Strain (c) Velocity (d) Pressure

8. The dimensional formula of Planck's constant (h) is:

- (a) $[\text{ML}^2\text{T}^{-1}]$ (b) $[\text{M L}^2 \text{T}^{-1}]$ (c) $[\text{ML}^2\text{T}^{-1}]$ (d) $[\text{MLT}^{-2}]$

9. Which of the following equations is dimensionally correct?

- (a) $v=u+at^2v$ (b) $v= u + at^2$ (c) $v=ut+at^2$ (d) $s=ut+1/2at^2$

10. If velocity v, time t, and acceleration a are fundamental quantities, the dimension of distance is:

- (a) vt (b) at (c) at^2 (d) va

11. If $\pi = 3.14$, then the value of π^2 is

- (a) 9.8596 (b) 9.860 (c) 9.86 (d) 9.9

12. The velocity of a particle v at an instant t is given by $v = at + bt^2$. The dimensions of b is

- (a) [L] (b) $[\text{LT}^{-1}]$ (c) $[\text{LT}^{-2}]$ (d) $[\text{LT}^{-3}]$

13. The dimensional formula for gravitational constant G is

- (a) $[\text{ML}^3\text{T}^{-2}]$ (b) $[\text{M}^{-1}\text{L}^3\text{T}^{-2}]$ (c) $[\text{M}^{-1}\text{L}^{-3}\text{T}^{-2}]$ (d) $[\text{ML}^{-3}\text{T}^2]$

14. If the force is proportional to square of velocity, then the dimension of proportionality constant is

- (a) $[\text{MLT}^0]$ (b) $[\text{MLT}^{-1}]$ (c) $[\text{ML}^{-2}\text{T}]$ (d) $[\text{ML}^{-1}\text{T}^0]$

15. 3.6453 and 3.6554 are rounded off to two decimal places. The results will be

- (a) 3.64, 3.65 (b) 3.65, 3.66 (c) 3.65, 3.65 (d) 3.6, 3.6

ANSWER (MCQ)

Q1 - Answer: a. kg^2 Explanation: Unit of $\frac{hC}{G} = \frac{(Js)(ms^{-1})}{Nm^2kg^{-2}} = \frac{(Nms)(ms^{-1})}{Nm^2kg^{-2}} = kg^2$

Q2 - Answer: Option(A) is correct.

Explanation: While units are symbolically represented, there should not be any dots. But if it appears at the ends of a sentence, then a dot may appear as a full-stop.

Q3 - Answer: c. Mass, length, and time

Explanation: Work = Force $\times$ Distance = (mass $\times$ acceleration) $\times$ distance $\text{kg}\cdot\text{m}^2/\text{s}^2$

Q4 - Answer: b. 5

Explanation: All the zeros on the right immediate after the decimal are not significant. All the zeros those are on right of the last non zero digit, after the decimal point, are significant

Q5 - Answer: c 7.00×10^2

Explanation: All digits (7, 0, 0) in 7.00 are significant because trailing zeros after a decimal are significant.

Q6 - Answer: c 663.8

Explanation: The final result after sum should have the same number of decimal places as the number with the least decimal places i.e 227.2 having one decimal place

Q7 - Answer: b

Explanation: Strain = change in length / original length = (L / L) = dimensionless

Q8 -Answer: a

Explanation: Energy = $h\nu$ $E = [M L^2 T^{-2}]$, $h = [M L^2 T^{-1}]$, $\nu = T^{-1}$ RHS = LHS

Q9 - Answer: b **Explanation:** LHS = RHS

Q10 - Answer: c **Explanation:** From $s = ut + \frac{1}{2}at^2$

Q11. Answer : c **Explanation:** $\pi^2 = 3.14 \times 3.14 = 9.8596$

As the input data (3.14) has 3 significant figure, the answer should be restricted to 3 significant figure by rounding off as ; $\pi^2 = 9.86$

12. Answer :d **Explanation :** By the principle of homogeneity, Dimension of v = Dimension of 'at' = Dimension of 'bt²' (i.e.) Dimension of 'bt²' = $[L T^{-1}]$

So the dimension of 'b' = $[L T^{-1}] t^2 = [L T^{-1}] [T^2] = [L T^{-3}]$

13. Answer :b $[M^{-1}L^3T^{-2}]$

14. Answer :d **Explanation** $F \propto v^2$ (or) $F = k v^2$ (where $k \rightarrow \text{constant}$) • Thus dimension of constant $k = F/v^2 = [M L T^{-2}] / [L T^{-1}]^2 = [M L T^{-2}] / [L^2 T^{-2}] = [M L^{-1}]$

15. Answer :b **Explanation:** If the digit from where the number is truncated is 5. The digit after it is 3 in one number and 4 in other number i.e., both are non-zero digits. So, as per rule the digit before 5 is to be increased by 1

CHALLENGE CORNER

Q1. 355.0 kg = 355000 g. Significant figures in the two numbers are

- (a) 4, 6 (b) 3, 6 (c) 3, 3 (d) 4, 4

Q2. Length, breadth and thickness of a strip having a uniform cross section are measured to be 10.5 cm, 0.05 mm, and 6.0 μm , respectively. Which of the following option(s) give(s) the volume of the strip in cm^3 with correct significant figures:

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- (a) 3.2×10^{-5} (b) 32.0×10^{-6} (c) 3.0×10^{-5} (d) 3×10^{-5}

Q3. If momentum (p), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula.

- (a) $(p^1 A^{-1} T^1)$ (b) $(p^2 A^1 T^1)$ (c) $(p^1 A^{-1/2} T^1)$ (d) $(p^1 A^{1/2} T^{-1})$

Q4. The numbers 2.745 and 2.735 on rounding off to 3 significant figures will give

- (a) 2.75 and 2.74 (b) 2.74 and 2.73 (c) 2.75 and 2.73 (d) 2.74 and 2.74

Q5. The mass and volume of a body are 4.237 g and 2.5 cm^3 , respectively. The density of the material of the body in correct significant figures is

- (a) 1.6048 g cm^{-3} (b) 1.69 g cm^{-3} (c) 1.7 g cm^{-3} (d) 1.695 g cm^{-3}

ANSWERS

1. d 2. D 3. d 4. d 5. C

ASSERTION REASON BASED QUESTION

In the following questions a statement of assertion(A) is followed by a statement of reason(R) mark the correct choice as

- (a) If both assertion and reason are true and reason is the correct explanation of the assertion
(b) If both assertion and reason are true but reason is the not correct explanation of assertion
(c) If assertion is true but reason is false
(d) If assertion is false but reason is true

1. **Assertion (A):** The principle of dimensional homogeneity is useful in checking the correctness of physical equations.

Reason (R): All terms in a correct physical equation must have the same dimensions.

2. **Assertion (A):** The dimensions of pressure and stress are the same.

Reason (R): Both are defined as force per unit area.

3. **Assertion (A):** The dimensional formula of gravitational constant G is $[M^{-1}L^3T^{-2}]$.

Reason (R): Gravitational force depends directly on the product of masses and inversely on the square of distance between them.

4. **Assertion (A):** Dimensional analysis can be used to derive numerical constants in equations.
Reason (R): Dimensional analysis gives relations between physical quantities only up to a dimensionless constant.
5. **Assertion (A):** If $v = u + at^2$, the equation is dimensionally incorrect.
Reason (R): Velocity and acceleration have different dimensions.

ANSWERS

1. a 2. a 3. a 4.d 5. b

SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

- Express 1mm and 1fm in meter
- Define A.U. and light year..
- Mention two physical quantities having same dimensional formulae but different units.
- Distinguish between fundamental units and derived units?
- How will you measure the diameter of the Moon using Parallax method?.
- Deduce the dimensional formula for the following physical quantities
 a) coefficient of viscosity b) surface tension
- State the principle of dimensional homogeneity.
- Check if the equation $v=u + at$ is dimensionally correct.
- Give the applications of the method of dimensional analysis
- If the unit of force is taken as 1 kilonewton (kN), the unit of length as 1 kilometre (km), and the unit of time as 100 seconds, determine the corresponding unit of mass in this system of units.

ANSWER(2 MARKS)

Q1. - **Answer:** 1mm = 10^{-6} m 1fm = 10^{-15} m

Q2 - **Answer:** A.U. is average distance of the Sun from the Earth. 1 A.U = 1.496×10^{11} m

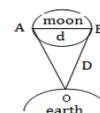
Light year is the distance that light travels with velocity of 3×10^8 m s⁻¹ in 1 year. 1 LY= 9.467×10^{15} m

Q3 -**Answer:** Following quantities have same dimensional formula: $[M^0L^0T^{-1}]$. But they have different units: (i) Angular velocity (rad s⁻¹) (ii) Velocity gradient (s⁻¹) (iii) Frequency (Hz)

Q4 - **Answer:** The units in which the fundamental quantities are measured are called fundamental or base units Examples- m, s, kg, A, mol

The units in which all the physical quantities which can be obtained by a suitable multiplication or division of power on fundamental units are called derived units Examples- m/s, kg/m³

Q5 - **Answer:** Diameter of Moon ; AB = d Parallax angle ; $\angle AOB = \theta$ Distance of Moon from Earth



OA = OB = D • Hence, $\theta = AB/OA = d/D$ $d = D \theta$

Q6 - **Answer:** a) $ML^{-1}T^{-1}$, b) ML^0T^{-2}

Q7 -**Answer:**The principle of dimensional homogeneity states that all terms in a physically meaningful equation must have the same dimensions.

Q8 -**Answer:** LHS: $[v] = [LT^{-1}]$ RHS: $[u] = [LT^{-1}]$

$[a \cdot t] = [LT^{-2}] [T] \Rightarrow$ Both sides = $[LT^{-1}] = [L T^{-1}] + [LT^{-2}] [T]$

Yes, the equation is dimensionally correct.

Q9 -**Answer:** To convert a physical quantity from one system of units to another.

To check the dimensional correctness of a given physical equation.

To establish the relation among various physical quantities.

Q10.As W.K.T $F = [MLT^{-2}] \Rightarrow M = \frac{F \times T^2}{L}$

Unit of force = 1 kN = 10^3 N Unit of length = 1 km = 10^3 m

Unit of time = 100 s = 10^2 s $M = \frac{10^3 \times (10^2)^2}{10^3} = 10^4 \text{Kg}$

CHALLENGE CORNER

1. Check whether the given equation $F.s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ is dimensionally correct, or not where, m is mass of the body, v is its final velocity, u is its initial velocity, F is applied force and s is distance covered.

2. How many significant figures are present in 0.02030?
3. Round off 4.376 to 3 significant figures.
4. Deduce the dimensional formula for R, using ideal gas equation $PV = nRT$?
5. Mention some physical quantities having same dimensional formulae but different units?

HINTS :-

Q1 - Ans: Yes correct L.H.S=R.H.S

Q2 - Ans: There are **4 significant figures**: 2, 0, 3, & the final 0 (after decimal & non-zero digits)

Q3 - Ans: Rounded value = **4.38**

Q4 - Ans: $[ML^2T^{-2}K^{-1} \text{ mol}^{-1}]$

Q5 - Ans: No.

COMMONLY MADE ERROR :

- Students should carefully use the dimension of all the quantities and then deduce the result
- Students may commit mistake while calculating the dimension of the given quantities.

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1. The Van der Waal equation for 1 mole of a real gas is $(P + \frac{a}{V^2})(V - b)$ where P is the pressure, V is the volume, T is the absolute temperature, R is the molar gas constant and a, b are Van der Waal constants. Determine the dimensional formulae of the constants a and b
2. From a point on the ground, the top of a tree is seen to have an angle of elevation 60° . The distance between the tree and a point is 50 m. Calculate the height of the tree?
3. What are significant figures? State any two rules for determining significant figures. Give one example.
4. Round off the following numbers to 3 significant figures:
(i) 0.004376 (ii) 56.987 (iii) 12345
5. Fill in the blanks by suitable conversion of units :
(a) $1 \text{ kg m}^2\text{s}^{-2} = \dots \text{g cm}^2\text{s}^{-2}$ (b) $1 \text{ m} = \dots \text{ly}$ (light year)
(c) $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} = \dots \text{cm}^3 \text{ s}^{-2} \text{ g}^{-1}$.
6. Answer the following :
(a) You are given a thread and a metre scale. How will you estimate the diameter of the thread ?
(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale ?
(c) The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only ?
7. If the value of universal gravitational constant in SI is $6.6 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$, then find its value in CGS System?.

ANSWER WITH SOLUTIONS(3 MARKS)

Q1. Van der Waals equation: $(P + \frac{a}{V^2})(V - b) = RT$ Dimensions of volume $V = [V] = [L^3]$ Dimensions of pressure $P = [P] = [ML^{-1}T^{-2}]$ Dimensions of a: From $\frac{a}{V^2} = P$ $a = [L^3]^2 [ML^{-1}T^{-2}] \Rightarrow [a] = [ML^5T^{-2}]$ Dimensions of b: From $V - b$ the terms must have the same dimension $[b] = [V] = [L^3]$

Q2. $\theta = 60^\circ$, $x = 50 \text{ m}$, $h =$

$$h = x \tan \theta = 50 \tan 60^\circ = 50 \times \sqrt{3} = 50 \times 1.732 \quad h = 86.6 \text{ m}$$

Q3 - Ans Significant figures are the digits in a measurement that are known accurately.

Two Rules for Determining Significant Figures:

All non-zero digits are always significant. Example: In 456, $\Rightarrow$ 3 significant figures.

Zeros between two non-zero digits are significant.

Example: In 4003, the zeros between 4 and 3 are significant $\Rightarrow$ 4 significant figures.

Example Combining the Rules: Number: 204.05 Digits: 2, 0, 4, 0, 5

All non-zero digits (2, 4, 5) are significant. Zeros between non-zero digits (the 0s) are also significant.

$\Rightarrow$ Number of significant figures = 5

Q4 - Ans (i) 0.004376 $\rightarrow$ 0.00438 (ii) 56.987 $\rightarrow$ 57.0 (iii) 12345 $\rightarrow$ 12300

Q5 – Ans (a) 10^7 (b) 1.053×10^{-16} (c) 6.67×10^{-8}

Q6 - Ans(a) By winding the thread in close turns on a pencil and using the formula i.e., Diameter = Average length No. of turns

(b) Yes, by increasing the number of divisions on circular scale as Least count = Pitch No. of divisions on circular scale

(c) Random error is reduced on taking large number of observations.

Q7 - In S I system, $G = 6.6 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$; In CGS system, CGS = ? Dimensional formula of gravitational constant ; $[M^a L^b T^c] = [M^{-1} L^3 T^{-2}]$ Hence, $a = -1, b = 3, c = -2$

$$\begin{aligned} G_{SI} [M_1^a L_1^b T_1^c] &= G_{CGS} [M_2^a L_2^b T_2^c] \\ G_{CGS} &= G_{SI} \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c \\ G_{CGS} &= 6.6 \times 10^{-11} \times \left[\frac{1 \text{ kg}}{1 \text{ g}} \right]^{-1} \left[\frac{1 \text{ m}}{1 \text{ cm}} \right]^3 \left[\frac{1 \text{ s}}{1 \text{ s}} \right]^{-2} \\ &= 6.6 \times 10^{-11} \times \left[\frac{1000 \text{ g}}{1 \text{ g}} \right]^{-1} \left[\frac{100 \text{ cm}}{1 \text{ cm}} \right]^3 [1]^{-2} \\ &= 6.6 \times 10^{-11} \times [10^3]^{-1} [10^2]^3 \\ &= 6.6 \times 10^{-11} \times 10^{-3} \times 10^6 \\ G_{CGS} &= 6.6 \times 10^{-8} \text{ dyne cm}^2 \text{ g}^{-2} \end{aligned}$$

CHALLENGE CORNER

1. A calorie is a unit of heat or energy and it equals about 4.2 J, where $1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$. Suppose we employ a system of units in which the unit of mass equals $\alpha \text{ kg}$, the unit of length $\beta \text{ m}$, the unit of time is $\gamma \text{ s}$. Show that a calorie has a magnitude $4.2 \alpha^{-1} \beta^{-2} \gamma^2$ in terms of new units.

2. State the number of significant figure in :

(a) 0.007 m² (b) 2.64×10^{24} (c) 0.2370 g cm⁻³ (d) 6.320 J (e) 6.032 Nm⁻² (f) 0.0006032 m²

3. Convert 76 cm of mercury pressure into Nm⁻² using the method of dimensions

4. Check the correctness of the equation $\frac{1}{2}mv^2 = mgh$ using dimensional analysis method.

5. The radius of circle is 3.12 m. Calculate the area of the circle regard to significant figures.

LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

Q1. Obtain an expression for the time period T of a simple pendulum. The time period T depends on (i) mass 'm' of the bob (ii) length 'l' of the pendulum and (iii) acceleration due to gravity g at the place where the pendulum is suspended. (Constant $k = 2\pi$)

Q2. A new system of units is proposed in which unit of mass is $\alpha \text{ kg}$, unit of length $\beta \text{ m}$ and unit of time $\gamma \text{ s}$. How much will 5 J measure in this new system?

Q3.(a) Explain any three important applications of dimensional analysis with suitable examples

(b) Using dimensional analysis, derive an expression for the speed v of a wave on a string. Assume that it depends on: Tension T in the string, and Mass per unit length μ .

ANSWERS(5 MARKS)

1. Hint: Assume, $T \propto m^a l^b g^c$ Use dimensions: $[T] = [M]^a [L]^b [LT^{-2}]^c$

Compare powers of M, L and T.

Solution: $a = 0, b = 1/2, c = -1/2$ Hence, $T = k\sqrt{\frac{l}{g}}$ Given $k = 2\pi$, $T = 2\pi\sqrt{\frac{l}{g}}$

2. Hint: Energy has dimensions: $[ML^2T^{-2}]$

New units: $M = \alpha \text{ kg}, L = \beta \text{ m}, T = \gamma \text{ s}$ Solution: New unit of energy = $\alpha\beta^2\gamma^{-2}$

Therefore, $5J = n\left(\frac{\alpha\beta^2}{\gamma^{-2}}\right)$ $n = 5\frac{\gamma^2}{\alpha\beta^2}$

3.(a) 1. Checking correctness of equations Example: $s = ut + 1/2 at^2$

2. Deriving formulae Example: $T = 2\pi\sqrt{\frac{l}{g}}$

3. Conversion of units Used for converting one system of units into another.

(b) Assume, $v \propto T^a \mu^b$ where $[T] = MLT^{-2}$, $[\mu] = ML^{-1}$

Compare dimensions. Solution: $a = 1/2$, $b = -1/2$ Hence, $v = \sqrt{\frac{T}{\mu}}$

SCIENTIST CORNER: CHANDRASEKHAR LIMIT

The Chandrasekhar Limit is the maximum mass of a stable white dwarf star, equal to about 1.4 times the mass of the Sun. If a star exceeds this limit, it collapses further to form a neutron star or a black hole. This concept was proposed by Indian astrophysicist Subrahmanyan Chandrasekhar, who received the Nobel Prize in Physics (1983) for his contribution to stellar evolution

CASE STUDY BASED QUESTIONS

CASE STUDY 1: Principle of Homogeneity and Dimensional Consistency

Read the following passage and answer the questions that follow: The principle of homogeneity of dimensions is extremely useful in checking the correctness of an equation in physics. According to this principle, if the dimensions of all the terms in an equation are not the same, the equation is wrong. If an equation fails this consistency test, it is proven wrong; however, if it passes, it is not necessarily proven right. Thus, a dimensionally correct equation need not be an correct equation, but a dimensionally inconsistent equation must be wrong.

Q1 The SI unit of energy is $J = kg\ m^2\ s^{-2}$, speed v is $m\ s^{-1}$, & acceleration a is $m\ s^{-2}$. Which of the following formulas for K.E (K) is WRONG on the basis of dimensional arguments?

- (a) $K = (1/2)mv^2$ (b) $K = m^2v^3$ (c) $K = (1/3)mv^2$ (d) $K = (1/5)mv^2$

Q2.. In the dimensional equation $T = k\ l^x\ g^y\ m^z$ (representing the time period of a simple pendulum), what is the correct value of Z?

- (a) $Z = 0$ (b) $Z = 1$ (c) $Z = 2$ (d) $Z = 1/2$

Q3. A famous physical relation relates the 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light c . This relation first arose as a consequence of:

- (a) The Principle of Homogeneity (b) Special relativity due to Albert Einstein
(c) Dimensional Analysis (d) Quantum Mechanics

Q4. A calorie is a unit of heat that equals about 4.2 J, where $1\ J = 1\ kg\ m^2\ s^{-2}$. If we employ a new system of units where the unit of mass equals $\alpha\ kg$, the unit of length equals $\beta\ m$, and the unit of time is $\gamma\ s$, what is the magnitude of a Calorie in this new system?

- (a) $\alpha^{-1}\ \beta^{-2}\ \gamma^2$ (b) $\alpha^{-1}\ \beta^2\ \gamma^3$ (c) $\alpha^{-2}\ \beta^{-2}\ \gamma^2$ (d) $\alpha^2\ \beta^2\ \gamma^{-2}$

CASE STUDY 2: PHYSICAL QUANTITIES AND PENDULUM MECHANICS

Read the following passage and answer the questions that follow: A researcher is studying the motion of a particle in a gravitational field. To understand the dependencies of different physical quantities, the researcher derives the mathematical expression for the time period of a simple pendulum. The derived formula is $T = 2\pi\sqrt{l/g}$, where T represents the time period in seconds, l represents the length of the pendulum in meters, and g represents the gravitational acceleration in m/s^2 .

Q1 In the derived formula $T = 2\pi\sqrt{\frac{l}{g}}$, what physical quantity does g represent?

- (a) Mass of the pendulum (b) Acceleration due to gravity
(c) Velocity of the pendulum (d) Time period

Q2 If the simple pendulum swings on the Moon, where gravity is weaker than on Earth, what happens to its time period T ?

- (a) It decreases (b) It increases (c) It remains unchanged (d) It becomes zero

Q3 If the length of the pendulum is quadrupled ($4l$), what happens to the time period?

- (a) It doubles (b) It becomes four times (c) It becomes half (d) It remains the same

Q4 Based on the given mathematical expression, which of the following physical quantities will NOT affect the time period of a simple pendulum?

- (a) Length of the pendulum (b) Acceleration due to gravity
(c) Mass of the pendulum bob (d) Location of the pendulum (Earth vs Moon)

CASE STUDY 3. TESTING A NEW SPORTS DRINK BOTTLE

A sports company is designing a new bottle for athletes. During testing, the physics team studies different physical quantities related to the bottle such as volume, force applied while squeezing, pressure inside the bottle, and speed of water coming out through the nozzle.

A student intern records the following observations:

- Radius of bottle opening = 2 cm Water speed from nozzle = 5 m s^{-1}
- Force applied while squeezing = 20 N Area of nozzle opening = $4 \times 10^{-4} \text{ m}^2$

The intern also proposes some equations during the experiment. The physics team checks whether the equations are dimensionally correct & whether proper SI units are used.

Q1. The pressure inside the bottle is calculated using: $P = \frac{F}{A}$

What are the dimensions of pressure?

- (a) $[M^1 L^{-1} T^{-2}]$ (b) $[M^1 L T^{-2}]$ (c) $[M^0 L^{-1} T^{-2}]$ (d) $[M^1 L^{-2} T^{-1}]$

2. A student says that the kinetic energy of flowing water can be written as: $E = \frac{1}{2} m v^2$

Which of the following represents the dimensions of energy?

- (a) $[M L^2 T^{-1}]$ (b) $[M L^2 T^{-2}]$ (c) $[M^2 L T^{-2}]$ (d) $[M L T^{-1}]$

3. During unit conversion, the intern writes: 1 litre = 1 m^3 . The statement is:

- (a) Correct because both measure volume (b) Incorrect because 1 litre = 10^{-3} m^3
(c) Incorrect because litre is not a unit of volume (d) Correct only in CGS system

4. The intern proposes the equation for distance travelled by water: $s = ut + \frac{1}{2} at^2$

Why is this equation considered dimensionally correct?

- (a) All terms have dimensions of force (b) All terms have dimensions of velocity
(c) All terms have dimensions of length (d) All terms are dimensionless

SOLUTIONS CASE STUDY-1

1. (b) $K = m^2 v^3$ **Hint:** According to homogeneity the dimensions of both sides must match.

2. **Answer:** (a) $Z = 0$

Hint: The time period of a simple pendulum is independent of its mass.

3. **Answer:** (b) Special relativity due to Albert Einstein

Hint: The text explicitly states that this mass-speed relation first arose as a consequence of special relativity developed by Albert Einstein.

4. **Answer:** (a) $\alpha^{-1} \beta^{-2} \gamma^2$ **Hint:** The dimensional formula for energy is $[M L^2 T^{-2}]$.

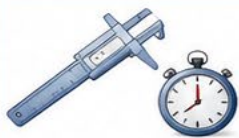
CASE STUDY-2

1. **Answer:** (b) Acceleration due to gravity 2. **Answer:** (b) It increases

3. **Answer:** (c) It increases

4. **Answer:** (d) Mass of the pendulum bob **Hint:** The term for mass (m) is completely absent from the formula $T = 2\pi \sqrt{\frac{l}{g}}$, indicating that the T is independent of the bob's mass

CASE STUDY-3 1.(a) 2.(b) 3.(b) 4.(c)



UNITS AND MEASUREMENT

CBSE CLASS XI PHYSICS

Accurate measurement is the first step towards scientific discovery.



1. PHYSICAL QUANTITIES

Physical quantities are of two types:

- Fundamental (Base) Quantities**
– They are independent of each other.
- Derived Quantities**
– They are expressed in terms of fundamental quantities.

Examples

Fundamental: length, mass, time, current, temperature, amount of substance, luminous intensity

Derived: area, velocity, force, energy, pressure, density, etc.



2. SI BASE (FUNDAMENTAL) QUANTITIES

Base Quantity	Symbol	SI Unit	Symbol	Dimension
Length	l	metre	m	[L]
Mass	m	kilogram	kg	[M]
Time	t	second	s	[T]
Electric Current	I	ampere	A	[I]
Thermodynamic Temperature	T	kelvin	K	[Θ]
Amount of Substance	n	mole	mol	[N]
Luminous Intensity	I_v	candela	cd	[J]

✓ SI (Système International) is the modern form of the metric system and is used worldwide.

3. DERIVED QUANTITIES

Derived quantities are obtained by algebraic combination of fundamental quantities.

Examples:

Physical Quantity	Formula	SI Unit	Dimension
Area	$l \times b$	m^2	$[L^2]$
Volume	l^3	m^3	$[L^3]$
Velocity	$\frac{l}{t}$	$m s^{-1}$	$[LT^{-1}]$
Acceleration	$\frac{l}{t^2}$	$m s^{-2}$	$[LT^{-2}]$
Force	ma	N ($kg m s^{-2}$)	$[MLT^{-2}]$
Density	$\frac{m}{V}$	$kg m^{-3}$	$[ML^{-3}]$
Energy	$F \times l$	J (N m)	$[ML^2T^{-2}]$
Pressure	$\frac{F}{A}$	Pa ($N m^{-2}$)	$[ML^{-1}T^{-2}]$

4. DIMENSIONAL ANALYSIS

- Every physical quantity can be expressed in terms of fundamental quantities.
- Dimensions help to check the correctness of equations.

Rules

- ✓ Quantities added or subtracted must have the same dimensions.
- ✓ Both sides of a physical equation must have the same dimensions.
- ✓ The dimensions of L, M, T, I, Θ, N, J are independent.

Dimensions of some important quantities:

Velocity (v)	$[LT^{-1}]$
Acceleration (a)	$[LT^{-2}]$
Force (F)	$[MLT^{-2}]$
Momentum (p)	$[MLT^{-1}]$
Energy (E)	$[ML^2T^{-2}]$
Power (P)	$[ML^2T^{-3}]$
Pressure (P)	$[ML^{-1}T^{-2}]$

Example

To find dimension of

$$G \text{ in } F = G \frac{m_1 m_2}{r^2}$$

Dimensions:

$$[MLT^{-2}] = [G] [M^2] [L^{-2}]$$

$$\Rightarrow [G] = [ML^3T^{-2}]$$

$$\text{Unit of } G = N m^2 kg^{-2}$$

5. SYSTEMS OF UNITS

(i) CGS System

- Based on centimetre (cm), gram (g) and second (s).
- Units of force: dyne (1 dyne = 1 g cm s⁻²)
- Not an SI unit.

(ii) FPS System

- Based on foot (ft), pound (lb) and second (s).
- Units of force: poundal (1 poundal = 1 lb ft s⁻²)
- Not an SI unit.

(iii) SI SYSTEM

- Based on 7 base units.
- Coherent system (no conversion factors).
- Widely accepted and used.

6. MEASUREMENT

Measurement is the comparison of an unknown quantity with a standard quantity of the same kind.

Characteristics of Good Measurement

- ✓ Accurate
- ✓ Precise
- ✓ Reproducible
- ✓ Least possible error

Types of Errors

- Systematic (or Determinate) Errors**
 - Caused by faulty instrument, wrong method, or personal bias.
 - Can be minimized.
- Random (or Indeterminate) Errors**
 - Cause fluctuations in readings.
 - Cannot be eliminated, only reduced.

7. SOME COMMON MEASURING INSTRUMENTS

Instrument	Quantity Measured	Least Count
Metre Scale	Length	1 mm or 0.1 cm
Vernier Calipers	External/Internal diameter, depth	0.1 mm or 0.01 cm
Micrometer Screw Gauge	Diameter, thickness	0.01 mm
Spherometer	Radius of curvature	0.01 mm
Stop Watch	Time	0.1 s
Ammeter	Electric current	(Varies with range)
Voltmeter	Potential difference	(Varies with range)

8. SIGNIFICANT FIGURES

Significant figures in a measurement include all certain digits plus the last uncertain digit.

Rules:

- 1 All non-zero digits are significant.
- 2 Zeros between non-zero digits are significant.
- 3 Leading zeros (before first non-zero digit) are not significant.
- 4 Trailing zeros to the right of decimal point are significant.
- 5 Trailing zeros in a whole number without decimal point are not significant.

Examples:

0.0078	→ 2 significant figures
102.03	→ 5 significant figures
200	→ 1 significant figure
200.0	→ 4 significant figures

9. ORDER OF MAGNITUDE

Order of magnitude of a quantity is the power of 10 to which the given quantity is nearly equal.

$$\text{If } x = a \times 10^n \quad (1 \leq a < 10)$$

$$\text{Order of magnitude} = 10^n$$

Examples:

- 0.045 = 4.5×10^{-2} → Order = 10^{-2}
- 5600 = 5.6×10^3 → Order = 10^3
- 0.0003 = 3×10^{-4} → Order = 10^{-4}

Useful in expressing very large or very small numbers conveniently.



10. EXPERIMENTAL UNCERTAINTIES (ERRORS)

Absolute Error (Δx)

Difference between the measured value (x_m) and the true value (x_t).

$$\Delta x = |x_m - x_t|$$

Relative Error

Ratio of absolute error to true (or measured) value.

$$\text{Relative error} = \frac{\Delta x}{x_t}$$

Percentage Error

$$\text{Percentage error} = \left(\frac{\Delta x}{x_t} \right) \times 100\%$$

Result of a Measurement

$$x = x_m \pm \Delta x$$

(Read as: x_m plus or minus Δx)

11. QUICK FORMULA BOX

- Velocity (v) = $\frac{d}{t}$
- Acceleration (a) = $\frac{\Delta v}{t}$
- Force (F) = ma
- Pressure (P) = $\frac{F}{A}$
- Density (ρ) = $\frac{m}{V}$
- Energy (E) = Fs
- Power (P) = $\frac{E}{t} = Fv$

PYQ FOCUS

- ☆ Dimensions of physical quantities.
- ☆ Units conversion.
- ☆ Significant figures.
- ☆ Order of magnitude.
- ☆ Errors in measurement.

KEY TAKEAWAYS

- ✓ Measurement is the backbone of all physical sciences.
- ✓ Use correct instrument and method to reduce errors.
- ✓ Always express result with proper unit and uncertainty.
- ✓ Dimensions help to check the validity of equations.

COMMON MISTAKES

- ✗ Ignoring zero error of instruments.
- ✗ Using wrong unit.
- ✗ Taking too few readings.
- ✗ Not reporting uncertainties.
- ✗ Rounding off too early in calculations.

REMEMBER THIS



"The more precise your measurement, the closer you are to the truth."



ACCURATE MEASUREMENT → RELIABLE RESULTS → SCIENTIFIC PROGRESS



CBSE Class XI Physics



Chapter-2: Motion in a Straight Line

"Understand Motion • Visualize Physics • Learn Smart"



1 FRAME OF REFERENCE

Frame of reference is the observer and object relation. Rest and motion are relative.

QUICK RECALL

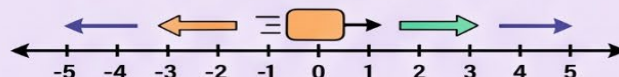
Motion depends on the observer's frame of reference.

Include real-life examples:



2 MOTION IN A STRAIGHT LINE

One-dimensional motion along the X-axis only. Positives with positive and negative direction.



✓ Distance ✓ Displacement ✓ Speed ✓ Velocity

3 DISTANCE vs DISPLACEMENT

Distance	Displacement
Scalar quantity	Vector quantity
Actual path length	Shortest path
Always positive	Can be positive, negative or zero

Memory trick: "Distance = Total path, Displacement = Final shift"

4 SPEED & VELOCITY

Most Important

to distinction between speed and velocity

Average Speed = $\frac{\text{Total Distance}}{\text{Total Time}}$

Average Velocity = $\frac{\text{Displacement}}{\text{Time}}$

Instantaneous Velocity = Velocity at a particular instant

5 UNIFORM & NON-UNIFORM MOTION

Uniform Motion
Equal distances in equal intervals of time



Non-Uniform Motion
Unequal distances in equal intervals



6 ACCELERATION

The rate of change of velocity with the formula

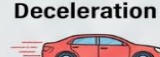
$$a = \frac{v - u}{t}$$

u = initial velocity
v = final velocity
t = time

Positive acceleration



Retardation/Deceleration



Negative acceleration



7 EQUATIONS OF MOTION

FORMULA MASTER BOX

These use x includes all 3 equations:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

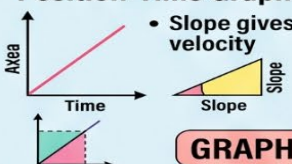
$$s = ut + 2as$$

$$v^2 = u^2 + 2as$$

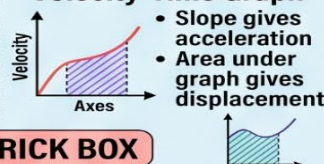
PYQ MOST IMPORTANT

8 GRAPHICAL REPRESENTATION OF MOTION

Position-Time Graph



Velocity-Time Graph



GRAPH TRICK BOX

9 CALCULUS IN MOTION

Visual introduction to elementary differentiation and integration

Velocity

Acceleration

Displacement

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$

$$s = \int v dt$$

Use beginner-friendly explanation suitable for school-level

10 QUICK REVISION ZONE

Important formulas

SI units

Graph relations

Key definitions

Common mistakes

Memory tricks

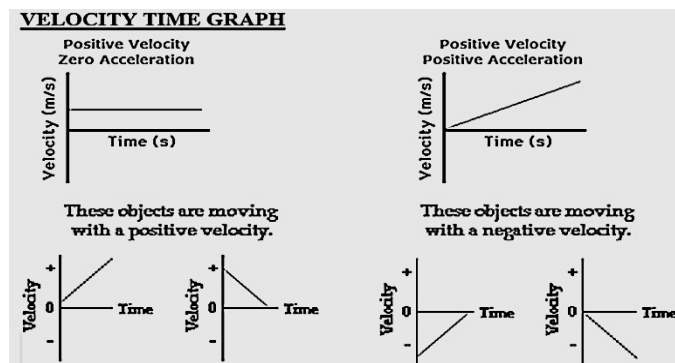
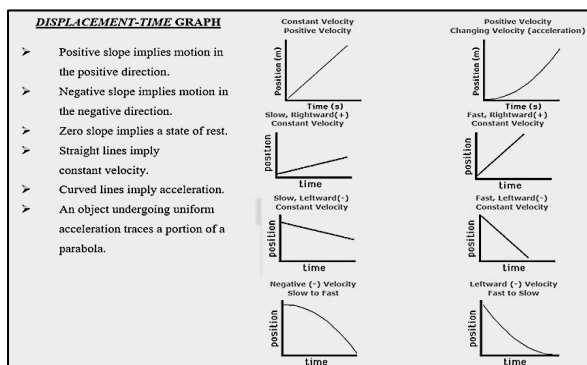
1-Minute Recall



Footer Section
"Physics is Easy When Motion Becomes Visual!"



The entire infographic is scientifically accurate and strictly according to the latest CBSE syllabus.



MULTIPLE CHOICE QUESTIONS (MCQ)

1. A passenger in an automobile observes that raindrops make an angle of 30° with the horizontal as the auto travels forward with a speed of 60 km/h. Compute the terminal (constant) velocity of the rain if it is assumed to fall vertically

- (a) 34.6 km/h (b) 44.6 km/h (c) 54.6 km/h (d) 24.6 km/h

2. Two objects of masses m_1 and m_2 fall from the heights h_1 and h_2 respectively.

The ratio of the magnitude of their momenta when they hit the ground is

- (a) $\sqrt{h_1 h_2}$ (b) $\sqrt{\frac{m_1 h_1}{m_2 h_2}}$ (c) $\frac{m_1}{m_2} \sqrt{\frac{h_1}{h_2}}$ (d) $\frac{m_1}{m_2}$

3. An athlete finishes a round of circular track of radius R in 40 sec. What is his displacement at the end of 2 min 20 sec?

- (a) $2R$ (b) $2\pi R$ (c) $7\pi R$ (d) Zero

4. A ball is dropped from height h . The velocity attained by the ball just before reaching the ground is

- (a) $\sqrt{2gh}$ (b) $\sqrt{3gh}$ (c) $\sqrt{gh}$ (d) zero

5. A car travels from A to B at a speed of 20 Km/hr and returns at a speed of 30 Km/hr the average speed of the car for the whole journey is

- (a) 5 Km/hr (b) 24 Km/hr (c) 25 Km/hr (d) 50 Km/hr

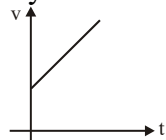
6. A 100 m long train is moving with a uniform velocity of 45 Km/hr. The time taken by the train to cross a bridge of length 1 Km is

- (a) 58 second (b) 68 second (c) 78 second (d) 88 second

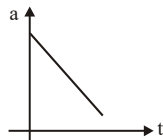
7. If $t = \sqrt{x} + 4$, then $\left(\frac{dx}{dt}\right)_{t=4}$ is

- (a) 4 (b) zero (c) 8 (d) 16

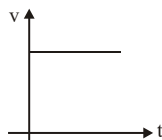
8. A body moves with uniform acceleration, then which of the following graphs is correct –



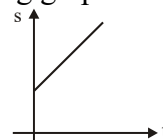
(a)



(b)

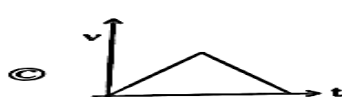
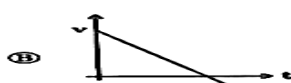
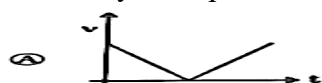


(c)



(d)

9. A ball is thrown vertically upward. Ignoring the air resistance, which one of the following plot represents the velocity-time plot for the period ball remains in air?



Q10. A stone falls freely such that the distance covered by it in the last second of its motion is equal to the distance covered by it in the first 5 seconds it remained in air for

- (a) 12 sec (b) 13 sec (c) 25 sec (d) 26 sec

CHALLENGE CORNER

Q11. A car travels first half of the distance between two places with a speed of 30 Km/hr and remaining half with the speed of 50 Km/hr the average speed of the car is

- (a) 37.5 Km/hr (b) 42 Km/hr (c) 40 Km/hr (d) 49 Km/hr

Q12. The motion of a particle is described by the equation $u=at$. The distance travelled by the particle in first 4 second is

- (a) 4a (b) 12a (c) 6a (d) 8a

Q13. A body A is thrown up vertically from the ground with a velocity v_0 and another body B is simultaneously dropped from a height H they meet at a height $H/2$, if v_0 is equal to

- (a) $\sqrt{2gH}$ (b) $\sqrt{gH}$ (c) $\frac{1}{2}\sqrt{gH}$ (d) $\sqrt{\frac{2g}{h}}$

Q14. A particle starts from rest with constant acceleration $=2\text{m/s}^2$. Find displacement in 5th sec

- (a) 9m (b) 18m (c) 15m (d) 6m

Q15. A ball falls from 20 m height on floor & rebounds to 5 m. Time of contact is 0.02 sec. Find acceleration of the ball during impact

- (a) 1200 m/s^2 (b) 1000 m/s^2 (c) 2000 m/s^2 (d) 1500 m/s^2

SOLUTIONS OF MCQ :-

1. (a) 34.6 km/h **Explanation** $\tan 30^\circ = \frac{v}{60}$ Since, $\tan 30^\circ = \frac{1}{\sqrt{3}}$

Therefore, $v = 60 \times \frac{1}{\sqrt{3}}$ $v = \frac{60}{1.732}$ $v \approx 34.6\text{ km/h}$

2. (a) If the displacement of an object is proportional to square of time, then the object is moving with constant acceleration

3 (a) 2R

4 (a) $\sqrt{2gh}$

5 (b) $v_{\text{avg}} = \frac{2 \times 20 \times 30}{20 + 30} = 24\text{ Km/hr}$

6 (d) Total Distance = Length of train + Length of Bridge

Total Distance = $(100 + 1000) = 1100\text{ m}$ Speed $45\text{ km/hr} = 45 \times \frac{5}{18} = \frac{25}{2}\text{ m/s}$

Time taken = Total distance / speed = $\frac{1100}{25/2} = 88\text{ sec}$

7. (b) $x = (t-4)^2 = t^2 - 8t + 16$ $\frac{dx}{dt} = 2t - 8$ At $t=4$, $\frac{dx}{dt} = 0$

8. (a) $v-t$ graph is a straight line $\rightarrow$ Correct

9. (a) During upward motion the magnitude of velocity decreases while during downward motion magnitude of velocity increases hence $v-t$ graph is a straight line with a negative slope

10 (b) $S_{\text{nth}} = u + \frac{g}{2}(2n - 1)$ For freely falling body $u=0$ Therefore, $S_{\text{nth}} = \frac{g}{2}(2n - 1)$

Distance travelled in first 5 sec $S = ut + \frac{1}{2}gt^2 = \frac{25}{2}g$ Given $S_{\text{nth}} = S$

ANSWERS OF CHALLENGE CORNER

11. (a) 37.5 Km/hr 12. (d) 8a 13. (b) $\sqrt{gH}$ 14. (a) 9m 15. (d) 1500 m/s^2

ASSERTION REASON BASED QUESTION

In the following questions a statement of assertion(A) is followed by a statement of reason(R) mark the correct choice as

- (e) If both assertion and reason are true and reason is the correct explanation of the assertion
- (f) If both assertion and reason are true but reason is the not correct explanation of assertion
- (g) If assertion is true but reason is false
- (h) If both assertion and reason are false

1. **ASSERTION:** The speed of a body can be negative.

REASON: If the body is moving in the opposite direction of positive motion, then its speed is negative.

2. **ASSERTION:** An object can have constant speed but variable velocity.

REASON: SI unit of speed is m/s.

3. **ASSERTION:** In real-life, in a number of situations, the object is treated as a point object.

REASON: An object is treated as point object, as far as its size is much smaller than the distance, it moves in a reasonable duration of time.

4. **ASSERTION:** The speedometer of an automobile measures the average speed of the automobile.

REASON: Average velocity is equal to total distance divided by total time taken.

5. **ASSERTION:** A body can have acceleration even if its velocity is zero at that instant of time.

REASON: The body will be momentarily at rest when reverses its direction of motion

6. **ASSERTION:** When a body is dropped or thrown horizontally from the same height it would reach the ground at the same time

REASON: Horizontal velocity has no effect on the vertical motion

7. **ASSERTION:** Displacement of a body may be zero when distance traveled by it is not zero

REASON: The displacement is the longer distance between initial and final positions

CHALLENGE CORNER

1. **ASSERTION:** Retardation is directly opposite to the velocity.

REASON: Retardation is equal to the time rate of decrease of speed

2. **ASSERTION:** In the presence of air resistance, if the ball is thrown vertically upward then time of ascent is less than the time of descent.

REASON: Force due to air friction always acts opposite to the motion of body.

3. **ASSERTION:** The area under acceleration-time graph is equal to velocity of object.

REASON: For an object moving with constant acceleration position-time graph is a straight line.

4. **ASSERTION:** It is not possible to have constant velocity and variable acceleration.

REASON: Accelerated body cannot have constant velocity.

5. **ASSERTION:** A body moving with decreasing speed may have increased acceleration.

REASON: The speed of body decreases when acceleration of body is opposite to velocity

SOLUTIONS OF A & R :-

1. (d) Speed can never be negative because it is a scalar quantity. So, if a body is moving in negative direction, then also the speed will be positive.

2. (b) Velocity is a vector quantity, so it has both direction and magnitude. Hence, an object can have variable velocity by keeping its magnitude constant, i.e. speed and by changing direction only.

3. (a) The approximation of an object as point object is valid only, when the size of the object is much smaller than the distance it moves in a reasonable duration of time.

4. (d) The assertion that the speedometer measures average speed is incorrect; it measures instantaneous speed. The reason, stating average velocity is total distance divided by total time, is also incorrect.

5. (a) When a body thrown up vertically will have zero velocity at the highest point but its acceleration will be equal to g .

6. (a) Horizontal velocity has no effect on time of flight. Therefore both assertion and reason are true and the reason is the correct explanation of the assertion
7. (c) As displacement is the shortest distance between the initial & final position.

ANSWERS OF CHALLENGE CORNER

1. (b) 2. (a) 3.(d) 4.(b) 5. (b)

SHORT ANSWER QUESTIONS (2 MARKS)

- Two balls of different masses are thrown vertically upward with the same initial speed. Which one will rise to a greater height?
- Can a body have a constant speed and still have a varying velocity?
- The position coordinate of a moving particle is given by $x=6+18t + 9t^2$ (Where x is in meters and t is in seconds) what is the velocity at $t=2$ second?
- Derive an equation for the distance travelled by a body moving with uniform acceleration during the nth second of its motion.
- Is it possible to have a constant rate of change of velocity when velocity changes both in magnitude and election if yes give one example

CHALLENGE CORNER

- A bowl hits a ball with a velocity of 30 m/s and re bounces with the same velocity what is the change in its velocity
- A student went on his bike from Delhi to Gurgaon at a speed of 60km/hr and came back at a speed of 40km/hr. What is his average speed for the entire journey?
- A particle is thrown upwards. It attains a height (h) after 5 seconds and again after 9s comes back. What is the speed of the particle at a height h?
- Displacement of a particle is given by the expression $x=3t^2+7t$, where x is in meters and t is in seconds. What is acceleration?
- The displacement x of a particle moving in one dimension under the action of the constant force is related to the time by the equation $t=\sqrt{x} - 3$ where x is in meters and t is in seconds. Find the velocity of the particle at $t = 3s$

SOLUTIONS (2 MARKS):-

- Ans: When two balls of different masses are thrown vertically upward with the same initial speed, both of them will rise to a greater height.
- Yes, a particle in uniform circular motion has a constant speed but varying velocity because of the change its direction of motion at every point
- Given $x=6+18t + 9t^2$

$$v = \frac{dx}{dt} = \frac{d}{dt} (6+18t + 9t^2) = 18+18t$$
At $t=2$ sec $v=18+18 \times 2=54\text{ms}^{-1}$
- Prove $S_n = u + a/2(2n- 1)$
- Yes, in projectile motion a body has uniform acceleration in the downward direction while its velocity changes both in direction and magnitude at every point of its trajectory.

ANSWERS OF EXERCISE QUESTIONS

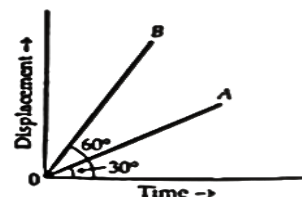
- 6: -60 ms^{-1} 7: 48 Km/hr 8 :19.6 m/s 9 : 6m/s^2 10: 12m/s

SHORT ANSWER QUESTIONS (3 MARKS)

1. A body is moving in a straight line and begins to slow down gradually. In such a situation:

- Identify the condition of motion.
- State the directions of velocity and acceleration vectors.
- Justify your answer with a suitable real-life example

2. Two straight lines drawn on the same displacement time graph makes angle 30° and 60° with time axis respectively as shown in the figure



- which line represents greater velocity
- what is the ratio of the two velocities
- What is the nature of motion represented by each line?

3. Draw following graphs representing motion of an object under freefall neglect air resistance

- variation of position with respect to time
- variation of velocity with respect to time
- variation of acceleration with time

4. A car moving along a straight highway with a speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?

5. A body travels from A to B at 40 ms^{-1} & from B to A at 60 ms^{-1} . Calculate the average speed and average velocity.

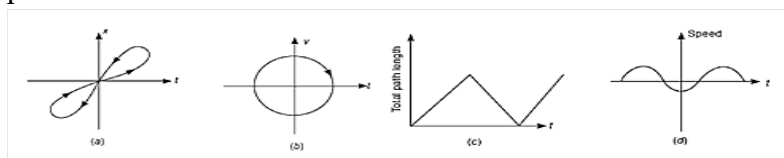
6. The position of an object moving along x-axis is given by $x = a + bt^2$, where $a = 8.5 \text{ m}$ and $b = 2.5 \text{ ms}^{-2}$ and (t) is measured in seconds. What is the velocity at $t = 0 \text{ s}$ and $t = 2.0 \text{ s}$? What is the average velocity between $t = 2.0 \text{ s}$ and $t = 4.0 \text{ s}$?

7. The displacement (in metre) of a particle moving along x-axis is given by $x = 18t + 5t^2$. Calculate (i) the instantaneous velocity $t = 2 \text{ s}$ (ii) average velocity between $t = 2 \text{ s}$ to $t = 3 \text{ s}$ (iii) instantaneous acceleration.

8. A body covers 12m in 2nd second and 20m in 4th second. How much distance will it cover in 4 seconds after the 5th second?

CHALLENGE CORNER

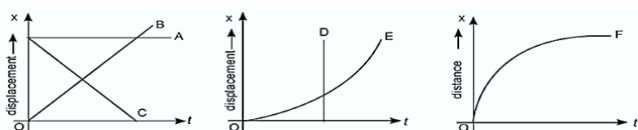
9. State with reasons which of these following graphs cannot possibly represent one-dimensional motion of a particle.



10. A body starts accelerating uniformly with a from a velocity 'u' and travels in a straight line. Prove that it covers a length of $u + \frac{a}{2}(2t-1)$ in the tth second of motion.

11. If the initial velocity of a particle is u and collinear acceleration at any time t is $a \propto t$. calculate the final velocity of the particle after time t.

12. Identify the types of motion in following cases :



13. Using calculus, derive the three equations of motion for an object moving in a straight line with uniform acceleration: $v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$

SOLUTIONS: -

Ans 1: (a) Condition of Motion:

The body is undergoing retardation or deceleration—a condition where the speed of the body decreases with time.

(b) Direction of Vectors: The velocity vector is in the direction of motion of the body.

The acceleration vector is opposite to the direction of velocity, as it acts to reduce the body's speed.

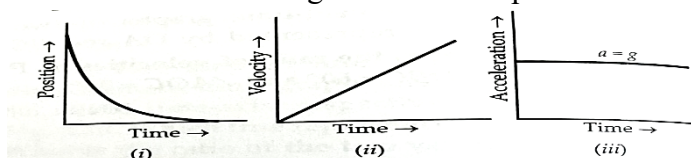
(c) Justification with Example: A common example is a car moving forward and gradually coming to a stop when the brakes are applied. While the car continues to move forward for a while, the acceleration due to braking acts in the opposite direction, slowing it down until it stops

Ans 2 .(a) The line with a greater slope (i.e., 60°) represents greater velocity

(b) Ratio of two velocities $= \frac{v_A}{v_B} = \frac{\tan 30}{\tan 60} = \frac{1}{3}$

(c) Nature of Motion : Each straight line on a displacement-time graph represents uniform motion.

3 :



4 : The initial velocity of the car = u ,Final velocity of the car = v

Distance covered by the car before coming to rest = 200 m

Using the equation, $v = u + at \Rightarrow t = (v - u)/a = 11.44$ sec.

5: Avg Speed $= \frac{\text{Total distance travelled}}{\text{Total time}} = \frac{2v_1v_2}{v_1+v_2} = 48$ Km/hr

Avg Velocity $= \frac{\text{Total displacement travelled}}{\text{Total time}}$ As body returns to its initial state Therefore net displacement is zero Hence Avg velocity $= 0$

6: $v(t) = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 0 + 2bt$ At $t=0$; $v=0$ & At $t=2$ s $v=2 \times 2.5 \times 2 = 10$ ms $^{-1}$

At $t=2$ s $x=a+4b$ & At $t=4$ s $x=a+16b$ Avg Velocity $= \frac{x_2 - x_1}{t_2 - t_1} = \frac{a+16b - a - 4b}{4 - 2} = 6b$

Hence, Avg Velocity $= 6 \times 2.5 = 15$ ms $^{-1}$

7 : As, $x=18t+5t^2$ $v(t) = \frac{dx}{dt} = 18 + 10t$ & $a(t) = \frac{dv}{dt} = 10$ ms $^{-2}$

vel at $t=2$ $v(2) = 18 + 10(2) = 38$ ms $^{-1}$ $v(3) = 18 + 10(3) = 48$ ms $^{-1}$ Since acceleration is constant

$V_{avg} = \frac{v(3) + v(2)}{2} = 43$ ms $^{-1}$ & instantaneous acceleration $a = 10$ ms $^{-2}$

8 : Case I: let initial velocity = u m/s ,distance $= s = 12$ m, Time $= t = 2$ sec

$S = u + a/2(2t-1) \Rightarrow 12 = u + a/2(2 \times 2 - 1) \Rightarrow 12 = u + 3a/2 \Rightarrow 2u + 3a = 24$ ------(1)

Case II: Time $= t = 4$ sec, distance $= s = 20$ m $20 = u + a(2 \times 4 - 1)/2 \Rightarrow 20 = u + 7a/2 \Rightarrow 2u + 7a = 40$ ---(2)

from 1 and 2: $-4a = -16 \Rightarrow a = 4$ m/s 2 substitute the value of a in equation 1 we get : $u = 6$ m/s

Case iii: distance covered in 4sec after 5second = distance covered in 9th s - distance covered in 5 sec $= (6 \times 9 + 1/2 \times 4 \times 9 \times 9) - (6 \times 5 + 1/2 \times 4 \times 5 \times 5) = 54 + 162 - (30 + 50) = 136$ m

ANSWERS OF CHALLENGE CORNER

9: (a) It is not possible because at the same time a body has two positions which is not possible.

(b) It is not possible because here a body at the same time possesses two different velocities.

(c) It is not possible as the total path length travelled by the particle cannot decrease with time.

(d) It is not possible as speed cannot be negative

10: Length covered in t^{th} second of motion $= u + a/2(2t-1)$

11. $v = \frac{Kt^2}{2}$

12. A-Body is at rest B - Body has uniform motion - velocity is constant

C - Uniform motion but -ve velocity D - Not possible, since x takes many values for same time. E - accelerated motion, since slope inc. F - decelerated motion, since velocity dec

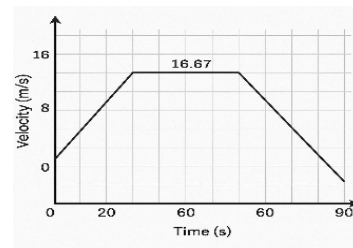
13 : Derivation using calculus method

(i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 - u^2 = 2as$ or $v^2 = u^2 + 2as$

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. While waiting at a metro station, a student records the motion of a train as it arrives. The velocity-time graph of the train is as follows:

- Draw a velocity-time graph for the entire motion.
- Calculate the total distance covered by the train.
- Find the total time taken and the average speed of the train.
- Identify the nature of acceleration in each phase and explain its physical meaning



2. Water drops fall freely from a tap at a height of 4.9 m. If time interval between successive drops is equal and the 4th drop is released when the first lands on the ground, find the separation between the second and third drops

3. A player throws a ball upwards with an initial speed of 29.4ms^{-1}

- What is the direction of acceleration during the upward motion of the ball?
- What are the velocity and acceleration of the ball at the highest point of its motion?
- Choose the $x = 0$ m and $t = 0$ s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion.
- To what height does the ball rise and after how long does the ball return to the player's hands? (Take $g = 9.8\text{ms}^{-2}$ and neglect air resistance).

4. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one-tenth of its speed. Plot the speed-time graph of its motion between $t = 0$ to 12 s

5. A point object is thrown vertically upwards at such a speed that it returns the thrown after 6 seconds. With what speed was it thrown up and how high did it rise? Plot speed-time graph for the object and use it to find the distance travelled by it in the last second of its journey.

CHALLENGE CORNER

6. Derive an equation for the distance covered by a uniformly accelerated body in nth second of its motion.

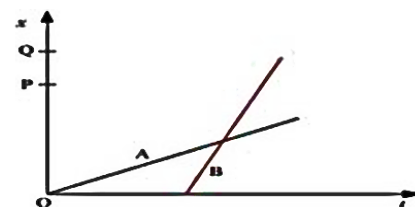
A body travels half its total path in the last second of its fall from rest. Calculate the time of its fall

7. The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig.

3.19.

Choose the correct entries in the brackets below;

- (A/B) lives closer to the school than (B/A)
- (A/B) starts from the school earlier than (B/A)
- (A/B) walks faster than (B/A)
- A and B reach home at the (same/different) time
- (A/B) overtakes (B/A) on the road (once/twice).



8. An aircraft needs a minimum speed of 108 km/h to take off. The runway is 500 meters long. A pilot begins from rest and accelerates uniformly.

- What is the minimum acceleration required for the aircraft to take off within the runway length?
- How long does it take to reach take off speed?
- Sketch the velocity-time graph.
- Suppose wind resistance causes a deceleration of 1 m/s^2 during motion. How would this affect the aircraft's ability to take off?
- Give one real-world precaution engineers must consider when designing runways based on this motion concept.

SOLUTIONS:

1:(a) Linearly rising from 0 to 16.67 m/s in 20 s Horizontal line at 16.67 m/s for 40 s
Linearly decreasing from 16.67 m/s to 0 in 30 s

(b) Total Distance Covered

During acceleration (0 to 20 s): $a = \frac{v-u}{t_1} = \frac{16.67-0}{20} = 0.8335 \text{ m/s}^2$ & $s_1 = ut + \frac{a}{2} t^2 = 166.7 \text{ ms}^{-1}$

During constant speed (20 to 60 s): $s_2 = v \times t_2 = 16.67 \times 40 = 666.8 \text{ ms}^{-2}$

During deceleration (60 to 90 s): $a = \frac{v-u}{t_3} = -0.556 \text{ m/s}^2$ $s_3 = vt_3 + \frac{a}{2} t_3^2 = 250.05 \text{ ms}$

Total distance: $s = s_1 + s_2 + s_3 = 166.7 + 666.8 + 250.05 = 1083.55 \text{ m}$

(c) Total time $t = 20 + 40 + 30 = 90 \text{ s} = 90 \text{ s}$

Avg speed or v_{avg} $= \frac{\text{Total distance travelled}}{\text{Total time}} = \frac{1083.55}{90} \approx 12.04 \text{ m/s}$

(d) Nature of acceleration:

First 20 s: Positive uniform acceleration (speed increasing)

Next 40 s: Zero acceleration (constant speed)

Last 30 s: Negative uniform acceleration (deceleration to rest)

2: As, $h = 4.9 \text{ m}$ Time for a drop to reach ground $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 4.9}{9.8}} = 1 \text{ sec}$

when the first drop lands on ground, the 4th is ready to come down. Therefore there are 3 equal time gaps of $\left(\frac{1}{3}\right)$ between two consecutive drops. Time of travel of 2nd and 3rd drops are $\frac{2}{3}$ & $\frac{1}{3}$ respectively. Separation

between 2nd and 3rd drops $= \frac{g}{2} \left[\left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2 \right] = 1.63 \text{ m}$

3.(a) Irrespective of the direction of the motion of the ball, acceleration (which is actually acceleration due to gravity) always acts in the downward direction towards the centre of the Earth.

(b) At maximum height, velocity of the ball becomes zero. Acceleration due to gravity at a given place is constant 9.8 ms^{-2}

(c) During upward motion, the sign of position is positive, sign of velocity is negative, and sign of acceleration is positive. During downward motion, the signs of position, velocity, and acceleration are all positive.

$x > 0$ for both upward and downward motions, $v < 0$ for upward and $v > 0$ for downward motion, $a > 0$ throughout the motion

Using $v^2 - u^2 = 2as \Rightarrow s = 44.1 \text{ m}$, & Using $v = u + at \Rightarrow t = 3 \text{ s}$

Total time of flight = time of ascent + time of descent = 6 s

4 :Let v be the speed on reaching the floor

$$v^2 - u^2 = 2as \Rightarrow v^2 - (0)^2 = 2 \times 9.8 \times 90 \Rightarrow v^2 = 1764 \Rightarrow v = 42 \text{ m s}^{-1}$$

Let t_1 be the time taken by the ball in reaching the ground.

$$v = u + at \Rightarrow 42 = 0 + 9.8t_1 \Rightarrow t_1 = 4.28 \text{ s.}$$

(i) The speed of the ball will go on increasing at a constant rate from 0 m/s to 42 m/s for 4.28 seconds. This is shown by the line OA. At A, the ball strikes the floor and its speed is decreased by $\frac{1}{10} \times 42 = 4.2$ (i.e.,

$42 - 4.2 = 37.8$) m/s This is shown by the line AB.

(ii) The ball goes up vertically with a speed $u = 37.8$ m/s and at the highest point its speed v becomes zero. Suppose it reaches the highest point in time t_2

$$v = u + at_2 \Rightarrow 0 = 37.8 - 9.8t_2 \Rightarrow t_2 = \frac{37.8}{9.8} = 3.85 \text{ s}$$

Therefore, the speed of the ball goes on decreasing at constant rate from 37.8 m/s to 0 for 3.85s. $v_2 = u_2 +$

$$2as \Rightarrow (0)^2 - (37.8)^2 = -2 \times 9.8 \times s \Rightarrow s = \frac{(37.8)^2}{2 \times 9.8} = 72.9 \text{ m}$$

Thus the ball rises to the height 72.9 m. The decrease in speed in the upward motion is shown by BC.

(iii) After reaching the highest point, the ball will take the same time as in (ii), i.e., 3.85 s to come down and strike the floor. Its speed will increase from 0 to 37.8 m/s. The speed with which it bounced back. This is shown by CD.

(iv) On hitting the floor, its speed will decrease by $\frac{1}{10}$ of 37.8 = 3.78 $\Rightarrow 37.8 - 3.78 = 34.02$ m/s

This is represented by DE.

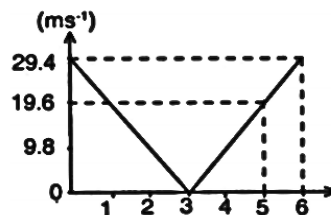
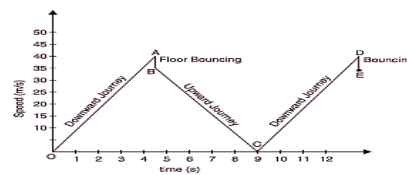
The total time since the ball was dropped = $4.28 \text{ s} + 3.85 \text{ s} + 3.85 \text{ s} = 11.98 \text{ s} \approx 12 \text{ s}$.

$$5: h = \frac{u^2}{2g} = \frac{(29.4)^2}{2 \times 9.8} = 44.1 \text{ m}$$

Initial speed = 29.4 ms^{-1} Height the object attained = 44.1 m

Distance covered in last second = area under speed-time graph

$$= \frac{1}{2} \times (6 - 5) \times (19.6 + 29.4) = 24.5 \text{ m}$$



ANSWERS OF CHALLENGE CORNER

6. Displacement in n th second = $S_n - S_{n-1} = u + \frac{a}{2}(2n-1)$. Let S be the complete length of fall & t be the time taken for it $S = \frac{1}{2}gt^2$ ($u=0$). Also $\frac{S}{2}$ is covered in last second $\frac{S}{2} = 0 + \frac{g}{2}(2t-1)$ On solving we get $t^2 - 4t + 2 = 0 \Rightarrow t = 2 \pm \sqrt{2} \text{ sec}$

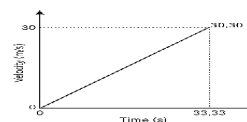
7. (a) A lives closer to school than B. (b) A starts from school earlier than B.

(c) B walks faster than A. (d) A and B reach home at the same time.

(e) B overtakes A once on the road

$$8.(a) a = 0.9 \text{ m/s}^2 \quad t = 33.33 \text{ s}$$

$$\text{Net acceleration } a_{\text{net}} = 0.9 - 1 = -0.1 \text{ m/s}^2$$



Runways are designed longer than the minimum required length to account for resistive forces like wind, friction, or emergency braking, ensuring safe takeoff

CASE STUDY BASED QUESTION

1. A person is standing on the terrace of a building 100 meters high. At time $t=0$, he throws a ball vertically upward with an initial velocity u . After a time interval Δt (where $\Delta t < 2 \text{ sec}$), he throws a second ball vertically upward with an initial velocity equal to half that of the first ball. At time $t=2$ seconds (measured

from the instant the first ball is thrown), the vertical separation between the two balls is observed to be +15 meters, with the first ball being above the second. Assume acceleration due to gravity $g=10 \text{ m/s}^2$, and neglect air resistance.

- (i) How does the gap between the balls change?
(a) Increases (b) Decreases (c) Becomes zero (d) Remains constant
- (ii) Find the initial velocity of ball 1.
(a) 10 m/s (b) 20 m/s (c) 30 m/s (d) 40 m/s
- (iii) With what velocity was second ball thrown?
(a) 10 m/s (b) 20 m/s (c) 30 m/s (d) 40 m/s
- (iv) Calculate the time interval between their throw.
(a) 0.01 s (b) 0.1 s (c) 1 s (d) 10 s

OR

- (iv) The type of motion of balls is
(a) accelerated (b) uniform (c) periodic (d) random

2. When an object is in motion, its position changes with time. So, the quantity that describes how fast is the position changing w.r.t. time and in what direction is given by average velocity. It is defined as the change in position or displacement $\Delta x/\Delta t$ in which that displacement occurs. However, the quantity used to describe the rate of motion over the actual path, is average speed. It is defined as the total distance travelled by the object divided by the total time taken.

- (i) A 250 m long train is moving with a uniform velocity of 45 km/h. The time taken by the train to cross a bridge of length 750 m is
(a) 80 s (b) 68 s (c) 56 s (d) 92 s
- (ii) A truck requires 3 hr to complete a journey of 150 km. What is average speed?
(a) 50 km/h (b) 25 km/h (c) 15 km/h (d) 10 km/h
- (iii) Average speed of a car between points A and B is 20 m/s, between B and C is 15 m/s and between C and D is 10 m/s. What is the average speed between A and D, if the time taken in the mentioned sections is 20s, 10s and 5s, respectively?
(a) 19 m/s (b) 16 m/s (c) 23 m/s (d) 17.14 m/s
- (iv) A cyclist is moving on a circular track of radius 40 m completes half a revolution in 40 s. Its average velocity (in m/s) is
(a) zero (b) 2 (c) 4π (d) 8

CHALLENGE CORNER

3. If you tossed an object either up or down and could somehow eliminate the effects of air on its flight you would find that the object accelerates downward at a certain constant rate. The rate is called the free fall acceleration and its magnitude is represented by g . The acceleration is independent of the objects characteristics such as mass density or shape it is the same for all objects.

- (i) The time of ascent of a body thrown vertically upward with initial velocity u is
(a) $t = u/2g$ (b) $t = 2u/g$ (c) $t = u/g$ (d) $t = u/3g$
- (ii) The time of descent of a body thrown vertically upward with initial velocity u is
(a) $t = u/4g$ (b) $t = u/g$ (c) $t = u/2g$ (d) $t = u/3g$
- (iii) The total time of flight to come back to the point of projection of a body thrown vertically upward with initial velocity u is
(a) $t = 2u/3g$ (b) $t = u/2g$ (c) $t = 2u/g$ (d) $t = u$
- (iv) Velocity of fall at the point of projection of a body thrown vertically upward with

initial velocity u is

(a) $v = u$

(b) $v = 2u$

(c) $v = 3u$

(d) $v = 4u$

SOLUTIONS

1:(i) (d) The gap between the balls remains constant as their acceleration is the same.

(ii) (b) Let velocity of first ball be u and time gap of second ball be x .

Thus first ball has time $(t + x)$ in upward motion.

$$S_1 = u(t + x) + \frac{1}{2}(-g)(t + x)^2 \quad \text{Similarly, for second ball, } S_2 = \frac{ut}{2} - \frac{1}{2}gt^2$$

$$\text{Taking difference, } S_1 - S_2 = u\left(t + x - \frac{t}{x}\right) - \frac{g}{2}[(t + x)^2 - t^2] = u\left(\frac{t}{x} + x\right) - \frac{g}{2}(2xt + x^2)$$

$$\text{Now, } S_1 - S_2 = 15 \text{ m at } t = 2\text{s} \quad \text{As } x < 2\text{s, } x = 1\text{s}$$

$$15 = u\left(\frac{2}{1} + 1\right) - \frac{1}{2} \times 10\{2 \times 1 \times 2 + 1(1)^2\} \Rightarrow u = 20 \text{ m/s (velocity of first ball)}$$

(iii) (a) Velocity of second ball = 10 m/s

(iv) (c) Time interval = $x = 1\text{s}$. Or (iv) (a) accelerated

2. (i) a 80 sec (ii) a 50 Km/hr (iii) d 17.14 sec (iv) b 2m/s^2

HINTS OF CHALLENGE CORNER

3.(i) c

(ii) b

(iii) c

(iv) a



MOTION IN A STRAIGHT LINE

CBSE CLASS XI PHYSICS

The path may be straight, but the understanding makes it powerful."



1. SCALAR AND VECTOR QUANTITIES

Scalar Quantities

Have magnitude only.

Examples: distance, speed, time, mass, temperature.

Vector Quantities

Have both magnitude and direction.

Examples: displacement, velocity, acceleration, force.

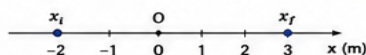
In motion along a straight line, direction is taken as either +ve or -ve.

2. POSITION, DISPLACEMENT AND DISTANCE

- Position (x): Location of a particle at a given time with respect to a reference point (origin).
- Displacement (Δx): Shortest distance from initial to final position in a specified direction.

$$\Delta x = x_f - x_i$$

- Distance: Actual length of the path travelled (always positive).



If a body comes back to its initial position, displacement = 0 but distance $\neq$ 0.

3. VELOCITY AND SPEED

Average Velocity

$$\bar{v} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{\Delta x}{\Delta t}$$

Instantaneous Velocity

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Speed = |Velocity|

(Speed is always positive)

Average Speed

$$\bar{s} = \frac{\text{Total distance}}{\text{Total time}}$$

4. ACCELERATION

Average Acceleration

$$\bar{a} = \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t}$$

Instantaneous Acceleration

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

SI Unit: m s^{-2}

Significance of sign of acceleration

- $a > 0$: Velocity increases with time
- $a < 0$: Velocity decreases with time
- $a = 0$: Velocity is constant

5. EQUATIONS OF MOTION (For uniformly accelerated motion)

Let initial velocity = u
Final velocity = v
Acceleration = a (constant)
Time = t
Displacement = s

SUVAT Equations

$$1 \quad v = u + at$$

$$2 \quad s = ut + \frac{1}{2}at^2$$

$$3 \quad v^2 = u^2 + 2as$$

$$4 \quad s = \frac{(u + v)}{2}t$$

Derivation Hint

From equation (1),

$$a = \frac{v - u}{t}$$

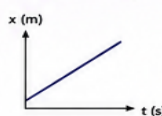
Substitute in equation (2)

$$\text{or use } v = \frac{dx}{dt}, a = \frac{dv}{dt}$$

6. GRAPHS IN MOTION

(i) Position-Time ($x-t$)

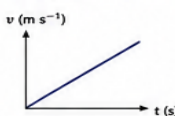
Slope = velocity



Straight line
 $\Rightarrow$ constant velocity

(ii) Velocity-Time ($v-t$)

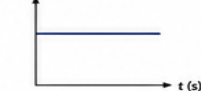
Slope = acceleration



Straight line
 $\Rightarrow$ constant acceleration

(iii) Acceleration-Time ($a-t$)

a (m s^{-2})

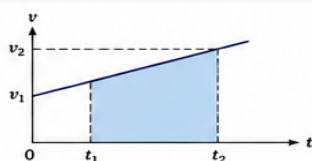


Horizontal line
 $\Rightarrow$ acceleration = constant

7. SPECIAL CASES

- | | |
|---|--|
| (i) Uniform Motion | $a = 0$
$v = \text{constant}$ |
| (ii) Motion with Positive Acceleration | $a > 0$
increases with time |
| (iii) Motion with Negative Acceleration | $a < 0$
decreases with time |
| (iv) Free Fall (near Earth) | $a = g = 9.8 \text{ m s}^{-2}$
downward |

8. RELATION BETWEEN $v-t$ GRAPH AND MOTION



- Area under $v-t$ graph between t_1 and t_2 = displacement between t_1 and t_2
- Area above time axis $\Rightarrow$ +ve displacement
- Area below time axis $\Rightarrow$ -ve displacement

9. COMPOUND MOTION (IMPORTANT CONCEPTS)

(i) Overtaking

When a faster body catches up a slower body ahead.
Relative velocity = $v_1 - v_2$

(ii) Head-on Collision

When two bodies move towards each other.
Relative velocity = $v_1 + v_2$

(iii) Crossing a Point

Time taken by a body to reach a point.
Use equations of motion.

10. IMPORTANT FORMULAS

- Average velocity = $\frac{\text{Total displacement}}{\text{Total time}}$
- Average speed = $\frac{\text{Total distance}}{\text{Total time}}$
- Instantaneous velocity = $\frac{dx}{dt}$
- Instantaneous acceleration = $\frac{dv}{dt} = \frac{d^2x}{dt^2}$
- Change in velocity = $v_f - v_i$
- Displacement (constant a) = $ut + \frac{1}{2}at^2$

11. TYPICAL NUMERICAL EXAMPLES

Example 1

A train starts from rest and attains a velocity of 72 km h^{-1} in 10 s with uniform acceleration. Find (i) acceleration (ii) distance travelled.

Solution:

$$u = 0, v = 72 \text{ km h}^{-1} = 20 \text{ m s}^{-1}, t = 10 \text{ s}$$

$$(i) \quad a = \frac{v - u}{t} = \frac{20 - 0}{10} = 2 \text{ m s}^{-2}$$

$$(ii) \quad s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(2)(10)^2 = 100 \text{ m}$$

Example 2

A body moving with velocity 10 m s^{-1} is uniformly retarded to rest in 5 s. Find the distance travelled.

Solution:

$$u = 10 \text{ m s}^{-1}, v = 0, t = 5 \text{ s}$$

$$a = \frac{v - u}{t} = \frac{0 - 10}{5} = -2 \text{ m s}^{-2}$$

$$s = ut + \frac{1}{2}at^2 = (10)(5) + \frac{1}{2}(-2)(5)^2 = 25 \text{ m}$$

Example 3

A car has a uniform acceleration of 3 m s^{-2} . Find its velocity after 8 s if it starts from rest.

Solution:

$$u = 0, a = 3 \text{ m s}^{-2}, t = 8 \text{ s}$$

$$v = u + at = 0 + (3)(8) = 24 \text{ m s}^{-1}$$

KEY TAKEAWAYS

- ✓ Motion in a straight line can be described completely by one dimension.
- ✓ Displacement is a vector; distance is a scalar.
- ✓ Velocity is the rate of change of displacement.
- ✓ Acceleration is the rate of change of velocity.
- ✓ Equations of motion are valid only for constant acceleration.
- ✓ Graphs provide visual understanding of motion.

COMMON MISTAKES

- ✗ Using distance instead of displacement in equations.
- ✗ Ignoring the sign of acceleration.
- ✗ Confusing average speed with average velocity.
- ✗ Forgetting unit conversions.
- ✗ Assuming acceleration is zero without checking the condition.

PYQ FOCUS

- ☆ Derive equations of motion.
- ☆ Find acceleration, velocity, time or displacement from given data.
- ☆ Interpret $v-t$ and $x-t$ graphs.
- ☆ Concept based: Scalars vs vectors, significance of negative value.

BOARD EXAM TIPS

- ✓ Draw neat diagrams and graphs.
- ✓ Write knowns clearly before solving.
- ✓ Write formula first, then substitute.
- ✓ Always write units with final answer.
- ✓ Check the sign of acceleration and velocity.

REMEMBER THIS

Equations of motion are powerful tools.



Understand the concepts, visualize the graphs, and problems become easy!



CONCEPT + PRACTICE + GRAPHS = MASTER MOTION IN A STRAIGHT LINE



CHAPTER-3

MOTION IN A PLANE

Understanding 2D Motion through Vectors, Components and Scientific Thinking

1. INTRODUCTION TO MOTION IN A PLANE

Motion in a plane (two dimensions) occurs when an object moves so that its position changes along two mutually perpendicular directions, usually X and Y axes.

Examples

- Projectile motion of a ball
- A bird flying in the sky
- An airplane taking off
- A boat crossing a river



2. SCALARS AND VECTORS

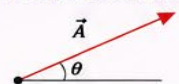
SCALAR
(magnitude only)

Examples: distance, speed, time, mass, temperature, energy.

VECTOR
(magnitude and direction)

Examples: displacement, velocity, acceleration, force, momentum.

Vector Representation



$|A|$ = magnitude

θ = direction
(angle with +X axis)

A vector is fully described by its magnitude and direction.

3. POSITION VECTOR

The position of a particle P at any time is given by the position vector:

$$\vec{r} = x\hat{i} + y\hat{j}$$

where,

x = distance along X-axis

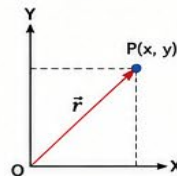
y = distance along Y-axis

$\hat{i}, \hat{j}$ = unit vectors along X and Y axes

Unit Vectors

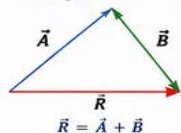
$\hat{i} = 1\hat{i} + 0\hat{j}$ (along X)

$\hat{j} = 0\hat{i} + 1\hat{j}$ (along Y)



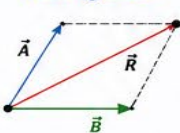
4. VECTOR ADDITION & RESOLUTION

Triangle Law

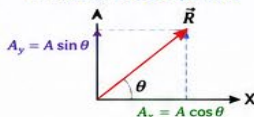


$$\vec{R} = \vec{A} + \vec{B}$$

Parallelogram Law



Resolution into Components



$$A_x = A \cos \theta$$

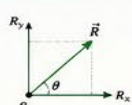
$$A_y = A \sin \theta$$

Resultant Components

If $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2}, \quad \phi = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



5. KINEMATICS IN TWO DIMENSIONS

If $\vec{v}$ is velocity and $\vec{a}$ is acceleration, then motion in X and Y directions are independent.

Along X-direction

$$v_x = u_x + a_x t$$

$$x = x_0 + u_x t + \frac{1}{2} a_x t^2$$

$$v_x^2 = u_x^2 + 2a_x(x - x_0)$$

Along Y-direction

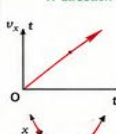
$$v_y = u_y + a_y t$$

$$y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

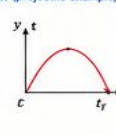
$$v_y^2 = u_y^2 + 2a_y(y - y_0)$$

Graphs (for constant acceleration)

X-direction



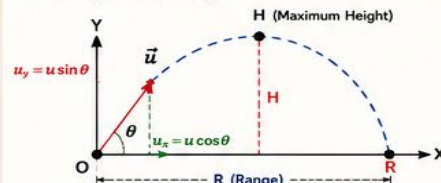
Y-direction (projectile example)



Key Idea: Break a complex 2D motion into two simple 1D motions along X and Y. Solve separately, combine results.

6. PROJECTILE MOTION

A projectile follows a parabolic path when projected with initial speed u at an angle θ .



Key Equations (g = acceleration due to gravity)

Components of Initial Velocity

$$u_x = u \cos \theta$$

$$u_y = u \sin \theta$$

Time of Flight (T)

$$T = \frac{2u \sin \theta}{g}$$

Maximum Height (H)

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

Range (R)

$$R = \frac{u^2 \sin 2\theta}{g}$$

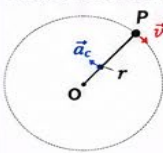
Special Case:

Maximum range when $\theta = 45^\circ$

$$\text{Then, } R_{\max} = \frac{u^2}{g}$$

7. UNIFORM CIRCULAR MOTION (UCM)

An object moves in a circle of radius r with constant speed v .



Important Formulae

Speed (v) = constant

Centripetal Acceleration:
(towards center) $a_c = \frac{v^2}{r}$

Angular Velocity: $\omega = \frac{v}{r}$

Time Period: $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$

Frequency: $f = \frac{1}{T} = \frac{\omega}{2\pi}$

Curiosity!

- Even though speed is constant, velocity is changing because direction changes continuously.
- Examples: Ceiling fan, Earth around Sun, a stone whirled in a string.

8. SCIENTIFIC THINKING

Think Like a Scientist

- Break complex motion into simple components.
- Analyze X and Y directions independently.
- Observe patterns in graphs and trajectories.
- Use equations to predict and verify results.
- Ask questions. Explore. Explain!

Curiosity Questions

- Why is projectile motion parabolic?
- Why does gravity affect only vertical motion?
- Why is range maximum at 45° ?
- What happens if there is air resistance?
- How does changing angle or speed affect the path?

9. LEARNING SUPPORT

For Bright Students

Challenge Yourself!

- A boat crosses a river of width d with speed u_b . River flows with u_r . Find resultant path.
- An airplane flies with velocity $\vec{v}_a$ in still air. Find ground velocity with wind $\vec{v}_w$.
- Explore relative velocity:
 $\vec{v}_{rel} = \vec{v}_A - \vec{v}_B$

Think Beyond:

How does changing angle θ affect range in projectile motion? Prove it!

For Slow Learners

Take it Step by Step

1. Draw X and Y axes.
2. Break motion into X and Y.
3. Write known values.
4. Use correct formula.
5. Solve one direction at a time.

Remember This

- X $\rightarrow$ Horizontal $\rightarrow$ constant speed (if $a_x = 0$)
- Y $\rightarrow$ Vertical $\rightarrow$ affected by gravity ($a_y = -g$)
- Always use units!

10. REAL-LIFE APPLICATIONS



Sports

Analyze path of ball in various games.



Missile Launch

Projectile principles used in targeting and trajectory.



Satellite Motion

Satellites in orbit move in uniform circular motion.



Airplane Navigation

Wind and velocity vectors affect flight path.



Rain at an Angle

Raindrops fall vertically, but from moving bus appear slanted.

11. SUMMARY



Motion in a Plane

is the combination of two perpendicular motions.



Vectors

help us represent magnitude and direction.



Break into Components

Solve along X and Y independently.



Projectile Motion

follows a parabolic path under gravity.



Uniform Circular Motion

occurs when speed is constant along a circle.



Science in Action

Observe, Question, Analyze, Understand, Apply!

EXTRA FACTS & FUN CORNER

Did You Know?

- A bullet fired horizontally can travel hundreds of meters!
- The Apollo missions used precise calculations of motion in a plane to reach the Moon.
- A Frisbee's flight is a combination of circular and translational motion!



Interesting Comparisons

- Car moving on a straight road $\rightarrow$ (1D)
- Car turning on a curve $\rightarrow$ (2D curved path)
- Fan blade tip $\rightarrow$ (UCM)
- Ball thrown $\rightarrow$ (Projectile: 2D + gravity)



Common Mistakes

- Forgetting vector direction
- Mixing up X and Y components
- Using wrong signs in equations
- Not considering units



Study Tips

- Practice diagrams daily.
- Solve numericals step-by-step.
- Understand concepts deeply, not just formulas.
- Revise and revisit regularly.



Remember the Big Picture

Physics is not about formulae. It is about understanding how nature works and using logic to explain the world!

THINK • QUESTION • ANALYZE • CALCULATE • UNDERSTAND • APPLY

MOTION IS EVERYWHERE. SCIENCE IS IN YOU!

MULTIPLE CHOICE QUESTIONS (MCQ)

Q1. Which physical quantity remains unchanged during projectile motion?

- (a) Momentum (b) Kinetic energy
(c) Vertical component of velocity (d) Horizontal component of velocity

Q2. A coin is tossed into the air at some acute angle with the ground. At a certain moment during its motion, the angle between its velocity and the downward pull of gravity is θ . What is the value of θ ?

- (a) $\theta = 90^\circ$ (b) $\theta = 0^\circ$ (c) $90^\circ < \theta < 180^\circ$ (d) $0^\circ < \theta < 90^\circ$

Q3. The square of resultant of two equal forces is three times of their dot product, the angle between the forces is

- (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

Q4. A person can throw a ball up to a maximum range of 100 meter how high above the ground we can pour the same ball ?

- (a) 5 m (b) 200 m (c) 100 m (d) 50 m

Q5. A particle moves in a circle of radius 25 cm at two revolutions per second. The acceleration of the particle in m/s^2 is

- (a) π^2 (b) $2\pi^2$ (c) $4\pi^2$ (d) $8\pi^2$

Q6. If a particle has negative velocity and negative acceleration, its speed

- (a) Zero (b) decreases (c) remains same (d) increases

Q7. Identify the unit vector in the following.

- (a) $\hat{i} + \hat{j}$ (b) $\frac{\hat{i}}{\sqrt{2}}$ (c) $\hat{k} - \frac{\hat{j}}{\sqrt{2}}$ (d) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

Q8. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. The ratio of centripetal acceleration to acceleration due to gravity is ($g = 9.8 \text{ m/s}^2$)

- (a) 6.38 (b) 9.98 (c) 11.33 (d) 12.13

Q9. If the velocity is $\vec{v} = 2\hat{i} + t^2\hat{j} - 9\hat{k}$ then the magnitude of acceleration at $t = 0.5 \text{ s}$ is

- (a) 1 ms^{-2} (b) 2 ms^{-2} (c) zero (d) -1 ms^{-2}

Q10. Two bullets are fired simultaneously horizontally and with different speeds from the same place which bullet will hit the ground first

- (a) The faster one (b) depends on their mass
(c) The slower one (d) both will reach simultaneously

Q11. In uniform circular motion

- (a) Both velocity and acceleration are constant
(b) Acceleration and speed are constant but velocity changes
(c) Both acceleration and velocity change (d) both acceleration and speed are constant

Q12. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle the motion of the particle takes place in a plane it follows that

- (a) Its velocity is constant (b) Its acceleration is constant
(c) Its kinetic energy is constant (d) It moves along a straight line

Q13. Which one of the following is the correct expression of centripetal acceleration?

- (a) $4\pi^2 v^2 R$ (b) $\frac{4\pi^2 v^2}{R}$ (c) $2\pi v R$ (d) $\frac{2\pi v R}{R}$

Q14. An object is moving in a circle of radius 100m with a constant speed of 31.4m/s. What is its average speed for one complete revolution?

- (a) 0 (b) 314m/s (c) 3.14m/s (d) 31.4m/s

Q15. A particle moves in xy plane according to the equations $x = a \sin(\omega t)$ and $y = a \cos(\omega t)$ then, particle follows

- (a) An elliptical path (b) A circular path
(c) A Parabolic path (d) straight line path inclined equally to x and y axis

Q16. Two bullets are fired horizontally and simultaneously towards each other from roof tops of two buildings 100 m apart and of same height of 200m with the same velocity of 25 m/s. When and where will the two bullets collide? ($g = 10 \text{ m/s}^2$) [NEET – 2019]

- (a) After 2s at a height 180 m (b) after 2s at a height of 20 m
(c) After 4s at a height of 120 m (d) they will not collide

Q17. A car starts from rest and accelerates at 5 m/s^2 . At $t = 4 \text{ s}$, a ball is dropped out of a window by a person sitting in the car. What is the velocity and acceleration of the ball at $t = 6 \text{ s}$? (Take $g = 10 \text{ m/s}^2$) [NEET-2021]

- (a) 20 m/s , 0 (b) $20\sqrt{2} \text{ m/s}$, 0 (c) $20\sqrt{2} \text{ m/s}$, 10 m/s^2 (d) 20 m/s , 5 m/s^2

Q18. If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is: (NEET- 2016)

- (a) 0° (b) 90° (c) 45° (d) 180°

EXERCISE QUESTIONS

Q19. A projectile is launched into the air. At the highest point of its motion, its speed becomes half of what it was at the moment of release. At what angle to the ground was the projectile thrown?

- (a) 60° (b) 15° (c) 30° (d) 45°

Q20. The angle between $\vec{P} = \hat{i} + \hat{j}$ and $\vec{Q} = \hat{i} - \hat{j}$ is

- (a) 45° (b) 90° (c) -45° (d) 180°

Q21. A child throws a stone at an angle of 60° with the ground, giving it kinetic energy K . When the ball reaches the topmost point of its flight, what will be its kinetic energy?

- (a) 0 (b) $\frac{K}{2}$ (c) K (d) $\frac{K}{4}$

Q22. A hunter fires bullets at 30° and 45° with the horizontal respectively, reach the maximum height in the same time the ratio of their initial velocities is

- (a) $1 : \sqrt{2}$ (b) $2 : 1$ (c) $\sqrt{2} : 1$ (d) $1 : 2$

Q23. A ball is thrown into the air in such a way that the farthest horizontal distance it travels before landing is exactly equal to the highest vertical height it reaches. At what angle was the ball thrown?

- (a) $\theta = \left(\frac{1}{4}\right)$ (b) $\theta = (4)$ (c) $\theta = (2)$ (d) $\theta = 45$

SOLUTIONS OF MCQ:

Ans1. Option (d) is correct. Explanation: There is no acceleration in horizontal direction. Hence, the horizontal component of velocity remains unchanged.

Ans2: Option (d) here velocity is acting upwards when projectile is going upwards and acceleration is downwards. The angle between v and g is more than 0° and less than 180° .

Ans3: Option (d) As, $P=Q$ $R^2 = 3PQ$

$$\text{Also } R^2 = P^2 + Q^2 + 2PQ \cos \theta \Rightarrow 3P^2 = P^2 + Q^2 + 2PQ \cos \theta \Rightarrow \cos \theta = \frac{1}{2}$$

$$\text{Hence, } \theta = 60^\circ = \frac{\pi}{3}$$

$$\text{Ans4. Option (d) } R_{\max} = \frac{u^2}{g} = 100 \text{ m} \quad H_{\max} = \frac{u^2}{2g} = 50 \text{ m}$$

$$\text{Ans 5. Option (c) } r=25 \text{ cm}=0.25 \text{ m, } f=2 \text{ s}^{-1} \text{ and } a=\omega^2 r=(2\pi f)^2 r=(4\pi)^2 \times 0.25=16\pi^2 \times 0.25=4\pi^2$$

Ans 6. Option (d) An object which moves in the negative direction has a negative velocity. If the object is speeding up then its acceleration vector is directed in the same direction as its motion (in this case a negative acceleration) also increases

Ans 7. Option (d) Explanation magnitude of Unit vector= 1

$$\text{Magnitude of vector is given by; } |A^2| = \sqrt{A_x^2 + A_y^2 + A_z^2}; |\hat{i} + \hat{j}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\left| \frac{\hat{i}}{\sqrt{2}} \right| = \sqrt{\frac{1^2}{2}} = \frac{1}{2}; \left| \hat{k} - \frac{\hat{j}}{\sqrt{2}} \right| = \sqrt{1^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{3}{2}}; \quad (\text{D}) \left| \frac{\hat{i}+\hat{j}}{\sqrt{2}} \right| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

$$\text{Ans 8. Option (a) } a = \frac{v^2}{r} = 62.5 \text{ m/s}^2; a_c/g = 6.38$$

Ans 9. Option (a) Acceleration; $a = \frac{dv}{dt} = \frac{d}{dt} (2\hat{i} + t^2\hat{j} - 9\hat{k}) = 2t\hat{j}$

Magnitude of acceleration; $a = 2$; When $t=0.5$ s; $a = 2(0.5) = 1 \text{ m s}^{-2}$

Ans 10. Option (a) both will reach simultaneously As $t = \frac{\sqrt{2h}}{\sqrt{g}}$ Hence, t does not depend on horizontal speed

Ans 11. Option (c) In circular motion with constant speed, acceleration is always inward, its magnitude is constant but direction changes, hence acceleration changes, so does velocity

Ans 12. (C) Its kinetic energy is constant.

Ans 13. Option (A) is correct. As, $a_c = R\omega^2$ Putting $\omega = 2\pi v$ $a_c = 4\pi^2 v^2 R$

Ans 14. (D) Average speed = constant speed = 31.4 m/s. As during the circular motion speed remains constant but velocity changes continuously.

Ans 15. (B) As $x^2 + y^2 = a^2$ thus the particle follows a circular path

Ans 16. Option (a) Rel. speed = 25+25=50 m/s, $t = 1/2gt^2 = 2$ s: Vertical fall in 2 s: $1/2gt^2 = 20$ m

Ht. at collision = 200-20=180 m: meet midway (50 m from each building) aft 2 s at 180 m ht.

Ans 17. Option (c) Initial vel. of car = 0, $a = 5 \text{ m/s}^2$; at $t=4$ s; $v = 0+5(4) = 20 \text{ m/s}$; After 2s : $v_x = 20 \text{ m/s}$ (as $a_x=0$) And $v_y = 0+10(2) = 20 \text{ m/s}$ so mag of $v = 20\sqrt{2} \text{ m/s}$; at $t=6$ s, $g=10 \text{ m/s}^2$ (ball under free fall)

Ans 18. Option (b) $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ Applying $A^2+B^2+2AB\cos\theta = A^2+B^2-2AB\cos\theta$
 $\Rightarrow 4AB\cos\theta = 0$. Since A, B $\neq 0$ So $\cos\theta = 0 \Rightarrow \theta = 90^\circ$

ANSWERS OF EXERCISE QUESTIONS

Ans 11:(a)

Ans 12.: (b)

Ans 13: (d)

Ans 14: (c)

Ans 15: (b)

ASSERTION REASON BASED QUESTION

In the following questions a statement of assertion (A) is followed by a statement of reason(R) mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion

(b) If both assertion and reason are true but reason is the not correct explanation of assertion

(c) If assertion is true but reason is false

(d) If both assertion and reason are false

Q1. ASSERTION :- Imagine you throw a ball into the air. If you throw it twice as fast, it will stay in the air for twice as long before hitting the ground. For a Projectile, the time of flight becomes n times the original value if its speed of projection is made n times.

REASON: Because you threw it faster, the ball will also land twice as far away from where you started. For a projectile, due to this the range of the projectile which becomes n times

Q 2. ASSERTION : For uniform circular motion, the displacement and acceleration are directed towards the center along the radius.

REASON: Centripetal acceleration is represented as $a_c = \frac{\omega^2}{R}$

Q 3. ASSERTION: To throw a javelin as far as possible, the athlete should try to throw it at 45° angle with the ground. (Ignore height of the thrower and air resistance)

REASON: The distance a javelin travels doesn't change no matter what angle you choose to throw it at.

Q 4. ASSERTION: If $\vec{P} \cdot \vec{Q} = \vec{P} \cdot \vec{R}$ Then $\vec{P}$ may not always be equal to $\vec{R}$.

REASON: The dot product of two vectors involves sine of the angle between the two vectors.

Q5. ASSERTION: Two balls of different masses are thrown vertically upward with same speed, they will pass through their point of projection in the downward direction with the same speed.

REASON: The maximum height and downward velocity attained at the point of Projection are dependent of the mass of the ball.

Q 6. ASSERTION: The path of a projectile on earth is parabolic but for a satellite it is elliptical.

REASON: Acceleration due to gravity decreases with height from earth's surface.

Q 7. ASSERTION: Minimum number of non-equal vectors in a plane required to give zero resultant is three.

REASON: If $\vec{P} + \vec{Q} + \vec{R} = 0$, then they must lie in one plane.

Q 8. ASSERTION: If there were no gravitational force, the path of the projected body always is a straight line.

REASON: Gravitational force makes the path of projected body always parabolic.

Q 9. ASSERTION : In non-uniform circular motion, the total acceleration of the particle is the vector sum of the tangential acceleration and the centripetal acceleration.

REASON : The tangential acceleration (along the tangent) and the centripetal acceleration (along the radius) act perpendicular to each other.

Q 10. Assertion (A): A body moving with uniform speed in a circular path is said to be accelerating.

Reason (R): Acceleration is defined only as the rate of change of speed with time.

EXERCISE QUESTIONS

Q11. ASSERTION. For angular projection, when angle of projection $\theta = (1)$, the horizontal range Is four times the maximum height.

REASON. The horizontal range of projectile is directly proportional to square of velocity and inversely proportional to acceleration due to gravity.

Q12. ASSERTION (A): A projectile is projected at an angle θ and then at an angle $(90^\circ - \theta)$, keeping the initial velocity same. In both the cases, the range will be same.

REASON (R): Range of a projectile $R = u^2 \sin 2\theta / g$.

Q13. ASSERTION (A): The horizontal range of a projectile is maximum when projected at an angle of 45° with the horizontal.

REASON (R): At 45° , the horizontal and vertical components of the initial velocity are equal.

Q14. ASSERTION (A): A stone thrown vertically upward from a moving train follows a parabolic path as seen by an observer on the ground.

REASON (R): The stone has a horizontal velocity equal to that of the train when it is thrown.

Q15. ASSERTION (A): If dot product and cross product of $\vec{P}$ and $\vec{Q}$ are zero, it implies that one of the vector $\vec{P}$ and $\vec{Q}$ must be a null vector.

REASON (R): Null vector is a vector with zero magnitude..

SOLUTIONS

Ans 1. Option (c) If you double the speed, the ball stays in the air twice as long.

$$\text{As } T_f = \frac{2u \sin \theta}{g} \Rightarrow T'_f = \frac{2(nu) \sin \theta}{g} = nT_f$$

If you throw the ball twice as fast, it actually goes four times as far, not just twice as far.

$$R = \frac{u^2 \theta}{g} \Rightarrow R' = \frac{n^2 u^2 \theta}{g} \Rightarrow n^2 R, \text{ Hence A is true R is false}$$

Ans 2. Option (d) As during uniform circular motion the centripetal acceleration directed towards the center and velocity is along the tangent and in reason the formula is incorrect. Therefore both are incorrect

$$\text{Ans 3. Option (c) When a body is projected its range depends on } R = \frac{u^2 \sin 2\theta}{g}$$

Ans 4. Option (c) As the dot product's dependence on the angle between vectors.

Ans 5. Option (c) As motion is governed by force of gravity and acceleration due to gravity (g) is independent of mass of object

Ans 6. Option(b) As both assertion and reason are correct but reason is the explaining the assertion for both cases.

Ans 7. Option (a) is correct. The minimum number of non-equal vectors in a plane required to give a zero resultant is three and they must be coplanar.

Ans 8. Option (a) is correct . In the absence of gravity, once a body is projected, no vertical acceleration acts on it. The combination of horizontal constant velocity and vertical acceleration due to gravity results in a parabolic trajectory.

Ans 9. Option (a) In non-uniform circular motion, the total acceleration has two perpendicular components: tangential (speed change) and centripetal (direction change). Their vector sum gives the net acceleration.

Answer 10: Option (c) Assertion is true but Reason is false.

In uniform circular motion, velocity changes due to changing direction, so the body accelerates. Acceleration depends on change in velocity (not speed alone).

ANSWERS OF EXERCISE QUESTIONS

Ans 11: (b) Ans 12.: (a) Ans 13: (a) Ans 14: (a) Ans 15: (b)

SHORT ANSWER QUESTIONS (2 MARKS)

Q1. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

Q2. Check whether the vectors are orthogonal or not $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 4\hat{i} - 5\hat{j}$

Q3. Calculate the angle θ subtended by the two adjacent wooden spokes of a bullock cart wheel. Express the angle in both radian and degree.

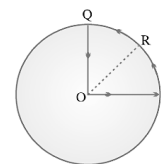
Q4. A football is kicked into the air vertically upwards. Find its

(a) Acceleration (b) velocity at the highest point?

Q5. A projectile achieves a maximum range that is $\frac{2}{\sqrt{3}}$ times its actual range. What is the angle of projection corresponding to the actual range?

Q6. What is the angle between the direction of velocity and acceleration at the highest point of projectile path?

Q7. A cyclist starts from center O of a circular park of radius 1 km and moves along the path OPRQO as shown. If he maintains constant speed of 10 m/s, what is his acceleration at point R in magnitude & direction?



Q8. What do you mean by rectangular components of a vector? Explain how a vector can be resolved into two rectangular components in a plane?

Q9. A fountain installed at ground level sprays water uniformly in horizontal direction with speed v from height h . Neglecting air resistance, what ground area around it gets wet?

Q10. Find the angle between two vectors $\vec{P}$ & $\vec{Q}$ if resultant over the vector is given by $R^2 = P^2 + Q^2$

EXERCISE QUESTIONS

Q11. On a certain day rain was falling vertically with the speed of 35 ms^{-1} a wind started blowing after some time with the speed of 12 ms^{-1} in east to west direction in which direction should a boy waiting at a bus stop hold his umbrella.

Q12. Find the area of a parallelogram formed by vectors $\vec{A} = 3\hat{i} + 2\hat{j}$ and $\vec{B} = -3\hat{i} + 7\hat{j}$

Q13. A stone tied at the end of the string is whirled in a circle. State the reason why the stone flies away tangentially when the string break

Q14. Define uniform circular motion? Also define time period, frequency and angular velocity? Write the relationship between them.

Q15. A basketball player takes two shots at the same angle. In the second shot, the ball is thrown with twice the initial speed as the first. If both land at the same height, by what factor does the horizontal range change, and why?

SOLUTIONS

Ans 1. As, $= \frac{u^2 \theta}{g}$; $R = R_{\max}$, $\theta = 45^\circ$, i.e., $\theta = \sin 90^\circ = 1$; $R_{\max} = \frac{u^2}{g}$

Suppose H = height upto which the ball goes when the cricketer throws it with velocity u . Since, the final velocity of the ball, $v = 0$.

$v^2 - u^2 = 2as$ here, $v = 0$, $a = -g$, $s = H$; $H = \frac{1}{2} \frac{u^2}{g} = 50 \text{ m}$

Ans 2. Condition for orthogonal vectors; $\vec{A} \cdot \vec{B} = 0$

$\vec{A} \cdot \vec{B} = (2\hat{i} + 3\hat{j}) \cdot (4\hat{i} - 5\hat{j}) = 8(\hat{i} \cdot \hat{i}) - 15(\hat{j} \cdot \hat{j}) = \vec{A} \cdot \vec{B} = 8 - 15 = -7$

As, $\vec{A} \cdot \vec{B} \neq 0$. Hence $\vec{A}$ and $\vec{B}$ are not orthogonal to each other.

Ans 3. 12 spokes divide the wheel into 12 equal arcs.

Central angle of full circle $= 2\pi$ rad. $\theta = 2\pi/12 = \pi/6$ rad $= 30^\circ$ so Angle between adjacent spokes $= 30^\circ$ & $\pi/6$ rad.

Ans 4. (a) Acc. will always be vertical downward & is called acceleration due to gravity (g).

(b) When football reaches the highest point it is momentarily at rest velocity, $v=0$. As, it is continuously retarded by acceleration due to gravity (g).

Ans 5. $R_{\max} = \frac{2}{\sqrt{3}} R \Rightarrow \frac{u^2}{g} = \frac{2}{\sqrt{3}} \frac{u^2 \theta}{g} \Rightarrow \theta = \frac{\sqrt{3}}{2} = \sin 60^\circ \therefore \theta = 30^\circ$

Ans 6. 90° . At the highest point the vertical component of velocity becomes zero the projectile has only a horizontal velocity while the acceleration due to gravity acts vertical downwards

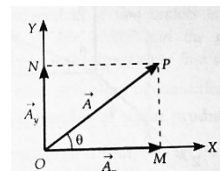
Ans 7. Centripetal acceleration, $a = \frac{v^2}{r} = \frac{10^2}{1000} = 0.1 \text{ ms}^{-2}$ along RO

Ans 8. When a vector is resolved along 2 mutually perpendicular directions the components so obtained are called a rectangular component of the given vector as shown in the figure

Here $\vec{A}_x = A_x \hat{i}$ & $\vec{A}_y = A_y \hat{j}$ Also, $\vec{A} = A_x \hat{i} + A_y \hat{j}$

$$A_x = A \cos \theta \quad \& \quad A_y = A \sin \theta \quad \Rightarrow \quad A = \sqrt{A_x^2 + A_y^2}$$

$$\tan \theta = \frac{A_y}{A_x}$$



Ans 9. Let R be the maximum Horizontal range, then $R = \frac{v^2}{g}$

Since the water sprinkle on the ground around the fountain in circular area of radius $= \pi R^2$

$$= \pi \left(\frac{v^2}{g} \right)^2 = \frac{\pi v^4}{g^2}$$

Ans 10. As we know that $R^2 = P^2 + Q^2 + 2PQ \cos \theta$; Given $R^2 = P^2 + Q^2$

$$P^2 + Q^2 = P^2 + Q^2 + 2PQ \cos \theta ; 2PQ \cos \theta = 0 \Rightarrow \cos \theta = 0 \Rightarrow \theta = 90^\circ$$

ANSWERS OF EXERCISE QUESTIONS

Ans 11. $v = 37 \text{ ms}^{-1}$ & $\theta \cong 19^\circ$

Ans 12. 27 sq units

Ans 13: When a stone is tied at the end of a string that moves in a circular path, its velocity is tangent to the circle.

Ans 14: The motion is considered to be a uniform circular motion when an object rotates in a circular path with the same speed. The total time taken by the object to complete one revolution is considered a time period. The total number of revolutions completed per second is termed "frequency." The angular velocity is defined as the time rate of change of angular displacement.

Ans 15: Becomes 4 times

SHORT ANSWER QUESTIONS (3 MARKS)

Q1. The sum & difference of two vectors are equal in magnitude. Mathematically prove that they are mutually perpendicular to each other.

Q2. Prove that the horizontal range is same when angle of projection is

(i) greater than 45° by certain value and (ii) less than 45° by the same value.

Q3. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r .

Q5. Consider the x-axis as representing east, the y-axis as north and z-axis as vertically upwards. Give the vector representing each of the following points.

(a) 5 m north east and 2 m up (b) 4 m south east and 3 m up (c) 2 m north west and 4 m up

Q6. The resultant of two forces whose magnitudes are in the ratio 3:5 is 28 N. If the angle of their inclination is 60° , then find the magnitude of each force

Q7. At what angle do the two forces $(P + Q)$ and $(P - Q)$ act so that the resultant is $\sqrt{3P^2 + Q^2}$.

Q8. Find the angle between the vectors $\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = -\hat{i} + \hat{j} - \hat{k}$?

Q9. A boy stands at 39.2 m from a building and throws a ball which just passes through a window 19.6 m above the ground. Calculate the velocity of projection of the ball.

EXERCISE QUESTIONS

Q10. Find the angle of projection for which the horizontal range & maximum height are equal

Q11. A football is kicked at an angle of 30° with the vertical if the horizontal component of its velocity is 19.6 ms^{-1} find the maximum height and horizontal range.

Q12. Calculate the angular speed of the (i) hour hand of a watch (ii) the Earth about its own axis. Also find the ratio of the two speeds

Q13. An insect is trapped in a circular groove of radius 12 cm. It moves with uniform speed along the groove and completes 7 revolutions in 100 s.

(i) Calculate the angular speed and the linear speed of the insect.

(ii) Determine the acceleration vector acting on the insect. Is it a constant vector? Also calculate its magnitude.

(iii) What is the displacement of the insect after 100 s?

Q14. A fighter plane is flying horizontally at an altitude of 1.5 km with speed 720 kmh^{-1} . At what angle of sight (w.r.t horizontal) when the target is seen, should the pilot drop the bomb in order to attack the target? (Take $g=10 \text{ ms}^{-2}$)

SOLUTIONS: -

Ans 1: Given $\vec{A} + \vec{B} = \vec{A} - \vec{B}$

Taking square on both sides $(\vec{A} + \vec{B})^2 = (\vec{A} - \vec{B})^2 \Rightarrow A^2 + B^2 + 2\vec{A}\vec{B} = A^2 + B^2 - 2\vec{A}\vec{B}$

$4\vec{A} \cdot \vec{B} = 0 \Rightarrow \vec{A} \cdot \vec{B} = 0$; Therefore these two vectors are perpendicular to each other.

Ans 2. (i) When the angle of projection is $\theta = 45 + \alpha$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin(45 + \alpha)}{g} = \frac{u^2 \sin \alpha \cos(90 + 2\alpha)}{g} = \frac{u^2 \cos \alpha \cos(2\alpha)}{g}$$

(ii) When the angle of projection is $\theta = 45 - \alpha$

$$R' = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin(45 - \alpha)}{g} = \frac{u^2 \sin \alpha \cos(90 - 2\alpha)}{g} = \frac{u^2 \cos \alpha \cos(2\alpha)}{g} \quad \text{Clearly } R = R'$$

Ans 3. Centripetal Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration. Magnitude of centripetal acceleration

$$|\vec{a}| = \frac{|\Delta \vec{v}|}{\Delta t} = \frac{1}{\Delta t} \frac{v |\Delta \vec{r}|}{r} = \frac{v}{r} \frac{|\Delta \vec{r}|}{\Delta t}$$

$$|\vec{a}| = \frac{v}{r} \left(\because v = \frac{|\Delta \vec{r}|}{\Delta t} \right)$$

Ans 5.(a) From figure x - component $= 5 \cos 45^\circ \hat{i} = 5 \sqrt{2} \hat{i}$

y - component $= 5 \sin 45^\circ \hat{j} = 5 \sqrt{2} \hat{j}$

z - component $= 2 \hat{k}$

Then position vector $\vec{r} = \frac{5}{\sqrt{2}} (\hat{i} + \hat{j}) + 2 \hat{k}$

(b) ||y, vector $\vec{r} = \frac{4}{\sqrt{2}} (\hat{i} - \hat{j}) + 3 \hat{k}$

(c) ||y, vector $\vec{r} = \sqrt{2} (\hat{i} + \hat{j}) + 4 \hat{k}$

Ans6. Let F_1 and F_2 be the two forces.

Then $F_1 = 3x, F_2 = 5x, R = 28 \text{ N}$ and $\theta = 60^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

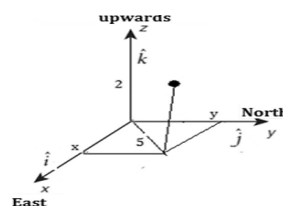
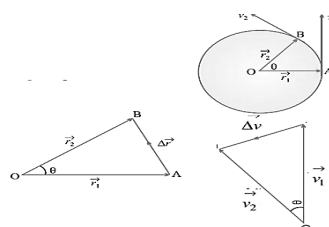
$$\Rightarrow 28 = \sqrt{(3x)^2 + (5x)^2 + 2(3x)(5x) \cos 60^\circ} \Rightarrow 7x = 28 \Rightarrow x = 4; \therefore F_1 = 12 \text{ N}, \quad F_2 = 5 \times 4 = 20 \text{ N}$$

Ans 7. Let F_1 and F_2 be the two forces.

$$\text{Then } F_1 = P + Q, F_2 = P - Q, \quad R = \sqrt{3P^2 + Q^2} \quad R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \theta$$

$$\Rightarrow P^2 - Q^2 = 2(P^2 - Q^2) \cos \theta \Rightarrow \cos \theta = \frac{1}{2} \therefore \theta = 60^\circ$$

$$\text{Ans 8. } |\vec{A}| = \sqrt{1^2 + 2^2 + (-1)^2} = \sqrt{6}; \quad |\vec{B}| = \sqrt{(-1)^2 + 1^2 + (-2)^2} = \sqrt{6}$$



$$\vec{A} \cdot \vec{B} = 1 \times (-1) + 2 \times 1 + (-1) \times (-2) = -1 + 2 + 2 = 3$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{3}{\sqrt{6} \sqrt{6}} = \frac{3}{6} = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

Ans 9. $H=19.6$ m $R=39.2+39.2=78.4$ m

$$\frac{H}{R} = \frac{u^2 \theta}{2g} \times \frac{g}{u^2 \theta} = \frac{1}{4} \tan \theta \Rightarrow \tan \theta = \frac{4H}{R} = \frac{4 \times 19.6}{78.4} = 1$$

$$\therefore \theta = 45^\circ \quad \text{As, } H = \frac{u^2 \theta}{2g} \quad 19.6 = \frac{u^2 \times 45}{2 \times 9.8} \Rightarrow u^2 = 4 \times 19.6 \quad u = \sqrt{78.4} \cong 8.85 \text{ ms}^{-1}$$

ANSWERS TO EXERCISE QUESTIONS

Ans 10. $\theta = 75^\circ 58'$ Hint : Horizontal Range = maximum height

Ans 11. Maximum height = 58.8 m

Horizontal Range = 135.8 m

Hint: $\theta = 90^\circ - 30^\circ = 60^\circ$ & find u

Ans 12. $\omega_{\text{hour hand}} : \omega_{\text{Earth}} = 2 : 1$

Hint : Hour hand of a watch completes one revolution in 12 hours and Earth complete one revolution in 24 hours & use $\omega = \frac{\theta}{t}$

Ans 13. Angular speed = 0.44 rad s^{-1} ; Linear speed = 5.28 ms^{-1} ; acceleration = 2.32 cms^{-1}

Linear displacement = 0

Ans 14. Angle of sight $\alpha = \left(\frac{\sqrt{3}}{4} \right)$ Hint: $\alpha = \left(\frac{H}{R} \right)$

LONG ANSWER TYPE QUESTIONS (5 MARKS)

Q1. (a) Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r .

(b) A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

Q 2. The position vector of a particle is given by $\vec{r} = 3t\hat{i} - 2t^2\hat{j} + 4\hat{k}$ m where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.

(a) Find the $\vec{v}$ and $\vec{a}$ of the particle?

(b) What is the magnitude and direction of velocity of the particle at $t = 2.0$ s?

Q 3. Two vectors $\vec{A}$ and $\vec{B}$ are inclined to each other at an angle θ . Using parallelogram law of vector addition, find the magnitude and direction of their resultant. Calculate the magnitude and gives direction of R when

(i) $\vec{A}$ and $\vec{B}$ act along the same straight line in the same direction.

(ii) $\vec{A}$ and $\vec{B}$ act along the same straight line but in opposite directions.

(iii) $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Q 4. (a) Define resolution of a vector with an example ?

(b) A projectile can have the same range R for two angles of projection. If t_1 & t_2 be the time of flight in the two cases then prove that $t_1 t_2 = \left(\frac{2R}{g} \right)$

Q 5. (a) An object is thrown with an initial velocity upwards and follows a curved path under the influence of gravity alone. What motion is used to describe motion of such an object? State the key assumptions for analyzing motion of object.

(b) Derive the mathematical equation representing the trajectory of the object. obtain expressions (i) maximum vertical height (ii) horizontal distance (iii) Time of flight

EXERCISE QUESTIONS (5 MARKS)

Q6 (a) Define centripetal acceleration. What will be the direction and velocity?

(b) Using diagram derive an expression for the centripetal acceleration of a particle moving with uniform velocity v along a circular path of radius r .

(c) A cyclist is riding with speed of 27km/h. As he approaches a circular turn on the road of radius 80m, he applies breaks and reduces his speed at constant rate 0.50m/s^2 . What is the magnitude of the net acceleration of the cyclist on the circular turn?

Q 7. Prove the following:

(a) For a projectile launched with the same speed v , but at two complementary angles of projection θ & $90 - \theta$ (i) Range is same, (ii) Heights are in the ratio $\tan 2\theta : 1$.

(b) If the range and maximum height are equal, then angle of projection is $\tan^{-1}(4)$.

Q 8. Using analytical method of resolution of vectors deduced the resultant of two vectors.

The position of a particle is given by $\vec{r} = (7t\hat{i} - 5t^2\hat{j} + 4t\hat{k})$ m where t is in seconds and $\vec{r}$ to be in meters. (a) Find the velocity and acceleration

(b) What is the magnitude and direction of velocity at $t=2\text{s}$?

SOLUTIONS

Ans 1. (a) Solution given on Q 4 of SAQ (3 M)

(b) Radius of the horizontal circle, $r = 80\text{ cm} = 0.80\text{ m}$, frequency $= f = \frac{14}{25}\text{ rps}$

Angular speed of revolution of the stone is $\omega = 2\pi f \Rightarrow \omega = 2 \times 22.7 \times \frac{14}{25} \Rightarrow \omega = \frac{88}{25}\text{ rad/s}$ Magnitude of acceleration produced in the stone will be equal to the magnitude of centripetal acceleration. $= r\omega^2 = 0.80 \times \left(\frac{88}{25}\right)^2 = 9.90\text{ ms}^{-2}$

We know that, the direction of the acceleration is towards the centre of the circle along its radius.

Ans 2. Ans 2.(a) As $\vec{r} = 3t\hat{i} - 2t^2\hat{j} + 4t\hat{k}$ Also, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3t\hat{i} - 2t^2\hat{j} + 4t\hat{k}) = 3\hat{i} - 4t\hat{j}\text{ ms}^{-1}$

Also, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(3\hat{i} - 4t\hat{j}) = -4\hat{j}\text{ ms}^{-2}$

(b) velocity of the particle $\vec{v} = 3\hat{i} - 4t\hat{j}\text{ ms}^{-1}$ At $t=2\text{ s} \Rightarrow 3\hat{i} - 4 \times 2\hat{j} = 3\hat{i} - 8\hat{j}$

Magnitude $\vec{v} = \sqrt{3^2 + (-8)^2} = \sqrt{9 + 64} = \sqrt{73} = 8.544\text{ ms}^{-1}$

Direction $\theta = \left(\frac{v_y}{v_x}\right) = \left(\frac{-8}{3}\right) = -70$ with x axis

Ans 3. When two vectors A and B are inclined at an angle θ , the magnitude R of their resultant is given by $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Direction is given by $\theta = \left(\frac{\vec{B} \sin\theta}{\vec{A} + \vec{B} \cos\theta}\right)$

1. $\theta = 0^\circ$ (Vectors in the same direction):

Magnitude: $R = \sqrt{A^2 + B^2 + 2AB}$ Direction: Same as A or B

2. $\theta = 180^\circ$ (Vectors in opposite directions):

Magnitude $R = \sqrt{A^2 + B^2 - 2AB}$ Direction: Along the larger vector

3. $\theta = 90^\circ$ (Perpendicular vectors):

Magnitude: $R = \sqrt{A^2 + B^2}$ Direction $\theta = \left(\frac{\vec{B}}{\vec{A}}\right)$

Ans 4. (a) Resolution of a Vector

Resolution of a vector is the process of decomposing a vector into two or more components along specified directions (e.g., horizontal and vertical). $A_x = A\cos\theta$ & $A_y = A\sin\theta$

Example:-Flying of bird, Motion of catapult etc.

(b) Projectile Range and Time of Flight Relationship

$\theta_1 = \theta$ & $\theta_2 = 90 - \theta$ $t_1 = \frac{2u\sin\theta}{g}$, $t_2 = \frac{2u\sin\theta_2}{g} = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$

$t_1 t_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g} = \frac{4u^2 \sin\theta \cos\theta}{g^2} = \frac{2}{g} \left(\frac{u^2 \sin 2\theta}{g}\right)$; $t_1 t_2 = \frac{2R}{g}$

Ans 5.(a) Projectile motion Assumptions:-Acceleration due to gravity (g) is constant and Air resistance is negligible.

(b) Derivation $H = \frac{u^2 \sin^2\theta}{2g}$; $T = \frac{2u\sin\theta}{g}$; $R = \frac{u^2 \sin 2\theta}{g}$

EXCERSISE SOLUTION

Ans 6. (a) Correct definition (b) Correct derivation (c) 0.86m/s^2 Hint $a = \sqrt{a_c^2 + a_t^2}$

Ans 7. (a) Correct Proof Hint $R = \frac{u^2 \sin 2\theta}{g}$ (b) Correct Proof Hint $H = \frac{u^2 \theta}{2g}$

(c) Correct Proof Hint $\alpha = \left(\frac{H}{R}\right)$

Ans 8. (a) Velocity: $\vec{v}(t) = 7\hat{i} - 10t\hat{j} \text{ m/s}$ Acceleration: $\vec{a}(t) = -10\hat{j} \text{ m/s}^2$

(b) Velocity at $t=2 \text{ s}$; Magnitude of velocity: 21.19 m/s ; Direction: 70.9° below the x-axis

CASE STUDY BASED QUESTIONS

Q 1. Projectile Motion in the era of war:

During a battle, soldiers need to hit a bunker on top of a hill. The bunker is 200 m away and 50 m higher than where the mortar is placed. They fire a mortar shell with a speed of 80 m/s at an angle of 30° above the ground. Take $g = 10 \text{ m/s}^2$ and ignore air resistance.

Q1.(i) What is the maximum height attained by the mortar shell?

- (a) 80 m (b) 160 m (c) 40 m (d) 320 m

(ii) What would be the horizontal range of the mortar if launched on level ground (same velocity and angle) ?

- (a) 3203 m (b) 554.2 m (c) 640 m (d) 160 m

(iii) What is the time of flight for the mortar shell if launched on level ground?

- (a) 8 s (b) 16 s (c) 4 s (d) 10 s

(iv) To ensure the mortar shell reaches the elevated bunker (50 m height), which adjustment is most critical?

- (a) Increase the launch angle (b) Decrease the launch angle
(c) Increase the launch velocity (d) Both B and C

Q2. While practicing on the field, a student launches a javelin high into the air like a flying spear. It leaves the hand at a speed of 32 m/s, tilted 35° above the ground, from a height of 2 m. Use $g = 9.8 \text{ m/s}^2$, ignore air resistance, and take $\cos(35^\circ) \approx 0.8192$.

(i) What is the horizontal component of the initial velocity?

- (a) 26.2 m/s (b) 22.6 m/s (c) 18.7 m/s (d) 20.1 m/s

(ii) What is the maximum height (from the ground) reached by the javelin during its flight?

- (a) 15.7 m (b) 17.2 m (c) 19.3 m (d) 21.6 m

(iii) At the max height of the javelin's trajectory, which of the following statements is TRUE?

- (a). Both horizontal and vertical components of velocity are zero
(b). Only the vertical component of velocity is zero
(c). Only the horizontal component of velocity is zero
(d). The javelin comes to a complete stop momentarily

(iv) Which factor determines how long the javelin stays in the air (time of flight), assuming no air resistance?

- (a) Only the horizontal component of velocity (b) Only the angle of projection
(c) Only the vertical component of velocity and initial height
(d) Both horizontal and vertical components equally

SOLUTIONS

Ans 1(i) Option A; $H = \frac{u^2 \theta}{2g} = \frac{80^2 30}{2 \times 10} = 80\text{m}$.

(ii) Option (b) is correct. $R = \frac{u^2 \sin 2\theta}{g} = \frac{80^2 (30)}{10} = \frac{6400 \sqrt{3}}{10 \times 2} = 554.2\text{m}$.

(iii) Option (a) is correct. $T = \frac{2u \sin \theta}{g} = \frac{2 \times 80 \sin 30}{10} = 8\text{s}$

(iv) Option (d) is correct. For an elevated target, reducing the angle or increasing the velocity compensates for the additional vertical displacement. A lower angle prioritizes horizontal distance, while higher velocity extends the range.

Ans 2 (i) (a) $u_x = u \cos \theta = 32 \cos(35^\circ) \approx 26.2 \text{ m}$

2(ii) (c) $H = \frac{u^2}{2g} - \frac{(18.4)^2}{2 \times 9.8} \approx 17.3 \text{ m}$; total Height from Ground = $17.3 + 2 = 19.3 \text{ m}$

2(iii) (b) because no external force acts in the horizontal direction

2(iv) (c) Only the vertical component of velocity and initial height

EXERCISE QUESTION

A basketball shot is a projectile and follows a parabolic path. Gravity pulls the ball down while the player's push sends it forward. The path up and down is similar unless the ball hits the backboard or a player. Knowing the player's height, distance from the hoop, and hoop height helps choose the right angle and speed: At 2 m distance (player height 1.27 m, hoop 3.05 m): 55° , 7 m/s At 6 m distance: 60° , 9.5 m/s Players add backspin so that if the ball hits the rim during a soft shot, it drops into the hoop instead of bouncing out.

1. The trajectory of a basketball is always a

(A) Straight line (B) Circle (C) Parabola (D) May be a circle, may be a parabola

2. When the basketball deviates from its parabolic path?

(A) If the ball collides with the backboard

(B) If the ball collide with another player interfering with its path

(C) If a backspin is applied to the ball (D) Both (A) and (B)

3. To calculate the speed to be imparted to the ball and the angle for an accurate shot the required information are

(A) Height of the player

(B) Horizontal distance of the hoop

(C) Height of the hoop

(D) All of the above.

4. If the horizontal distance of the hoop from a player increases

(A) Velocity and angle both should increase (B) Velocity and angle both should decrease

(C) Velocity should increase but angle should decrease

(D) Velocity should decrease but angle should increase

Solutions

1. C

2. D

3. D

4. A

Physics is **NOT** about memorising. It's about understanding **MOTION!**

MOTION IN A PLANE

CLASS XI PHYSICS – FORMULA SHEET

QUICK RECALL
Motion in a plane involves two or more independent motions combined using **VECTOR ALGEBRA**

1 INTRODUCTION TO MOTION IN A PLANE

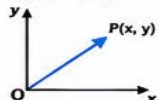
Scalars vs Vectors

- ✓ Scalars → have magnitude only.
- ✓ Vectors → have magnitude and direction.



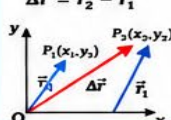
Position Vector ($\vec{r}$)

Vector from origin O to point P(x, y)



Displacement ($\Delta\vec{r}$)

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$



Velocity Vector ($\vec{v}$)

Rate of change of position

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

Acceleration Vector ($\vec{a}$)

Rate of change of velocity

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

Magnitude of Vector

If $\vec{A} = A_x\hat{i} + A_y\hat{j}$
 $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

Quick Recall

Vector → Directed quantity
Add using rules of vector algebra.



2 VECTOR ALGEBRA & VECTOR RESOLUTION

Unit Vector

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

Rectangular Components

$$\begin{aligned}\vec{A} &= A_x\hat{i} + A_y\hat{j} \\ A_x &= A \cos \theta \\ A_y &= A \sin \theta\end{aligned}$$

Vector in $\hat{i}, \hat{j}$ form

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

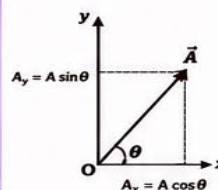
Dot Product

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

Resultant of Two Vectors

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

Resolution of Vector



Triangle Law



Parallelogram Law



Direction (ϕ) of Resultant

$$\tan \phi = \frac{B \sin \theta}{A + B \cos \theta}$$

Memory Trick

"Cos → x-axis
Sin → y-axis"

3 MOTION IN A PLANE

Independent x-y Motion

Motion along each axis is independent.



Resultant Velocity

$$v = \sqrt{v_x^2 + v_y^2}$$

Equations of Motion

Along x-axis (no acceleration)

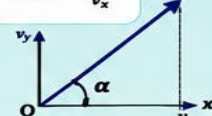
- $x = x_0 + u_x t$
- $v_x = u_x$ (constant)

Along y-axis (acceleration = -g)

- $y = y_0 + u_y t - \frac{1}{2}gt^2$
- $v_y = u_y - gt$
- $a_y = -g$

Direction

$$\tan \alpha = \frac{v_y}{v_x}$$

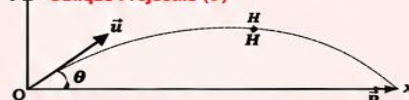


Key Point

Horizontal motion → uniform
Vertical motion → uniformly accelerated by g downwards.

4 PROJECTILE MOTION ★ MOST IMPORTANT Ⓢ PYQ Favourite

Oblique Projectile (θ)



Initial Components

$$\begin{aligned}u_x &= u \cos \theta \\ u_y &= u \sin \theta\end{aligned}$$

Equations of Motion

- $x = u \cos \theta t$
- $y = u \sin \theta t - \frac{1}{2}gt^2$
- $v_x = u \cos \theta$ (constant)
- $v_y = u \sin \theta - gt$
- $v = \sqrt{v_x^2 + v_y^2}$
- $\tan \alpha = \frac{v_y}{v_x}$

Important Results

$$\begin{aligned}\text{Time of Flight: } T &= \frac{2u \sin \theta}{g} \\ \text{Maximum Height: } H &= \frac{u^2 \sin^2 \theta}{2g} \\ \text{Range: } R &= \frac{u^2 \sin 2\theta}{g} \\ \text{Trajectory Equation: } y &= x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}\end{aligned}$$

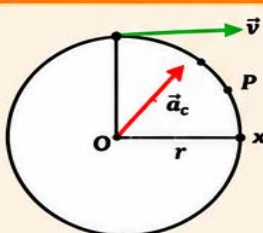
Special Cases

- ✓ $\theta = 45^\circ \rightarrow R$ is maximum
- ✓ $\theta = 90^\circ \rightarrow H$ is maximum
- ✓ θ and $(90^\circ - \theta)$ give same R.

Other Types

- ✓ Horizontal ($\theta = 0^\circ$):
 $x = ut, y = 0$
- ✓ Vertical ($\theta = 90^\circ$):
 $x = 0, y = ut - \frac{1}{2}gt^2$

5 UNIFORM CIRCULAR MOTION



Angular Displacement (θ)
 $\theta = \frac{s}{r}$ (radians)

Angular Velocity (ω)
 $\omega = \frac{d\theta}{dt} = \frac{v}{r}$

Linear Velocity (v)
 $v = r\omega$

Centripetal Acceleration (a_c)
 $a_c = \frac{v^2}{r} = r\omega^2$ (towards centre)

Time Period (T)

$$T = \frac{2\pi}{\omega}$$

Frequency (f)

$$f = \frac{1}{T}$$

Relation

$$\omega = 2\pi f$$



Remember: In uniform circular motion, speed is constant but velocity & acceleration keep changing direction.

6 QUICK RECALL FORMULA BOX

Projectile Motion

- $x = u \cos \theta t$
- $y = u \sin \theta t - \frac{1}{2}gt^2$
- $v_x = u \cos \theta$
- $v_y = u \sin \theta - gt$
- $v = \sqrt{v_x^2 + v_y^2}$
- $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$

Circular Motion

- $v = r\omega$
- $a_c = \frac{v^2}{r}$
- $T = \frac{2\pi}{\omega}$
- $f = \frac{1}{T}$
- $\omega = 2\pi f$

Vector Formulas

- $\vec{A} = A_x\hat{i} + A_y\hat{j}$
- $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$
- $\vec{A} \cdot \vec{B} = AB \cos \theta$
- $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$
- $\tan \phi = \frac{B \sin \theta}{A + B \cos \theta}$

Graph Relations

- Slope (x-t) = v
- Slope (v-t) = a
- Area under v-t graph = displacement
- Area under a-t graph = change in v

7 MEMORY TRICKS + COMMON MISTAKES



- ✓ Cos with x-axis
- ✓ Sin with y-axis
- ✓ θ & $(90^\circ - \theta) \rightarrow$ same R
- ✓ Resultant uses cosine (cos)

Common Mistakes

- ✗ Forgetting direction of vectors
- ✗ Confusing speed with velocity
- ✗ Using $g = +9.8 \text{ m/s}^2$ in equations
- ✗ Not resolving vectors into components



PYQ Focus Areas

- Master:
- Oblique projection
 - Time of flight
 - Maximum height
 - Range

★ MASTER THE BASICS. UNDERSTAND THE MOTION. ★

Newton's Laws of Motion

The Mechanics of Momentum and Friction

THE THREE PILLARS: NEWTON'S LAWS OF MOTION



The First Law: The Law of Inertia

An object remains at rest or in uniform motion in a straight line unless acted upon by an external, unbalanced force.

The Second Law:

$$F = ma$$



The net force on an object is equal to its mass multiplied by its acceleration; acceleration is directly proportional to force and inversely proportional to mass.



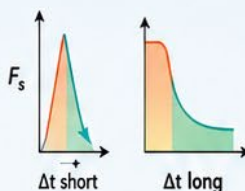
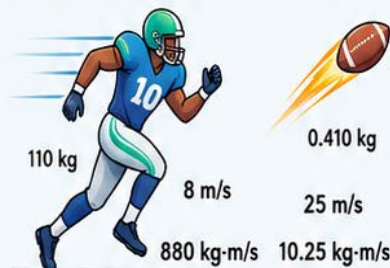
The Third Law: Action and Reaction

For every action, there is an equal and opposite reaction; forces always exist as interactions between two different objects.

MOMENTUM AND THE POWER OF IMPULSE

Linear Momentum ($p = mv$)

Described as the "quantity of motion," momentum is the product of a system's mass and its velocity.



Impulse-Momentum Theorem

$$\Delta p = F_{net} \cdot \Delta t$$

Impulse is the product of the average net force and the time it acts ($\Delta p = F_{net} \cdot \Delta t$), which is equal to the change in an object's momentum.

Impulse-Momentum Theorem

$$\Delta p = F_{net} \cdot \Delta t$$

Airbags save lives by increasing the time (Δt) over which a stop occurs, which significantly decreases the net force acting on the occupant to action its momentum.

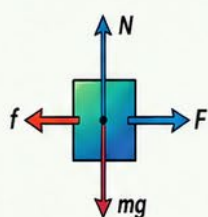
Engineering Safety: The Airbag Effect

Airbags save lives by increasing the time (Δt) over which a stop occurs, which significantly decreases the net force acting on the occupant.



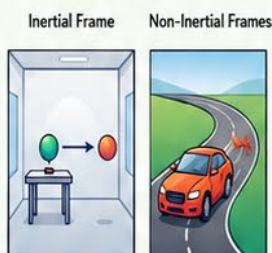
ANALYZING SYSTEMS: TOOLS AND FRAMES

Free Body Diagrams (FBD)



Visual tools used to show the relative magnitude and direction of all forces—like gravity (mg), normal force (N), and friction (f)—acting on a single object.

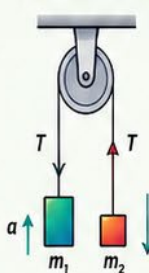
Inertial vs. Non-Inertial Frames



Inertial vs. Non-Inertial Frames

Inertial frames are non-accelerating environments where Newton's laws hold true; non-inertial frames are accelerating (like a turning car) where "fictitious" forces arise.

The Atwood Machine



The Atwood Machine

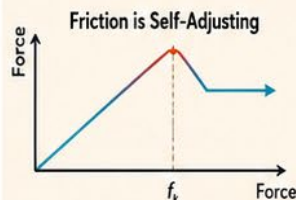
A classic laboratory device consisting of two masses connected by a string over a pulley, used to verify laws of constant acceleration and measure gravity (g).

FRICITION: THE FORCE OF RESISTANCE

The Friction Hierarchy: $f_s < f_k < f_r$

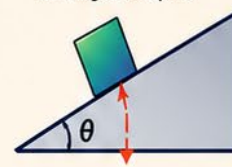


Rolling friction is the weakest, followed by kinetic sliding friction; static friction is strongest—friction being the strongest force to overcome.



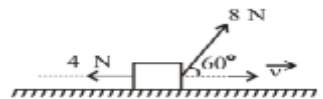
Static friction increases to match the applied force until it reaches its "limiting force," f_s , which point the object begins to move.

The Angle of Repose



The critical angle of an inclined plane at which an object just begins to slide down due to gravity overcoming static friction.

MULTIPLE CHOICE QUESTIONS-

- 1) A body of mass 10 kg is moving with a velocity of 5m/s. The momentum of the body is-
(a) 10 kgm/s. (b) 5 kgm/s. (c) 50 kgm/s. (d) 25 kgm/s
- 2) When the velocity is tripled the banking angle should be
(a) 1/3 (b) 1/9 (c) 3 times (d) 9 times
- 3) A stone of mass 0.5kg tied to the end of the string in whirled round in the circle of radius 0.5m with a speed of 4rps in a horizontal plane. The tension in the string-
(a) 2N (b) 4N (c) 5N (d) 20N
- 4) A body of mass m is moving with an acceleration ' a ' under the action of a force ' $g/4$ '. The weight of the body is-
(a) $g/4a$ (b) $g/4a$ (c) $g^2/2a$ (d) $g^2/4a$
- 5) Force produces an acceleration of a_1 in a body and the same force produces an acceleration of a_2 which is twice to a_1 in another body. If the two bodies are combined and the same force is applied on the combination, the acceleration produced in it is-
(a) $(3/2) a_1$ (b) $(2/3) a_1$ (c) $(3/2) a_2$ (d) $(2/3) a_2$
- 6) A cyclist takes a turn on a circular road of radius 20m with a speed of 10m/s. The centripetal acceleration of the cyclist is
(a) 2 m/s^2 (b) 5 m/s^2 (c) 10 m/s^2 (d) 20 m/s^2
- 7) Two forces of magnitudes 4 N and 8 N are acting on a box, when the box moves rightward across a frictionless horizontal support. The speed of the box at time t is 1m/ s the change in kinetic energy of the box is-
(a) Zero (b) 80 J (c) 8 J (d) 4
- 
- 8) An object of mass 5 kg is attached to the hook of a spring balance and the balance is suspended vertically from the roof of a lift. The reading on the spring balance, when the lift is going up with an acceleration of 0.25 ms^{-2} ($g = 10 \text{ m/s}^2$)
(a) 51.25 N (b) 48.75 N (c) 52.75 N (d) 47.25N
- 9) An object of mass 2 kg is at rest. Now a force of $F = 12t^2 + 4t$ is applied on the object. Calculate the velocity of object at $t = 3\text{s}$.
(a) 144 (b) 63 (c) 60 (d) 32
- 10) A horizontal force of 130.2 N is applied on a 20 kg block which rests on a horizontal surface. If the coefficient of friction is 0.4, then the net force on the block is ($g = 9.8 \text{ m/s}^2$)
(a) 196 N (b) 78.4N (c) 51.8 (d) 208.6
- 11) Which one of the following statements is in correct ?
(a) Frictional force opposes the relative motion.
(b) Limiting value of static friction is directly proportional to normal reaction.
(c) Rolling friction is smaller than sliding friction.

(d) Coefficient of sliding friction has dimensions of length.

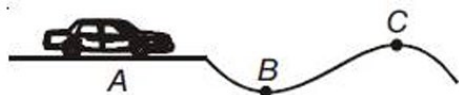
12) A bullet of mass 10 g moving horizontally with a velocity of 400 ms^{-1} strikes a wood block of mass 2 kg which is suspended by light inextensible string of length 5 m. As a result, the centre of gravity of the block found to rise a vertical distance of 10 cm. The speed of the bullet after it emerges out horizontally from the block will be

- (a) 100 ms^{-1} (b) 80 ms^{-1} (c) 120 ms^{-1} (d) 160 ms^{-1}

13) The linear momentum p of a body moving in one dimension varies with time according to the equation $p = a + bt^2$, where a and b are positive constants. The net force acting on the body is (a) A constant (b) Proportional to $2t$ (c) Inversely proportional to t (d) Proportional to t

14) Three equal weights A, B and C of mass 2 kg each are hanging on a string passing over a fixed pulley which is frictionless as shown in figure. The tension in the string connecting weight B and C is ? (a) zero (b) 13 N (c) 3.3 N (d) 19.6 N

15) A vehicle is moving on a track with constant speed as shown in figure. The apparent weight of the vehicle is



- (a) Maximum at A (b) Maximum at B (c) Maximum at C (d) Same at A, B and C

EXERCISE QUESTIONS

16) Maximum value of static friction is called _____

- (a) Limiting friction (b) Rolling friction (c) Normal reaction (d) Coefficient of friction

17) A body of mass m collides against a wall with velocity v and rebounds with same speed. Change in its momentum is

- (a) $2mv$ (b) mv (c) $-2mv$ (d) zero

18) Two bodies having masses in the ratio 2:3 fall freely under gravity from heights which are in the ratio 9:16. The ratio of their linear momentum on touching the ground is-

- (a) 2:9 (b) 3:16 (c) 1:2 (d) 3:2

19) The resultant force of 5N and 10N cannot be

- (a) 12N (b) 4N (c) 8N (d) 5

20) A car moving at a speed " v " is stopped by a retarding force F in a distance ' S '. If the retarding force were $3F$. The distance covered by the car before it comes to rest.

- (a) $S/3$ (b) $S/6$ (c) $S/9$ (d) $S/12$

ANSWERS OF MCQ

Q.1 Answer: (c) $50 \text{ kg} \cdot \text{m/s}$ Hint: Use momentum formula: $p = mv$

Q.2 Answer: (d) 9 times Hint: Banking angle depends on square of velocity.

Q.3 Answer: (b) 4 N Hint: Use centripetal force formula: $F = mv^2/r$

- Q.4 Answer: (d) $g^2/4a$ Hint: From Newton's second law, find mass using $F = ma$.
- Q.5 Answer: (b) $(2/3)a_1$ Hint: Same force acts on both bodies. Find masses and combine.
- Q.6 Answer: (b) 5 m/s^2 Hint: Use centripetal acceleration formula: $a = v^2/r$
- Q.7 Answer: (a) Zero Hint: Net force becomes zero, so kinetic energy does not change.
- Q.8 Answer: (a) 51.25 N Hint: Apparent weight in upward acceleration: $N = m(g+a)$
- Q.9 Answer: (b) 63 Hint: Use $F = ma$ and integrate acceleration to get velocity.
- Q.10 Answer: (c) 51.8 N Hint: Net force = Applied force – friction force.
- Q.11 Answer: (d) Hint: Coefficient of friction is dimensionless.
- Q.12 Answer: (c) 120 m/s Hint: Apply conservation of momentum and energy conservation.
- Q.13 Answer: (d) Proportional to t Hint: Use $F = dp/dt$.
- Q.14 Answer: (b) 13 N Hint: Apply Newton's laws for connected masses.
- Q.15 Answer: (b) Maximum at B Hint: Apparent weight is maximum at the lowest point.
- Q.16 Answer: (a) Limiting friction Hint: Maximum static friction is called limiting friction.
- Q.17 Answer: (a) $2mv$ Hint: Momentum changes from $+mv$ to $-mv$.
- Q.18 Answer: (c) $1 : 2$ Hint: Use momentum relation with free fall velocity.
- Q.19 Answer: (b) 4 N Hint: Resultant of 5 N and 10 N lies between 5 N and 15 N .
- Q.20 Answer: (a) $S/3$ Hint: Stopping distance is inversely proportional to retarding force.

ASSERTION AND REASONING:

Directions-Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

- (a) Assertion is correct and reason both are correct and reason is a correct explanation for assertion.
 (b) Assertion and reason both are correct and reason is not a correct explanation for assertion
 (c) Assertion is correct but reason is incorrect.
 (d) Both Assertion and reason is incorrect.

1) **Assertion:** The book is at rest on a study table means no net force is acting on the book

Reason: The weight of the book and reaction force are acting in the same direction.

2) **Assertion:** Due to friction between tyres and road there is wear and tear of tyres

Reason: Friction is a necessary evil

3) **Assertion:** A bullet fired by a gun with high speed causes more harm than the same bullet fired with moderate speed.

Reason: Change in momentum depends on mass and velocity.

4) **Assertion:** We know novice player tries to catch the cricket ball almost instantly and this hurts him.

Reason: Force not only depends on the change in momentum but also on how fast the change in momentum is brought about.

5) **Assertion:** When a fixed force is applied for a certain interval of time on two bodies of different masses initially at rest the lighter body picks up a greater speed than the heavier body

Reason: Same force for the same time causes same change in momentum for different bodies

6) **Assertion:** Driving at optimum speed on a banked road will cause little wear & tear of tyres.

Reason: Frictional force provides the necessary centripetal force.

7) **Assertion:** In a system of three forces in equilibrium, the resultant of any two forces obtained by the parallelogram law of forces must be equal and opposite to the third force.

Reason: These forces are non-concurrent forces.

8) **Assertion:** Frictional force is non conservative force

Reason: frictional force depends on the path not on initial and final position of the object.

9) **Assertion:** when external force exceeds the maximum limit of static friction, the body begins to move.

Reason: Static friction comes into play the moment when we apply a force on a body.

10) **Assertion:** Ball bearings are used in moving parts of a machine to reduce power dissipation

Reason: Rolling friction between ball bearing and surface in contact is small.

EXERCISE QUESTIONS-

11) **Assertion:** It is difficult to drive a car on slippery road than an ordinary road.

Reason: An ordinary road the friction between the tyres and the road provides the necessary external force to accelerate the car.

12) **Assertion:** Rolling friction is always lesser than the sliding friction

Reason: In rolling friction there is one point of contact between the body and the plane

13) **Assertion:** For a projectile under the vertical Gravitational force the horizontal component of velocity remains unchanged

Reason: the component of velocity normal to the force remains unchanged

14) **Assertion:** In ideal rolling even when effect of friction is ignored body must come to rest after certain time.

Reason: During collision between point of contact and floor kinetic energy of the body gradually decreases.

15) **Assertion:** Action is the cause and reaction is always the effect of it on same body.

Reason: Action and reaction forces cancel each other

ANSWERS:

1Q	2Q	3Q	4Q	5Q	6Q	7Q	8Q	9Q	10Q	11Q	12Q	13Q	14Q	15Q
c	b	a	a	a	b	c	a	b	a	a	b	a	a	d

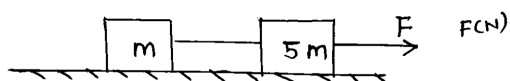
SHORT ANSWER TYPE QUESTIONS[2M]

Q1 A monkey seems to climb up and another monkey seems to move down at same rope hanging over a pulley with same uniform acceleration. If the respective masses of monkeys are in the ratio 1:3. Find the common acceleration?

Q2 Can a body have zero velocity and non-zero acceleration? Explain with two examples.

Q3 A particle moves in a circle of radius 20 cm at 120 revolutions per minute. Calculate the linear acceleration of the particle.

Q4 The tension in the string shown in fig



Q5 A scooter is moving with speed of 10m/s on a circular track of radius 25m. Its speed is increasing at the rate of 3 m s^{-2} . Calculate its resultant acceleration.

Q6 What is a centripetal force. How it is different from centrifugal force. Identify source of the centripetal force acting on respective objects in the following cases?

(i) Electron revolving around the nucleus.

(ii) Earth revolves around the sun.

Q7 A batsman hits a ball of mass 50kg moving with a speed of 20m/s is hit by a batsman with a speed of 10m/s. If time of contact is 5

seconds. Calculate the force exerted by the bat as well calculate impulse imparted on the ball.

Q8 A bullet of mass 30 g penetrates 5cm inside a wooden block with an initial velocity of 500m/s and finally comes to rest. Calculate the average force exerted by the bullet.

Q9 A force $F=4t^3+20$ is applied on an object of mass 50 kg moving with initial velocity of 10m/s. Calculate the velocity of the object after 10 seconds.

Q10 Justify the reason behind banking of tracks. A cyclist takes a turn of circular path of radius 10m with a speed of 10m/s. Through what angle he should bend his cycle without skidding.

EXERCISE QUESTIONS

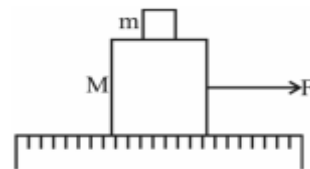
Q11 A rocket is projected upwards with a lift mass of 20,000 kg with an initial acceleration of 10m/s. Calculate the initial thrust imparted to it.

Q12 Why does a fruit fall down when the branch of the tree is shaken? State and explain the law involved in this phenomena.

Q13 A horse can pull a heavy cart on a road but on muddy field it is very difficult. Justify using a suitable diagram.

Q14 How linear velocity and angular velocity are different from each other. Deduce a relation between them.

Q15 A system of two blocks of masses $m=2\text{kg}$ and $M=8\text{ kg}$ is placed on a smooth table as shown in fig. The coefficient of static friction between two blocks is 0.5. What is the maximum horizontal force F that can be applied to the block of mass M so that the blocks move together.



ANSWERS-

Q1: Answer: $a = g/2$ Hint: Apply Atwood machine formula using mass ratio 1:3.

Q2: Answer: Yes Hint: In circular motion, speed remains constant but direction changes continuously.

Q3: Answer: $a = 320\pi^2 \text{ cm s}^{-2}$ Hint: Use $a = \omega^2 r$ and $\omega = 2\pi f$.

Q4: Answer: $T = F/6$ Hint: Draw Free Body Diagram and apply Newton's Second Law.

Q5: Answer: Resultant acceleration = 5 m/s^2 Hint: Use $a = \sqrt{a_c^2 + a_t^2}$.

Q6: Answer: Centripetal force acts towards center; centrifugal force acts outward.

Hint: Electron $\rightarrow$ Electrostatic force, Earth $\rightarrow$ Gravitational force.

Q7: Answer: Impulse = 2000 kg m/s , Force = 400 N Hint: Use Impulse = $m(v-u)$ and $F = \Delta p/t$.

Q8: Answer: Retarding force = $1.25 \times 10^5 \text{ N}$ Hint: Use Work-Energy theorem.

Q9: Answer: Velocity after 10 s = 214 m/s Hint: Integrate $F dt = m(v-u)$.

Q10: Answer: $\theta = 45^\circ$ Hint: Use $\tan\theta = v^2/rg$.

Q11: Answer: Thrust = $4 \times 10^5 \text{ N}$ Hint: Use $F = m(g+a)$.

Q12: Answer: Fruit falls due to inertia of rest. Hint: Branch moves suddenly but fruit tends to remain at rest due to Newton's First Law.

Q13: Answer: Horse cannot move easily on muddy road because friction is very small.

Hint: Frictional force between horse and ground decreases on muddy surface.

Q14: Answer: Linear velocity is related to straight line motion and angular velocity to rotational motion. Relation: $v = r\omega$ Hint: Linear velocity depends on radius and angular velocity.

Q15: Answer: Maximum force $F = 50 \text{ N}$ Hint: Use $f = \mu mg$, then apply $F = (M+m)a$.

SHORT ANSWER TYPE QUESTIONS [3 MARK]

Q1 Think like a Cyclist : A biker takes a turn on a curved road of radius 25 m with speed 15 m/s. Explain why he bends inward and calculate the angle of inclination.

Q2 A gun of mass 4 kg fires a bullet of mass 40 g with velocity 300 m/s. Calculate recoil velocity of gun and state the law involved.

Q3 An athlete of mass 60 kg climbs a rope. The rope can withstand maximum tension of 700 N. Determine whether the rope breaks when athlete climbs upward with acceleration 4 m/s^2 .

Q4 Explain why friction is called a necessary evil. Write any three advantages and disadvantages from daily life.

Q5 A shell of mass 10 kg moving at 60 m/s explodes into two fragments of masses 4 kg and 6 kg. If smaller fragment moves at 80 m/s, find velocity of larger fragment.

Q6 A machine gun fires bullets of mass 25 g with velocity 500 m/s. A 50 kg animal runs toward hunter at 5 m/s. Find number of bullets required to stop the animal.

Q7 The force-time graph of a particle is triangular with base 4 s and height 30 N. Calculate impulse imparted to particle.

Q8 Derive expression for stopping distance of a vehicle moving on rough horizontal road with coefficient of friction μ .

Q9 A stone tied to a string of length 1 m moves in circular path with speed 5 m/s. Calculate centripetal acceleration and tension in string if mass is 0.2 kg.

Q10 Think like a Pilot ☹️: A jet plane loops in vertical circle of radius 500 m at speed 200 m/s. Compare centripetal acceleration with g .

EXERCISE QUESTIONS

Q11 Why newton's second law is called real law of motion. Deduce first and second law of motion from it to support your answer.

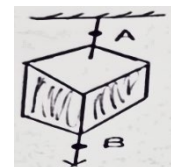
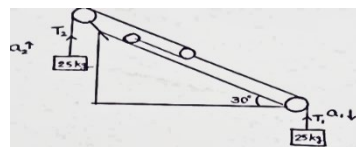
Q12 A balloon is descending down with 5 m/s^2 acceleration. Calculate number of gases should be taken out from it such that it starts ascending with same acceleration. Give logic behind your answer.

Q13 A child pulled a thread as shown in fig from a point, just below B.

i) He increases the force gradually ii) Gives a jerk

In both cases from where string shall break. Explain and state law of physics behind it.

Q14 Find tension a_1 , a_2 , T_1 and T_2 in the given figure-



ANSWERS-

Q1: Answer: Cyclist bends inward to obtain centripetal force. Angle $\approx 42^\circ$

Hint: Use $\tan\theta = v^2/rg$.

Q2: Answer: Recoil velocity = 3 m/s Hint: Apply conservation of momentum.

Q3: Answer: Rope tension = 840 N, hence rope breaks. Hint: Use $T = m(g+a)$.

Q4: Answer: Friction helps in walking, writing, braking but causes heat and wear.

Hint: Write any 3 advantages and disadvantages.

Q5: Answer: Velocity of larger fragment = 46.7 m/s Hint: Apply conservation of momentum.

Q6: Answer: Number of bullets required = 2 Hint: Bullet momentum = animal momentum.

Q7: Answer: Impulse = 60 Ns Hint: Area under force-time graph.

Q8: Answer: Stopping distance = $v^2/2\mu g$ Hint: Use $v^2 - u^2 = 2as$ and $a = -\mu g$.

Q9: Answer: Centripetal acceleration = 25 m/s^2 , Tension = 5 N Hint: Use $a = v^2/r$ & $T = mv^2/r$.

Q10: Answer: Centripetal acceleration = 80 m/s^2 , Ratio with $g \approx 8:1$

Hint: Compare v^2/r with g .

Q11: Answer: Newton's Second Law explains force quantitatively and includes first law.

Hint: $F = dp/dt$.

Q12: Answer: Gas must be removed to reduce weight so balloon rises.

Hint: Buoyant force becomes greater than weight.

Q13: Answer: Slow pull $\rightarrow$ upper thread breaks; sudden jerk $\rightarrow$ lower thread breaks.

Hint: Due to inertia.

Q14: Answer: Apply Newton's Laws on both masses separately to find T_1 , T_2 , a_1 , a_2 .

Hint: Draw Free Body Diagram.

LONG ANSWER TYPE QUESTIONS-

Q1 Give the magnitude and direction of the net force acting on

- (a) a drop of rain falling down with a constant speed,
- (b) a cork of mass 10 g floating on water,
- (c) a kite skillfully held stationary in the sky,
- (d) a car moving with a constant velocity of 30 km/h–130 km/h–1 on a rough road, and
- (e) a high-speed electron in space, far from all material objects, and free of electric and magnetic fields.

Q2 What is meant by banking of roads? Explain the need for it. Obtain an expression for the maximum speed with which a vehicle can safely negotiate a curved road banked at angle θ . The coefficient of friction between road and wheels is μ .

Q3 A plane of mass 500 kg rises with a vertical acceleration of 5 m/s^2 . The weight of pilot is 50 kg. Find-

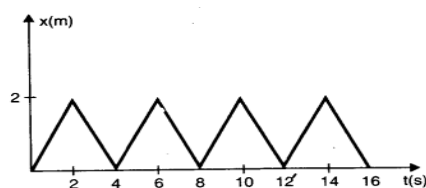
- (a) The magnitude of force on the floor of the plane by the pilot.
- (b) Action of the rotor of the helicopter on the surrounding air- ($g=10 \text{ m/s}^2$)
- (c) A car of mass 1000kg is moving with a speed of 36 km h^{-1} on a level road. Calculate the retarding force required to stop the car in a distance of 50 m .What is the need of banking?

Q4 What is the law of conservation of linear momentum? Write its two applications. How does the law of conservation of linear momentum related to Newton's third law of motion? A body of mass is 5kg initially at rest explodes and breaks into three fragments of masses in the ratio 1:1:3. The two pieces of equal masses fly off perpendicular to each other with a speed of 30 m s^{-1} each. What is the velocity of higher fragments?

EXERCISE QUESTIONS-

Q5 A machine gun of mass 10 kg fires 20 g bullets at the rate of 10 bullets per second with a speed of a 800 ms^{-1} . What force is required to hold the guns in position?

(b) What is impulse and how it is related to angular momentum. One dimensional motion of a body of mass 0.4kg is shown by the position time (x-t). Find the magnitude of each impulse.



Q6 (a) Write the laws of limiting friction.

(b) A truck moving at 54 km/h carries a steel beams which rests on a wooden floor. What is the minimum time in which truck can stop without the beams moving forward? Coefficient of static friction between steel and wood is 0.5.

(c) A mass of 1 kg is suspended by a string. It is first lifted up with an acceleration of 4.9 m/s^2 and then lowered down with same acceleration. Find the ratio of tensions in the string in the two cases.

ANSWERS-

Q1: Answer: Net force is zero in all cases. Hint: Constant velocity or rest $\Rightarrow$ net force = 0.

Q2: Answer: Banking helps provide centripetal force. Hint: Use banking of roads formula.

Q3(a): Answer: Force on floor = 750 N Hint: Use $F = m(g+a)$.

Q3(b): Answer: Rotor force on air = 8250 N Hint: Newton's Third Law.

Q3(c): Answer: Retarding force = 1000 N Hint: Use $v^2 - u^2 = 2as$.

Q4: Answer: Velocity of heavier fragment $\approx 14.14 \text{ m/s}$ Hint: Conservation of momentum.

Q5(a): Answer: Force required = 160 N Hint: Force = rate of change of momentum.

Q5(b): Answer: Impulse = change in momentum. Hint: $J = F\Delta t$.

Q6(a): Answer: Limiting friction $\propto$ normal reaction. Hint: $f_{\max} = \mu N$.

Q(b): Answer: Minimum stopping time = 3 Hint: Maximum retardation = μg .

Q6(c): Answer: Tension ratio = 3:1 Hint: $T = m(g \pm a)$.

CASE BASED STUDY QUESTIONS:

Q1 Conservation of Momentum & Recoil

A rifle fires a bullet with very high speed. According to Newton's Third Law, the gun experiences recoil in the opposite direction. This phenomenon explains conservation of momentum in isolated systems. Recoil is reduced by increasing the mass of the gun.

(i) A bullet of mass 25 g is fired from a gun of mass 5 kg with velocity 400 m/s. The recoil velocity of the gun is:

- (a) 1 m/s (b) 2 m/s (c) 4 m/s (d) 8 m/s

(ii) Which law explains recoil of gun?

- (a) Newton's First Law (b) Newton's Second Law (c)
Newton's Third Law (d) Law of Gravitation

(iii) Why is recoil velocity of gun smaller than bullet velocity?

(iv) **Think like an Engineer**  Why are heavy rifles preferred over lighter rifles?

Q2 Lift and Apparent Weight

A person standing inside a lift experiences changes in apparent weight depending upon the acceleration of the lift. The weighing machine actually measures the normal reaction force acting on the person.

(i) A lift moves upward with constant velocity. The apparent weight is:

- (a) Zero (b) Less than actual weight (c) Equal to actual weight (d) Greater than actual weight

(ii) A person feels lighter in a lift when it:

- (a) Accelerates upward (b) Accelerates downward
(c) Moves with constant speed (d) Stops suddenly

(iii) What happens to apparent weight during free fall?

(iv) **Think like a Scientist**  Why do astronauts feel weightless inside satellites?

Q3 Banking of Roads

Vehicles moving on curved roads require centripetal force for safe turning. To reduce dependence on friction and prevent skidding, roads are banked.

- (i) The angle of banking depends mainly on:
 (a) Radius and speed (b) Mass of vehicle (c) Colour of road (d) Atmospheric pressure
- (ii) A car moves safely on a banked road because:
 (a) Friction disappears (b) Weight becomes zero
 (c) Horizontal component of normal reaction provides centripetal force
 (d) Gravity increases
- (iii) What is the optimum speed on a banked road of radius 20 m & 45° ? (Take $g = 10 \text{ m/s}^2$)
 (a) 10 m/s (b) 14 m/s (c) 20 m/s (d) 40 m/s

(iv) **Think like an Engineer** 

Why are highways and racing tracks designed with banking at curves?

Q4 Impulse and Collision

During collisions, a large force acts for a very short time interval producing impulse. Airbags and helmets work on this principle.

- (i) Impulse is equal to:
 (a) Force $\times$ Distance (b) Force $\times$ Time (c) Mass $\times$ Velocity (d) Work done
- (ii) A batsman moves his hands backward while catching the ball to:
 (a) Increase force (b) Reduce collision time (c) Increase collision time (d) Increase momentum
- (iii) A ball experiences impulse of 20 Ns for 0.2 s. Average force is:
 (a) 10 N (b) 50 N (c) 100 N (d) 200 N

(iv) **Think like a Doctor** 

Why do airbags reduce injuries during accidents?

ANSWERS / HINTS

Q1 (i) (b) 2 m/s (ii) (c) Newton's Third Law (iii) Gun has larger mass (iv) To reduce recoil

Q2 (i) (c) Equal to actual weight (ii) (b) Accelerates downward (iii) Zero (iv) Due to free fall condition

Q3 (i) (a) Radius and speed (ii) (c) Horizontal component provides centripetal force
 (iii) (b) 14 m/s (iv) To avoid skidding and provide safe turning

Q4 (i) (b) Force $\times$ Time (ii) (c) Increase collision time (iii) (c) 100 N
 (iv) They increase impact time and reduce force

LAWS OF MOTION

ONE-PAGE REVISION | SMART LEARN | SCORE HIGH

★ MOST IMPORTANT

- NCERT Based
- Formula Based Questions
- PYQ Favourite

1. NEWTON'S LAWS OF MOTION

First Law (Law of Inertia)

A body continues in its state of rest or of uniform motion in a straight line unless acted upon by an external unbalanced force.



Inertia: Tendency of a body to resist any change in its state of rest or of uniform motion.

Second Law

The rate of change of momentum of a body is proportional to the net external force acting on it and occurs in the direction of the force.

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \quad (\text{General Form})$$

For constant mass, $\vec{F}_{\text{net}} = m\vec{a}$

Third Law

To every action, there is an equal and opposite reaction.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

(Forces act on different bodies)

2. MOMENTUM AND IMPULSE

Momentum ($\vec{p}$)

$$\vec{p} = m\vec{v}$$

SI Unit: kg m s^{-1}

Impulse ($\vec{J}$)

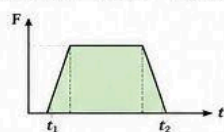
$$\vec{J} = \vec{F} \Delta t$$

SI Unit: N s

Impulse - Momentum Theorem

$$\vec{J} = \Delta\vec{p} = \vec{p}_2 - \vec{p}_1$$

Area under F-t graph = Impulse



Change in momentum = Area under F-t curve between t_1 and t_2

3. FRICTION

Types

- Static Friction
- Kinetic (Sliding) Friction
- Limiting Friction
- Rolling Friction

Laws of (Dry) Friction

- Friction acts tangentially at the surface of contact.
- It opposes relative (or impending) motion.
- For a given pair of surfaces, limiting friction is proportional to normal reaction.

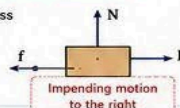
$$\text{Limiting Friction } f_{s,\text{max}} = \mu_s N$$

$$\text{Kinetic Friction } f_k = \mu_k N$$

($\mu_s > \mu_k$)

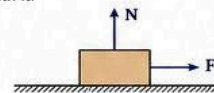
Coefficient of Friction (μ)

Dimensionless
No Unit



4. NORMAL REACTION (N)

Perpendicular force exerted by a surface on the body in contact with it.



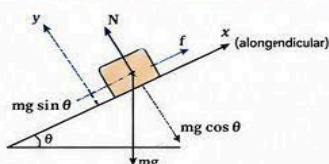
Special Cases

- Block on horizontal surface:
 $N = mg$
- Block on inclined plane (angle θ):
 $N = mg \cos \theta$
- Lift accelerating upward with a :
 $N = m(g + a)$
- Lift accelerating downward with a :
 $N = m(g - a)$

✗ COMMON MISTAKES

1. Confusing weight (mg) with normal reaction (N).
2. Ignoring direction of friction.
3. Applying Newton's 3rd law on same body.
4. Taking friction as always $\mu_s N$ (it is $\mu_k N$ during motion).

5. MOTION ON AN INCLINED PLANE

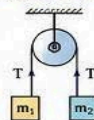


Key Formulas

- Component of weight along plane = $mg \sin \theta$
- Component of weight perpendicular to plane = $mg \cos \theta$
- Normal reaction: $N = mg \cos \theta$
- If moving up: $ma = mg \sin \theta + f$
- If moving down: $ma = mg \sin \theta - f$
(Adjust sign as per direction)

6. CONNECTED BODIES

Atwood Machine (Ideal string, frictionless pulley)

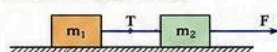


$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

$$T = \frac{2m_1 m_2}{m_1 + m_2} g$$

($m_2 > m_1$)

Two Blocks on Horizontal Surface (Connected by light string)



$$a = \frac{F}{m_1 + m_2} \quad T = \frac{m_1 F}{m_1 + m_2}$$

7. PSEUDO FORCE

An apparent force which arises in a non-inertial frame of reference.

$$\vec{F}_{\text{pseudo}} = -m\vec{a}_{\text{frame}}$$

(opposite to acceleration of frame)

Used in:

- Accelerating vehicles
 - Non-inertial frames
- (Always remember: Real forces cause motion, pseudo forces are just for analysis.)

💡 QUICK RECALL

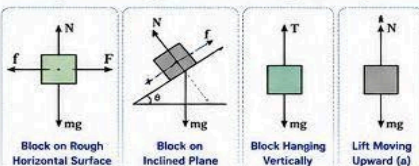
- Inertia → Resist change
- More mass → More inertia
- Net external force → changes motion
- Momentum is conserved if $\vec{F}_{\text{ext}} = 0$

🎯 PYQ TIPS

- Derivations: Impulse-Momentum, Atwood machine
- Numericals: Inclined plane, Connected bodies, Friction
- Concept Based: Newton's laws, Pseudo force, Inertia

8. FREE BODY DIAGRAM (FBD)

A diagram showing all the forces acting on a body.



★ Always show correct directions.

9. CONSERVATION OF MOMENTUM

If net external force on a system is zero, the total momentum remains constant.

$$\sum \vec{p}_{\text{initial}} = \sum \vec{p}_{\text{final}}$$

Examples

- Recoil of gun
- Explosion
- Collisions

10. WORK-ENERGY THEOREM (Useful Connection)

Work done by net external force on a body = Change in kinetic energy

$$W_{\text{net}} = \Delta K = \frac{1}{2} m(v^2 - u^2)$$

(Helpful in many Law of Motion numericals)

POWER PUNCH (Mnemonics)



"F = ma"

F → Force creates
m → Mass resists
a → Acceleration results

"Inertia is natural,
Change is external."

COMPARISON TABLE

Property	Inertia	Mass	Weight
Meaning	Resistance to change in motion	Measure of amount of matter	Gravitational force on a body
Depends on	-	Only on body	g (location)
Vector/Scalar	-	Scalar	Vector
SI Unit	-	kg	N
Changes with velocity?	No	No	No
Changes with location?	No	No	Yes (changes with g)

IMPORTANT FORMULAS BOX

$$\vec{F}_{\text{net}} = m\vec{a} \quad (\text{Newton's 2nd Law})$$

$$\vec{F}_{AB} = -\vec{F}_{BA} \quad (\text{Newton's 3rd Law})$$

$$\vec{p} = m\vec{v} \quad (\text{Momentum})$$

$$\vec{J} = \vec{F} \Delta t = \Delta\vec{p} \quad (\text{Impulse})$$

$$f_{s,\text{max}} = \mu_s N, \quad f_k = \mu_k N \quad (\text{Friction})$$

$$N = mg \cos \theta \quad (\text{Inclined Plane})$$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2} \quad (\text{Atwood Machine})$$

$$P = \frac{W}{t} \quad (\text{Power})$$

SYMBOLS & UNITS

Symbol	Quantity	Unit (SI)
m	Mass	kg
$\vec{v}$	Velocity	m s^{-1}
$\vec{a}$	Acceleration	m s^{-2}
$\vec{F}$	Force	N (kg m s^{-2})
$\vec{p}$	Momentum	kg m s^{-1}
$\vec{J}$	Impulse	N s
N	Normal Reaction	N
f	Friction	N
μ	Coefficient of Friction	No Unit
θ	Angle	rad or °

LAST MINUTE CHECKLIST ✓

- ✓ Newton's 3 Laws (Statements & Applications)
- ✓ FBD drawing practice
- ✓ Inclined plane components
- ✓ Atwood machine formulas
- ✓ Friction cases
- ✓ Impulse-Momentum theorem
- ✓ Concept clarity > Formula memorization



STUDY SMART

Concept → Formula → Diagram → Practice → Revise



PRACTICE DAILY

Solve PYQs & NCERT Examples



ANALYZE MISTAKES

Learn from errors & weak areas



REVISE REGULARLY

Short notes & Formula sheets

STAY CALM
STAY FOCUSED
YOU CAN DO IT! 🏆



MECHANICS ESSENTIALS: A COMPLETE VISUAL GUIDE TO WORK, ENERGY, & POWER

1. WORK, POWER, & INTEGRATION

Work Done by Constant Force

$\vec{v} \cdot \vec{d}$

$W = \vec{F} \cdot \vec{d} = Fd \cos(\theta)$
The product of the displacement magnitude and the force component parallel to displacement.

Work Done by Variable Force

$W = \int F(x) dx$
Represented as the area under a force-vs.-displacement graph.

Power

Power: The Rate of Doing Work:
 $P = dW/dt$ (instantaneous) or $P = W/\Delta t$ (average).
Measures how quickly work is done.

Light Wurb VS Large Engine

2. ENERGIIC AND POTENTIAL ENERGY

The Work-Energy Theorem

Initial velocity → Final velocity (K)

$W_{net} = \Delta K = K_{final} - K_{initial}$
states that net work done equals the change in kinetic energy.

General Notion of Potential Energy

U

Energy associated with the configuration of a system, like position in a gravitiani ($U_g = mgh$).

Potential Energy of a Spring

$U_s = \frac{1}{2} kx^2$
Stored energy dependent on the displacement or stretching) of the spring from its equilibrium.

Hooke's Law

Isolated System
 $EM_{mechanical} = K + U = \text{Constant}$
(No Non-Conservative Work)

3. FORCES, CLASSIFICATIONS AND MOTION

Conservative vs. Non-Conservative Forces

Conservative (e.g., Gravity, Spring) forces are path-independent; work depends only on endpoint. $W_{loop} = 0$

Non-conservative (e.g., Friction, Air Resistance) forces are dependent, resulting in non-mechanical energy loss (e.g., Heat, ΔE_{th}).

Motion in a Vertical Circle

Forces at key points:
(Top: $T_{top} = \frac{mv^2}{r}$; Bottom: $T_{bottom} = \frac{mv^2}{r}$)
Conservation of Mechanical Energy can relate speeds.

Min speed to complete loop: $v_{top} = \sqrt{gr}$

Elastic Collisions

vefo. → after

Kinetic Energy and Momentum are conserved ($\Delta K=0$, $\Delta p=0$).
The balls have velocities before/after.

Inelastic Collisions

Momentum conserved, Kinetic Energy not conserved ($\Delta K \neq 0$).
Energy lost to heat or deformation.
Perfectly Inelastic: Objects stick together ($\Delta p \neq 0$, max K loss).

One- vs. Two-Dimensional Collisions

Analysis uses system momentum components:
(1D: $p_{initial} = p_{final}$ along one axis;
2D: $\sum p_{ix} = \sum p_{fx}$ and $\sum p_{iy} = \sum p_{fy}$)

MULTIPLE CHOICE QUESTIONS

- 1) In which of the following work is being not done?
- (a) Shopping in the supermarket (b) Standing with a basket of fruit on the head
- (c) Climbing a tree (d) Pushing a wheel barrow
- 2) A man pushes a wall and fails to displace it, he does
- (a) negative work (b) positive but not maximum work
- (c) no work at all (d) maximum positive work
- 3) Two bodies of different masses are moving with same kinetic energy. Then the ratio of their momenta is equal to the ratio of their
- (a) masses (b) square of masses (c) square root of masses (d) cube root of masses
- 4) The speed of an object of mass m dropped from an inclined plane (frictionless), at the bottom of the plane, depends on:
- (a) height of the plane above the ground (b) angle of inclination of the plane
- (c) mass of the object (d) All of these
- 5) A particle is taken round a circle by application of force. The work done by the force is
- (a) positive non-zero (b) negative non-zero (c) Zero (d) None of the above
- 6) Work done by a conservative force is positive if
- (a) P.E. of the body increases (b) P.E. of the body decreases
- (c) K.E. of the body increases (d) K.E. of the body decreases
- 7) The total mechanical energy of a system is conserved if the force, doing work on it is
- (a) constant (b) variable (c) conservative (d) non-conservative
- 8) Which of the following must be known in order to determine the power output of an automobile?
- (a) Final velocity and height (b) Mass and amount of work performed
- (c) Force exerted and distance of motion (d) Work performed and elapsed time of work
- 9) Which one of the following statements is true?
- (a) Momentum is conserved in elastic collisions but not in inelastic collisions
- (b) Total kinetic energy is conserved in elastic collisions but momentum is not conserved in elastic collisions
- (c) Total kinetic energy is not conserved but momentum is conserved in inelastic collisions
- (d) Kinetic energy and momentum both are conserved in all types of collisions
- 10) Consider the elastic collision of two bodies A and B of equal mass. Initially B is at rest and A moves with velocity v . After the collision
- (a) the body A traces its path back with the same speed
- (b) the body A comes to rest and B moves away in the direction of A's approach with the velocity v
- (c) both the bodies stick together and are at rest

(d) B moves along with velocity $v/2$ and A retraces its path with velocity $v/2$.

11) In a one-dimensional elastic collision, the relative velocity of approach before collision is equal to

- (a) sum of the velocities of the bodies
- (b) e times the relative velocity of separation after collision
- (c) $1/e$ times the relative velocity of separation after collision
- (d) relative velocity of separation after collision

12) In case of elastic collision, at the time of impact.

- (a) total K.E. of colliding bodies is conserved. (b) total K.E. of colliding bodies increases
- (c) total K.E. of colliding bodies decreases (d) total momentum of colliding bodies decreases.

13) When two spheres of equal masses undergo glancing elastic collision with one of them at rest, after collision they will move

- (a) opposite to one another (b) in the same direction (c) together (d) at right angle to each other

14) In an inelastic collision

- (a) momentum is not conserved
- (b) momentum is conserved but kinetic energy is not conserved
- (c) both momentum and kinetic energy are conserved
- (d) neither momentum nor kinetic energy is conserved

15) In an elastic collision, what is conserved ?

- (a) Kinetic energy (b) Momentum (c) Both (a) and (b) (d) Neither (a) nor (b)

EXERCISE QUESTIONS

16) The work done in lifting water from a well of depth 6 m using a bucket of mass 0.5 kg and volume 2 litre, will be

- (a) 73.5 J (b) 147 J (c) 117.6 J (d) 98 J

17) A ball is thrown vertically downwards from a height of 20 m with an initial velocity v_0 . It collides with the ground and loses 50% of its energy in collision and rebounds to the same height. The initial velocity v_0 is : (Take $g = 10 \text{ m s}^{-2}$)

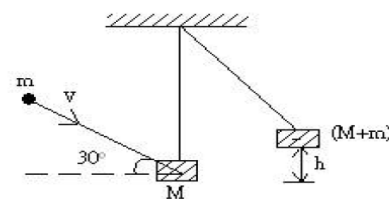
- (a) 20 m s^{-1} (b) 28 m s^{-1} (c) 10 m s^{-1} (d) 14 m s^{-1}

18) The potential energy of a certain spring when stretched through distance S is 10 joule. The amount of work done (in joule) that must be done on this spring to stretch it through an additional distance s , will be

- (a) 20 (b) 10 (c) 30 (d) 40

19) A bullet of mass 20 g hits at a block of 1.98 kg suspended from massless string of length 100 cm and sticks to it. The bullet flies down at an angle of 30° to the horizontal with a velocity of 200 m/s. Through what height the block will rise

- (a) 0.15 m (b) 0.30 m (c) 0.45 m (d) 0.75 m



20) A uniform chain is held on a frictionless table with one-fifth of its length hanging over the edge. If the chain has a length l and a mass m , how much work is required to pull the hanging part back on the table ?

- (a) $mg l / 10$ (b) $mg l / 5$ (c) $mg l / 50$ (d) $mg l / 2$

SOLUTIONS – MULTIPLE CHOICE QUESTIONS

- Q.1 Answer: (b) Hint: No displacement occurs while standing with a basket.
- Q.2 Answer: © Hint: Work done is zero because displacement is zero.
- Q.3 Answer: © Hint: For same kinetic energy, momentum is proportional to $\sqrt{\text{mass}}$.
- Q.4 Answer: (a) Hint: Final speed depends only on vertical height.
- Q.5 Answer: © Hint: Force is perpendicular to displacement in circular motion.
- Q.6 Answer: (b) Hint: Work done by conservative force decreases potential energy.
- Q.7 Answer: © Hint: Mechanical energy is conserved only under conservative forces.
- Q.8 Answer: (d) Hint: Power = Work / Time.
- Q.9 Answer: © Hint: Momentum is conserved in all collisions, KE only in elastic collisions.
- Q.10 Answer: (b) Hint: In elastic collision of equal masses, velocities are exchanged.
- Q.11 Answer: (d) Hint: Relative velocity of approach = relative velocity of separation.
- Q.12 Answer: (a) Hint: Kinetic energy remains conserved in elastic collision.
- Q.13 Answer: (d) Hint: After glancing elastic collision, equal masses move at right angles.
- Q.14 Answer: (b) Hint: Momentum is conserved but kinetic energy is not conserved.
- Q.15 Answer: © Hint: Both momentum and kinetic energy are conserved.
- Q.16 Answer: (b) 147 J Hint: Total mass = bucket + water = $0.5 + 2 = 2.5$ kg.
- Q.17 Answer: (a) 20 m/s Hint: Use conservation of energy before and after collision.
- Q.18 Answer: © 30 J Hint: Spring energy $\propto x^2$, doubling stretch increases energy four times.
- Q.19 Answer: (a) 0.15 m Hint: Use conservation of momentum and convert KE into PE.
- Q.20 Answer: © $mg l / 50$ Hint: Use center of mass of hanging part.

ASSERTION AND REASON TYPE QUESTIONS

Directions: In the following questions, Assertion (A) is followed by Reason (R). Choose the correct option:

- (a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false. (d) Both A and R are false.

1) **Assertion:** A delivery worker carrying a heavy parcel on his head while walking on a straight horizontal road does no mechanical work on the parcel.

Reason: Work done becomes zero when force & displacement are perpendicular to each other

2) **Assertion:** During a complete lap on a circular track, the total work done by gravitational force can become zero.

Reason: Conservative forces do not depend on the actual path followed.

3) **Assertion:** A loaded truck and a motorcycle can have same momentum but different kinetic energies.

Reason: Kinetic energy depends on both momentum and mass.

4) **Assertion:** Instantaneous power delivered by an engine depends on both applied force and velocity.

Reason: Instantaneous power is the dot product of force and velocity.

5) **Assertion:** The kinetic energy gained by a football after being kicked equals the net work done on it.

Reason: Work-energy theorem is valid only for single particle systems.

6) **Assertion:** A stretched rubber band stores potential energy.

Reason: External work done against restoring force gets stored as elastic potential energy.

7) **Assertion:** Satellites revolve around Earth without loss of total mechanical energy.

Reason: Gravitational force is conservative in nature.

8) **Assertion:** Two collisions with same change in momentum can produce different impact forces.

Reason: Force depends on rate of change of momentum.

9) **Assertion:** Work done by gravity while lifting a body depends only on initial and final positions.

Reason: Gravitational force is a conservative force.

10) **Assertion:** Work done over a closed path is always zero.

Reason: Work done depends on nature of force involved.

EXERCISE QUESTIONS

11) **Assertion:** The power of a pump which raises 100 kg of water in 10sec to a height of 100 m is 10 KW.

Reason: The practical unit of power is horse power.

12) **Assertion:** According to law of conservation of mechanical energy change in potential energy is equal and opposite of the change in kinetic energy.

Reason: Mechanical energy is not a conserved quantity.

13) **Assertion:** When the force retards the motion of a body, the work done may zero.

Reason: Work done depends on angle between force and displacement.

14) **Assertion:** In an elastic collision of two bodies, the momentum and energy of each body is conserved.

Reason: If two bodies stick to each other, after colliding, the collision is said to be perfectly elastic.

15) **Assertion:** A body cannot have energy without having momentum but it can have momentum without having energy.

Reason: Momentum and energy have same dimensions.

ANSWER - ASSERTION AND REASON

1Q	2Q	3Q	4Q	5Q	6Q	7Q	8Q	9Q	10Q	11Q	12Q	13Q	14Q	15Q
a	a	a	a	c	a	a	a	a	d	b	c	d	d	d

SHORT ANSWER QUESTIONS (2 MARKS)

- Q1 When a conservative force does negative work on a body, what happens to its potential energy?
- Q2 A body of mass 4 kg moves such that displacement is given by $S = t^3/2$. Find the work done in first 2 s.
- Q3 A pump of power 5 kW raises water to a height of 20 m. Find amount of water raised in 1 minute.
- Q4 Two bodies of mass 50 kg each move in opposite directions with speeds 12 m/s and 8 m/s. Find common velocity after collision.
- Q5 A body explodes into fragments in the ratio 1:2:3. Two smaller fragments move perpendicular to each other with speeds 10 m/s. Find velocity of heaviest fragment.
- Q6 A resistive force-distance graph forms a triangle of area 40 J. If initial kinetic energy is 100 J, find final kinetic energy.
- Q7 Find potential energy stored in a spring in terms of force F and extension x .
- Q8 A spring stores 200 J energy when stretched by 4 cm. Find energy stored when stretched by 8 cm.
- Q9 If momentum of a body becomes double, how much does kinetic energy increase?
- Q10 Two springs of constants 1000 N/m and 2000 N/m are stretched by same force. Find ratio of energies stored.
- Q11 Which Spring stores more energy when stretched equally: soft spring or stiff spring?
- Q12 A force $F=(3+x)$ acts along x -axis. Find work done from $x=1$ m to $x=3$ m.
- Q13 If kinetic energy increases by 200%, by what percent does momentum increase?
- Q14 In velocity-time graph, during which interval is work done negative?
- Q15 A body of mass 1 kg moving at 2 m/s compresses a spring of constant 200 N/m. Find maximum compression.

ANSWERS / HINTS

- Q1: Potential energy increases. Hint: $W = -\Delta U$
- Q2: Work done = 64 J Hint: Use $v = ds/dt$ and KE theorem.
- Q3: Mass of water = 1500 kg Hint: $P = mgh/t$
- Q4: Velocity = 2 m/s Hint: Conservation of momentum.
- Q5: Velocity of heavier fragment = 7.45 m/s Hint: Resultant momentum of smaller fragments.
- Q6: Final KE = 60 J Hint: $KE_{\text{final}} = KE_{\text{initial}} - \text{work done}$.
- Q7: $PE = 1/2 Fx$ Hint: Elastic potential energy formula.
- Q8: Energy = 800 J Hint: $PE \propto x^2$
- Q9: Kinetic energy becomes four times Hint: $KE \propto p^2$
- Q10: Ratio = 2 : 1 Hint: $PE \propto 1/k$ for same force.
- Q11: Stiff spring Hint: Larger k stores more energy for same extension.
- Q12: Work done = 8 J Hint: Integrate force over displacement.

Q13: Momentum increases by 100% Hint: $KE \propto p^2$

Q14: During decreasing velocity interval Hint: KE decreases.

Q15: Maximum compression = 0.14 m Hint: KE = Spring PE

SHORT ANSWER QUESTIONS (3 MARKS)

Q1 A worker pushes a box with force 120 N for 8 m on a rough floor. Later the force decreases uniformly to 60 N for next 8 m. Find total work done by worker and friction force of 40 N.

Q2 A block attached to a spring compresses it by x . Find minimum mass required to just lift another block placed on spring platform.

Q3 A particle rotates in a vertical circle of radius 1.5 m. If ratio of maximum to minimum tension is 3:1, find speed of particle.

Q4 A body starts from height $5R$ and moves in frictionless loop of radius R . Find normal reaction at top of loop.

Q5 A pendulum bob released from horizontal position loses 20% energy due to air resistance. Find speed at lowest point.

Q6 A particle of mass 1 kg moves with velocity $v = 3x^2$. Find work done from $x=0$ to $x=2$ m.

Q7 Show that for a car moving with constant power, displacement squared is proportional to cube of time.

Q8 State and prove work-energy theorem. For which forces is it applicable?

Q9 State whether following are true or false with reason:

(a) Momentum of each body is conserved in elastic collision.

(b) Work done in closed path is always zero.

(c) Kinetic energy decreases in inelastic collision.

Q10 A force $F = (3i + 4j - 2k)$ N displaces a body from $A(1,2,3)$ to $B(4,-1,5)$. Find work done.

Q11 A force $F=(8+2x)$ acts on a body along x -axis. Find work done from $x=1$ m to $x=4$ m.

Q12 A car of mass 1200 kg moving at 20 m/s collides with spring of constant 8×10^3 N/m. Find maximum compression.

Q13 A particle dropped from height H has kinetic energy double its potential energy at some point. Find height from ground.

Q14 A bob attached to flywheel falls through height h . Find angular speed of flywheel in terms of h and r .

Q15 A cyclist moving at 15 m/s applies brakes and stops in 5 s. If retarding force is 300 N, find work done by brakes.

ANSWERS / HINTS

Q1: Work done by worker = 1440 J Hint: Use area under force-displacement graph.

Q2: Minimum mass = $m/2$ Hint: Spring PE = gravitational PE.

Q3: Speed = $3\sqrt{g}$ m/s Hint: Use tension ratio equations.

Q4: Normal reaction = $4mg$ Hint: Apply conservation of mechanical energy.

Q5: $v = \sqrt{1.6gL}$ Hint: Only 80% energy remains.

Q6: Work done = 72 J Hint: $W = \Delta KE$.

Q7: $x^2 \propto t^3$ Hint: Use $P = dW/dt$.

Q8: Applicable for all forces. Hint: Work done equals change in kinetic energy.

Q9: (a) False (b) False (c) True Hint: Conservation laws apply to system.

Q10: Work done = 14 J Hint: $W = F \cdot S$.

Q11: Work done = 33 J Hint: Integrate force over displacement.

Q12: Maximum compression = 7.75 m Hint: KE converts into spring PE.

Q13: Height from ground = $H/3$ Hint: Total energy remains constant.

Q14: $\omega = (1/r)\sqrt{(4gh/3)}$ Hint: Use rotational kinetic energy.

Q15: Work done = -22500 J Hint: Retarding force does negative work.

LONG ANSWER QUESTIONS (5 MARKS)

Q1 (a) Show that in case of one-dimensional elastic collision of two bodies, the relative velocity of separation after the collision is equal to the relative velocity of approach before collision.

(b) Hence write expressions for the velocities of two bodies in terms of their initial velocities before collision.

Q2 A body tied to one end of a string is made to revolve in a vertical plane. Derive the expression for the velocity of the body and tension in the string at any point. Hence find (i) tension at the bottom and top of the circle. (ii) Minimum velocity at the lowest point so that it is just able to loop the vertical loop and (c) the minimum velocity at the top.

Q3 (a) Draw the graph of equation $F_s = -kx$ where F_s is the spring force and x is the displacement of the block from equilibrium position. Using the graph, show that maximum work done by the spring at x_m is $W_s = -\frac{1}{2}kx_m^2$ (k = spring constant)

(b) What will be the work done by external force if the spring is displaced from x_1 to x_2 ?

Q4 (a) Define work done. How can we calculate work done by

(i) Constant force

(ii) Variable force and

(iii) From graph if an object is displaced along a straight line.

(b) Find work done by a force $F = 2x$ (SI unit) if it displaces an object from origin to a

Point (1, 2, -2)

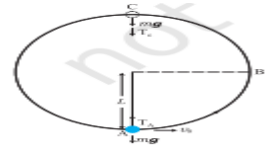
Q5 Define perfect elastic collision. Let one object of mass m_1 moving with u undergoes perfect elastic collision with a stationary object of mass m_2 in one dimension. After collision both stick together and move with velocity ' V '. Prove that there will be some loss of kinetic energy in the collision.

EXERCISE QUESTIONS

Q6 A block of mass 2 kg is pulled up on a smooth incline of angle 30° with the horizontal. If the block moves with an acceleration of 1 m/s^2 , find the power delivered by the pulling force at a time 4 s after the motion starts. What is the average power delivered during the 4 s after the motion starts?

Q7 A block of mass m is pulled along a horizontal surface by applying a force at an angle θ with the horizontal. The friction coefficient between the block and the surface is μ . If the block travels at a uniform velocity, find the work done by this applied force during a displacement d of the block.

Q8 A bob of mass m is suspended by a light string of length L . It is imparted a horizontal velocity v_0 at the lowest point A such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack on reaching the topmost point C as shown in fig. Obtain an expression for



- (i) V_0 (ii) The speeds at point B and C
(iii) The ratio of kinetic energy is K_B / K_C at B and C.

Comment on the nature of trajectory of the bob after it reaches the point c.

ANSWERS / HINTS

Q1: Relative velocity of separation = Relative velocity of approach.

Hint: Use conservation of momentum and coefficient of restitution $e = 1$.

Q2: At bottom: $T = mg + mv^2/r$

At top: $T = mv^2/r - mg$ Minimum velocity at top = $\sqrt{gr}$

Minimum velocity at bottom = $\sqrt{5gr}$ Hint: Apply conservation of energy and centripetal force.

Q3: Maximum work done by spring = $-\frac{1}{2}kx^2$

External work from x_1 to $x_2 = \frac{1}{2}k(x_2^2 - x_1^2)$ Hint: Work done equals area under F-x graph.

Q4: Work done by constant force: $W = F \cdot S$ Variable force: $W = \int F dx$

From graph: Area under F-x graph Hint: For vector force use dot product.

Q5: Perfect elastic collision conserves both momentum and KE.

If bodies stick together, collision becomes inelastic and KE decreases.

Hint: Compare initial and final kinetic energies.

EXERCISE

Q6: Instantaneous power = 24 W Hint: $P = Fv$ and $v = u + at$.

Q7: Work done = $(\mu mgd \cos\theta) / (\cos\theta + \mu \sin\theta)$

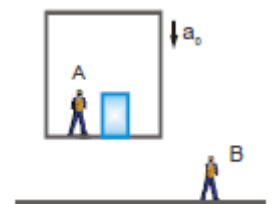
Hint: Resolve forces horizontally and vertically.

Q8: $V_0 = \sqrt{5gL}$, $V_B = \sqrt{3gL}$, $V_C = \sqrt{gL}$,

$K_B : K_C = 3 : 1$ Hint: Use conservation of energy at points A, B and C.

CASE BASED QUESTIONS:

Q 1. A block of mass m is kept in an elevator which starts moving downward with an acceleration a_0 as shown in figure. The block is observed by two observers A and B for a time interval t_0 .



(i) The observer B finds that the work done by gravity is

- (a) $\frac{1}{2}mg^2t_0^2$ (b) $-\frac{1}{2}mg^2t_0^2$ (c) $\frac{1}{2}mgat_0^2$ (d) $-\frac{1}{2}mgat_0^2$

(ii) The observer B finds that work done by normal reaction N is

- (a) Zero (b) $-Nat_0^2$ (c) $\frac{Nat^2}{2}$ (d) none of these

(iii) The observer B finds that work done by pseudo force is

- (a) Zero (b) $-ma^2t_0$ (c) $+ma^2t_0$ (d) $-mga t_0$

(iv) According to observer B, the net work done on the block is

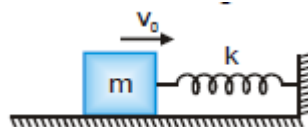
- (a) $\frac{1}{2}ma^2t_0^2$ (b) $-\frac{1}{2}ma^2t_0^2$ (c) $\frac{1}{2}mga t_0^2$ (d) $-\frac{1}{2}mga t_0^2$

OR

(v) According to the observer A

- (a) The work done by gravity is zero (b) the work done by normal reaction is zero
(c) The work done by pseudo force is zero (d) all the above

Q2. A block of mass m moving with a velocity v_0 on a smooth horizontal surface strikes and compresses a spring of stiffness k till mass comes to rest as shown in the figure. This phenomenon is observed by two observers:



A: Standing on the horizontal surface;

B: Standing on the block

(i) To an observer A, the work done by spring force is-

- (a) Negative but nothing can be said about its magnitude (b) $-\frac{1}{2}mv_0^2$
(c) Positive but nothing can be said about its magnitude (d) $+\frac{1}{2}mv_0^2$

(ii) To an observer A, the work done by the normal reaction N between the block and the spring on the block is

- (a) Zero (b) $-\frac{1}{2}mv_0^2$ (c) $-\frac{1}{2}mv_0^2$ (d) None of these

(iii) To an observing A, the net work done on the block is

- (a) $-mv_0^2$ (b) $+mv_0^2$ (c) $-\frac{1}{2}mv_0^2$ (d) zero

(iv) According to the observer A

- (a) The kinetic energy of the block is converted into the potential energy of the spring
(b) The mechanical energy of the spring-mass system is conserved
(c) The block loses its kinetic energy because of the negative work done by the conservative force of spring
(d) All the above

OR

(v) To an observer B, when the block is compressing the spring

- (a) Velocity of the block is decreasing (b) retardation of the block is increasing
(c) Kinetic energy of the block is zero (d) all the above

EXERCISE CASE STUDY BASED QUESTION

Q3. Two ice hockey pucks A (0.2 kg) and B (0.3 kg) move on a frictionless surface. Puck A moves at 4 m/s while B is at rest; after a perfectly elastic head-on collision, A rebounds with a velocity of 1 m/s to the left.

(i) What is the velocity of puck B after the collision? (ii) Verify whether kinetic energy is conserved. (iii) What type of collision is demonstrated here, and why?

(iv) What is the change in momentum of puck A?

ANSWERS: Q1 : (i) c (ii) d (iii) a (iv) b (v) d

Q2 : (i) b (ii) b (iii) c (iv) d (v) c

Q3 (i) 3.3 m/s (ii) yes (initial = final = 1.6 J) (iii) perfectly elastic collision (iv) -1 kg m/s

1. WORK DONE (W)

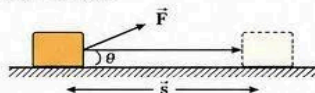
Work done by a constant force $\vec{F}$ on a body when displacement is $\vec{s}$

$$W = \vec{F} \cdot \vec{s} = Fs \cos \theta$$

Where, θ = angle between $\vec{F}$ and $\vec{s}$

SI Unit: joule (J)

1 J = 1 N·m



Special Cases

- $\theta = 0^\circ \Rightarrow W = +Fs$ (Maximum +ve work)
- $\theta = 90^\circ \Rightarrow W = 0$ (No work)
- $\theta = 180^\circ \Rightarrow W = -Fs$ (Maximum -ve work)

2. WORK DONE BY VARIABLE FORCE

Infinitesimal work done

$$dW = \vec{F} \cdot d\vec{s} = F ds \cos \theta$$

Total work done in moving from position 1 to 2

$$W_{1 \rightarrow 2} = \int_1^2 \vec{F} \cdot d\vec{s}$$

For a force along x-direction:

$$W_{x_1 \rightarrow x_2} = \int_{x_1}^{x_2} F(x) dx$$



QUICK RECALL

- Work is a scalar quantity
- Can be positive, negative or zero
- Only the component of force along displacement does work

3. KINETIC ENERGY (K.E.)

Work-Energy Theorem

Net work done on a body = Change in its kinetic energy

$$W_{\text{net}} = \Delta K = K_f - K_i$$

Kinetic Energy

$$K = \frac{1}{2} mv^2$$

Change in kinetic energy

$$\Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$$

Where, m = mass, v = speed

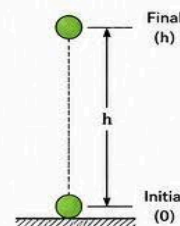
SI Unit: joule (J)

4. WORK DONE BY GRAVITATIONAL FORCE

Work done in raising a body of mass m through a height h

$$W_g = -mgh$$

(Negative, as force is opposite to displacement)



5. POTENTIAL ENERGY (P.E.)

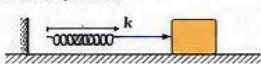
Gravitational Potential Energy (at height h from reference level)

$$U = mgh$$

Elastic Potential Energy (in a spring of spring constant k stretched/compressed by x)

$$U = \frac{1}{2} kx^2$$

SI Unit: joule (J)



6. CONSERVATION OF MECHANICAL ENERGY

If only conservative forces act on a body, total mechanical energy remains constant.

$$E_{\text{mechanical}} = K + U = \text{Constant}$$

For a body falling freely from rest through height h:

$$mgh = \frac{1}{2} mv^2 \Rightarrow v = \sqrt{2gh}$$

Examples of Conservative Forces

- Gravitational force
- Elastic force

Examples of Non-conservative Forces

- Frictional force
- Air resistance
- Applied force



MEMORY TRICK

Mechanical Energy (K + U) is conserved only when NO energy is wasted (no friction, no air resistance).

7. POWER (P)

Power is the rate of doing work.

$$P = \frac{dW}{dt}$$

For constant power

$$P = \frac{W}{t}$$

SI Unit: watt (W)

1 W = 1 J s⁻¹

POWER QUICK VIEW



Instantaneous Power

$$P = \vec{F} \cdot \vec{v}$$

($\vec{v}$ = instantaneous velocity)



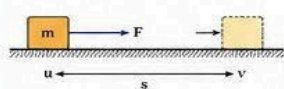
For constant force

$$P = Fv \cos \theta$$

θ = angle between $\vec{F}$ and $\vec{v}$

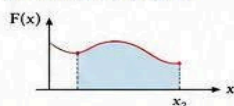
8. WORK-ENERGY THEOREM APPLICATIONS (NET WORK = CHANGE IN KINETIC ENERGY)

(a) Constant Force



$$Fs \cos \theta = \frac{1}{2} m (v^2 - u^2)$$

(b) Variable Force (1D)



$$W_{1 \rightarrow 2} = \int_{x_1}^{x_2} F(x) dx = \Delta K$$

★ IMPORTANT NOTES

- Work has no direction (scalar)
- K.E. cannot be negative
- P.E. depends on the reference level
- Power can be zero even when work is being done (e.g., circular motion with constant speed)

✗ COMMON MISTAKES

1. Ignoring direction ($\cos \theta$) in work formula.
2. Confusing work and energy.
3. Taking gravitational work as +mgh instead of -mgh while raising a body.
4. Forgetting that kinetic energy depends on speed, not velocity ($K = \frac{1}{2} mv^2$).
5. Using final speed instead of velocity in power formula.

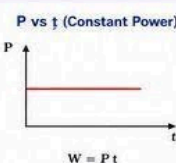
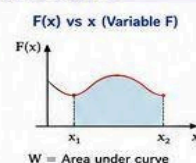
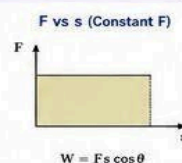
9. UNITS & DIMENSIONS

Quantity	Dimension	SI Unit
Work (W)	ML^2T^{-2}	Joule (J)
Energy (K, U)	ML^2T^{-2}	Joule (J)
Power (P)	ML^2T^{-3}	Watt (W)
Force (F)	MLT^{-2}	Newton (N)

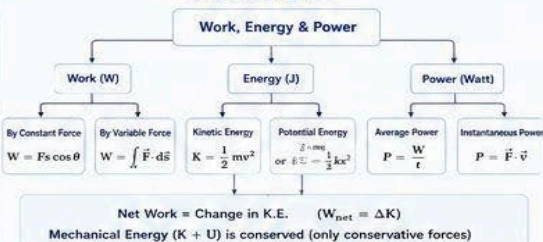
10. PYQ FAVOURITE TOPICS

- ✓ Work done by variable force (graph based)
- ✓ Work-Energy theorem applications (block on rough surface, U-incline, etc.)
- ✓ Conservation of mechanical energy (problems on height, speed)
- ✓ Power in vehicles, machines, and variable force motion

11. KEY GRAPHS



12. CONCEPT MAP



13. MNEMONICS

W – Work is done (in Joule)
E – Energy is stored (in Joule)
P – Power is rate (Joule per sec)
Think: WEP → Do – Store – Rate

• Another Trick:
“W on the move becomes E,
E at the rate becomes P.”



NEP 2020 FOCUS

- Conceptual Understanding
- Real-life Applications
- Critical Thinking
- Interactive & Experiential Learning

14. REAL LIFE APPLICATIONS



- Work-Energy theorem in cars (acceleration, braking, climbing a hill)
- Power in machines (cranes, lifts, engines)
- Potential energy in springs, roller coasters
- Energy transformation in sports (running, jumping, cycling)
- Mechanical energy conservation in satellite motion (ideal case)



STUDY SMART
Understand Concepts Deeply



PRACTICE DAILY
Solve PYQs & NCERT Examples



ANALYZE MISTAKES
Learn from errors & weak areas



REVISE REGULARLY
Short notes & formula sheets

**STAY CALM
STAY FOCUSED
YOU CAN DO IT!**



System of Particle and Rotational Motion

1. SYSTEM OF PARTICLES

RIGID BODY

- A rigid body is one in which the distances between different particles of the body do not change even though forces are acting on them.
- Axis of rotation:** The line along which the body is fixed is known as the axis of rotation.
- In pure rotation about a fixed axis, every particle of the rigid body moves in a circle in a plane perpendicular to the axis and has its centre on the axis. Every point has the same angular velocity at any time.



CENTRE OF MASS (C.M.)

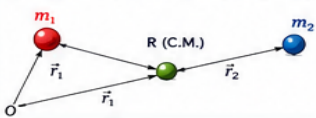
The centre of mass of a body is a point where the entire mass of the body can be supposed to be concentrated. The nature of motion executed by the body shall remain unaffected if all the forces acting on the body were applied directly at this point.

Two-particle system

For masses m_1, m_2 at position vectors $\vec{r}_1, \vec{r}_2$

$$\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

If $m_1 = m_2$, then $\vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{2}$
(Lies at the mid-point of the line joining them)



N-particle system

For masses m_i at position vectors $\vec{r}_i$ ($i = 1$ to N)

$$\vec{R} = \frac{\sum_{i=1}^N m_i \vec{r}_i}{\sum_{i=1}^N m_i}$$

Coordinates of C.M. (3-D)

$$x_{cm} = \frac{\sum m_i x_i}{M}, y_{cm} = \frac{\sum m_i y_i}{M}, z_{cm} = \frac{\sum m_i z_i}{M}$$

where $M = \sum m_i$

Continuous distribution of mass

$$x_{cm} = \frac{1}{M} \int x dm, y_{cm} = \frac{1}{M} \int y dm, z_{cm} = \frac{1}{M} \int z dm$$

$$\text{where } M = \int dm$$

Motion of Centre of Mass

$$\text{Velocity of C.M.: } \vec{V}_{cm} = \frac{d\vec{R}}{dt}$$

$$\text{Acceleration of C.M.: } \vec{a}_{cm} = \frac{d^2\vec{R}}{dt^2}$$



TORQUE (MOMENT OF A FORCE)

The turning effect of a force about an axis.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF \sin \theta$$

SI unit: N m | Dimensional formula: $[ML^2T^{-2}]$

ANGULAR MOMENTUM

$$\vec{L} = \vec{r} \times \vec{p}$$

For a rigid body about a fixed axis:

$$L = I\omega$$

SI unit: $kg m^2 s^{-1}$ | Dimensional formula: $[ML^2T^{-1}]$



LAW OF CONSERVATION OF ANGULAR MOMENTUM

If the net external torque acting on a system is zero, the total angular momentum of the system remains constant.

$$\vec{L}_{initial} = \vec{L}_{final}$$

Applications: Figure skating, collapsing of a star, rotations of planets, gyroscope.



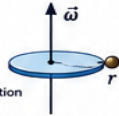
2. ROTATIONAL KINEMATICS

ANGULAR VELOCITY ($\vec{\omega}$)

$$\vec{\omega} = \frac{d\vec{\theta}}{dt}$$

- Directed along the axis of rotation (right-hand rule).

SI unit: $rad s^{-1}$ | Dimensional: $[T^{-1}]$

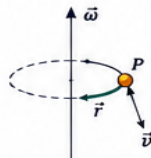


LINEAR VELOCITY AND ANGULAR VELOCITY

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$|\vec{v}| = \omega r$$

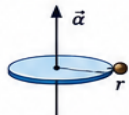
where $\vec{r}$ is the position vector of the particle from the axis.



ANGULAR ACCELERATION ($\vec{\alpha}$)

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

- SI unit: $rad s^{-2}$
- Dimensional: $[T^{-2}]$



EQUATIONS OF ROTATIONAL MOTION (Uniform angular acceleration)

- $\omega = \omega_0 + \alpha t$
- $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$
- $\omega^2 = \omega_0^2 + 2 \alpha \theta$
- $\theta = \frac{(\omega + \omega_0)}{2} t$

where ω_0 = initial angular velocity
 ω = final angular velocity
 θ = angular displacement
 t = time
 α = angular acceleration



TYPES OF MOTION OF RIGID BODY

- Pure Rotation:** Body rotates about a fixed axis. All points have same ω .



- Pure Translation:** All points have same $\vec{v}$.

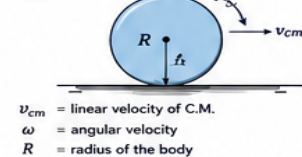


- General Plane Motion:** Combination of translation and rotation.



ROLLING MOTION (without slipping)

$$v_{cm} = \omega R$$



v_{cm} = linear velocity of C.M.
 ω = angular velocity
 R = radius of the body

3. ROTATIONAL DYNAMICS

EQUATION OF ROTATIONAL MOTION

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

For rotation about a fixed axis (I constant):

$$\tau_{net} = I \alpha$$

SI unit of τ : N m

Dimensional formula: $[ML^2T^{-2}]$



CONSERVATION OF ANGULAR MOMENTUM

If net external torque on a system is zero, angular momentum is conserved.

$$\vec{L}_i = \vec{L}_f$$

- Applies to a closed system.
- Useful in many applications.



EQUILIBRIUM OF RIGID BODIES

A rigid body is in equilibrium if

$$\sum \vec{F} = \vec{0} \quad (\text{Translational equilibrium})$$

$$\sum \vec{\tau} = 0 \quad (\text{Rotational equilibrium})$$

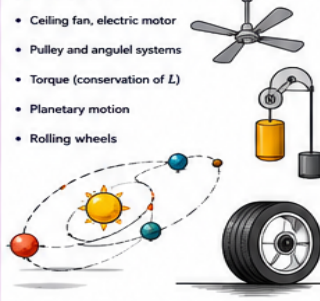


COMPARISON: LINEAR vs ROTATIONAL

	Linear Motion	Rotational Motion
Displacement	x	θ
Velocity	v	ω
Acceleration	a	α
Force	F	Torque (τ)
Mass (m)	m	Moment of inertia (I)
Newton's 2nd law	$F = ma$	$\tau = I \alpha$
Momentum	$p = mv$	$L = I \omega$
K.E.	$\frac{1}{2} m v^2$	$\frac{1}{2} I \omega^2$

APPLICATIONS

- Ceiling fan, electric motor
- Pulley and angular systems
- Torque (conservation of L)
- Planetary motion
- Rolling wheels



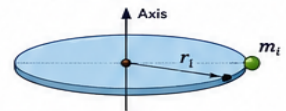
4. MOMENT OF INERTIA

MOMENT OF INERTIA (I)

I about a given axis = Sum of the product of masses of all particles and the square of their perpendicular distances from the axis.

$$I = \sum m_i r_i^2$$

- SI unit: $kg m^2$
- Dimensional formula: $[ML^2]$
- Depends on:
 - Position of axis of rotation
 - Orientation of axis
 - Shape, size and mass distribution



RADIUS OF GYRATION (K)

Distance from axis at which, if the whole mass were concentrated, moment of inertia would remain the same.

$$K = \sqrt{\frac{I}{M}}$$

- Also, root mean square distance of particles from the axis.

$$I = MK^2$$

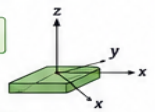
SI unit: m | Dimensional: $[L]$

THEOREMS

1. Perpendicular Axes Theorem (For Planar Lamina)

Moment of inertia about an axis perpendicular to its plane equals the sum about two perpendicular axes in the plane.

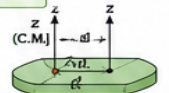
$$I_z = I_x + I_y$$



2. Parallel Axis Theorem

Moment of inertia about any axis is equal to the moment about a parallel axis through C.M. plus Md^2 .

$$I = I_{cm} + Md^2$$



MOMENT OF INERTIA (I) AND RADIUS OF GYRATION (K)

S.No.	Body	Axis of Rotation	I	$K = \sqrt{\frac{I}{M}}$	Diagram
1	Uniform circular ring (mass M , radius R)	(i) Centre, $\perp$ to plane (ii) Diameter in its plane	MR^2 $\frac{1}{2} MR^2$	R $\frac{R}{\sqrt{2}}$	
2	Solid disc (mass M , radius R)	(i) Centre, $\perp$ to plane	$\frac{1}{2} MR^2$	$\frac{R}{\sqrt{2}}$	
3	Solid sphere (mass M , radius R)	Any diameter	$\frac{2}{5} MR^2$	$\frac{2}{\sqrt{5}} R$	
4	Thin rod (length L , mass M)	(i) Centre, $\perp$ to length (ii) End, $\perp$ to length	$\frac{1}{12} ML^2$ $\frac{1}{3} ML^2$	$\frac{L}{\sqrt{12}}$ $\frac{L}{\sqrt{3}}$	
5	Rectangular plate ($l \times b$, mass M)	Centre, $\perp$ to plane	$\frac{1}{12} M(l^2 + b^2)$	$\frac{\sqrt{l^2 + b^2}}{\sqrt{12}}$	
6	Hollow cylinder (mass M , radius R)	Axis of cylinder	MR^2	R	

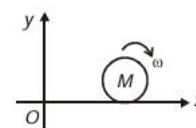
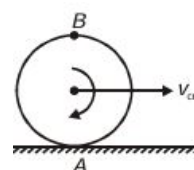
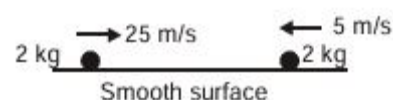
KEY POINTS

- In pure rotation, all points have same angular velocity.
- Linear and angular quantities are related through $v = \omega r$.
- Torque causes change in angular momentum.
- Moment of inertia depends on axis, shape and mass distribution.
- Conservation of angular momentum is valid for closed system.
- Rolling without slipping: $v_{cm} = \omega R$.



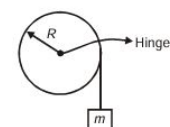
MULTIPLE CHOICE QUESTION

- A planet orbits the Sun in an elliptical path as shown in the figure. Let v_p and v_a be the speeds of the planet at perihelion and aphelion respectively. Which of the following relations is correct?
 (a) $r_p / r_a = v_a / v_p$ (b) $r_p / r_a = v_p / v_a$ (c) $r_p / r_a = \sqrt{v_p / v_a}$ (d) $r_p / r_a = \sqrt{v_a / v_p}$
- The centre of mass of a system of two particles of masses 2 kg and 3 kg located at (1, 0) m and (5, 0) m respectively is at:
 (a) (2, 0) m (b) (3.4, 0) m (c) (4, 0) m (d) (5, 0) m
- A disc and a ring, both of mass M and radius R , are released from rest at the top of an inclined plane. Which one reaches the bottom first and why?
 (a) Disc, because it has lower moment of inertia (b) Ring, because it has higher moment of inertia
 (c) Both reach together (d) cannot be determined
- If the net external force acting on a system is zero, which of the following must be true?
 (a) The centre of mass moves with constant velocity (b) The system is at rest
 (c) The angular momentum is conserved (d) The moment of inertia is zero
- A rod of length L and mass M is pivoted at one end. Its moment of inertia about the pivot is: (a)
 (1/3) ML^2 (b) (1/12) ML^2 (c) (1/2) ML^2 (d) ML^2
- A ring is rotating about a diameter. The radius = 5cm & mass of ring = 1kg. A force is applied on the ring such that it is perpendicular to the axis and vector AB as shown in the figure. The magnitude of force is 10N. Find the work done by the torque, when the ring rotates by 90° .
 (a) 0.25π J (b) 0.15π J (c) 0.2π J (d) 0
- Calculate the velocity of the centre of mass of the system of two particles each of mass 2 kg as shown in figure
 (a) 5 m/s (b) 10 m/s (c) 2.5 m/s (d) 15 m/s
- A disc is undergoing in pure rolling on a horizontal surface, then which of the statement about angular momentum of the disc is incorrect?
 (a) Angular momentum about A is conserved under any situation
 (b) Angular momentum about B is conserved only if surface is smooth
 (c) Angular momentum about any point on the disc is conserved
 (d) None of these
- If a spherical ball is under pure rolling on a table, then the fraction of its total kinetic energy associated with rotation is
 (a) 3/5 (b) 2/7 (c) 2/5 (d) 3/7
- A wheel starts from rest and attains an angular velocity of 20 rev/s after being uniformly accelerated for 20 s. The total angle in radian through which it has turned in 10 s is
 (a) 200π (b) 40π (c) 150π (d) 100π
- A solid cylinder, a solid sphere and a hollow sphere each of mass m and radius r are released from the top of a smooth inclined plane. Then which of the bodies has minimum acceleration down the plane?
 (a) Solid cylinder (b) Solid sphere (c) Hollow sphere (d) All have same acceleration
- A solid cylinder of mass m and radius r is rolling with angular speed ω on a horizontal plane. The magnitude of its angular momentum about origin O is
 (a) $mr^2\omega$ (b) $1.5 mr^2\omega$ (c) $2 mr^2\omega$ (d) $2.5 mr^2\omega$



13. Calculate the torque acting on the disc in the given arrangement (Radius of disc 1m and mass m)

- (a) $2mg/3$ (b) $mg/3$ (c) mg (d) $4mg/3$



14. Magnitude of angular momentum of a wheel changes from $2L$ to $3L$ in 5 sec. by a constant torque acting opposite to initial direction of rotation.

What is the magnitude of the torque?

- (a) $L/5$ (b) L (c) $2L/5$ (d) $3L/5$

15. A shell following a parabolic path explodes somewhere in its flight. The centre of mass of fragments will move in

- (a) Tangential direction (b) Radial direction (c) Horizontal direction (d) Same parabolic path

16. An elliptical disc shown in the figure is rotated in turn about $x-x'$, $y-y'$ and $z-z'$ axes passing through the centre of mass of the disc. Moment of inertia of the disc is

- (a) Same about all the three axes (b) Maximum about $z-z'$ axes
(c) Maximum about $y-y'$ axis (d) Same about $x-x'$ and $y-y'$ axis

17. Choose the incorrect statement

- (a) The centre of mass of a two particle system lies on the line joining the two particles, being closer to the heavier particle
(b) In rolling, the point of contact of the rolling body remains at rest relative to the surface on which it is rolling
(c) Parallel axis theorem is applicable only for laminar bodies
(d) A particle moving on a straight line may have non-zero angular momentum about a point

18. A stick of length L mass M initially upright on a frictionless floor, starts falling, then

- (a) Center of mass will fall vertically down (b) Center of mass will follow a circular path
(c) Center of mass will follow any curve path (d) All of these

19. A shell following a parabolic path explodes somewhere in its flight. The centre of mass of fragments will move in

- (a) Tangential direction (b) Radial direction (c) Horizontal direction (d) Same parabolic path

20. If torque acting on a system about an axis of rotation is zero, then the quantity that remains constant about that axis is

- (a) Force (b) Linear momentum (c) Angular momentum (d) Angular velocity

ASSERTION REASON TYPE QUESTIONS:

(a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion A

(c) Assertion (A) is true, but Reason (R) is false. (d) If both, Assertion (A) and Reason (R) are false.

21. Assertion (A): It is harder to open and shut the door if we apply force near the hinge.

Reason (R): Torque is maximum at hinge of the door.

22. Assertion (A): An ice-skater **stretches** out arms-legs during performance.

Reason (R): Stretching out arms-legs helps the performer to balance his or her body so that he or she does not fall.

23. Assertion (A): When you **lean** behind over the hind legs of the chair, the chair falls back after a certain angle.

Reason (R): Centre of mass lying outside the system makes the system unstable.

24. Assertion (A): The speed of a whirl wind in a tornado is alarmingly high.

Reason (R): If no external torque acts on a body, its angular velocity remains constant

25. Assertion (A): If two particles have equal mass, their centre of mass lies at the midpoint.
Reason (R): Centre of mass is inversely proportional to mass.
26. Assertion (A): Friction is necessary for a body to roll on a surface.
Reason (R): Friction provides the necessary tangential force and torque.
27. Assertion (A): A body tied to a string is moved in a circle with uniform speed. If the string suddenly breaks, the angular momentum of the body becomes zero.
Reason (R): The torque on the body equals the rate of change of angular momentum.
28. Assertion (A): If there is no external torque on a body about its centre of mass, then the velocity of the centre of mass remains constant.
Reason (R): The linear momentum of an isolated system remains constant.
29. Assertion (A): Two cylinders, one hollow and one solid (wood), having the same mass and identical dimensions are released simultaneously to roll without slipping down an inclined plane from the same height. The hollow cylinder reaches the bottom first.
Reason (R): By the principle of conservation of energy, the total kinetic energies of both cylinders are identical when they reach the bottom of the incline.
30. Assertion — If the cylinder rolling with angular speed ω suddenly breaks up into two equal halves of the same radius, the angular speed of each piece becomes 2ω .
Reason — If no external torque acts, the angular momentum of the system is conserved.

HINTS AND SOLUTIONS

1Q	2Q	3Q	4Q	5Q	6Q	7Q	8Q	9Q	10Q
A	B	A	A	A	C	B	C	B	D
11Q	12Q	13Q	14Q	15Q	16Q	17Q	18Q	19Q	20Q
D	B	B	B	D	B	C	A	D	C
21	22	23	24	25	26	27	28	29	30
C	C	C	C	C	A	D	D	D	A

SHORT ANSWER TYPE(2 MARKS QUESTION):

- Two blocks of masses 10 kg and 20 kg are placed on the x-axis. The first mass is moved on the axis by a distance of 2 cm. By what distance should the second mass be moved to keep the position of centre of mass unchanged?
- A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.
- (i) Why do we prefer to use a wrench, having a long arm?
(ii) Under what condition, the torque due applied can be considered to be zero?
- (i) Is there any possibility of coinciding with the geometrical centre and C.M. of a body?
(ii) Does the C.M of a body necessarily have to lie inside a body?
- (i) What are the factors on which the M.I of a body depends?
(ii) When a cat can land on its feet after a fall, which property of Physics is being used by her?
- (i) What is a rigid body?
(ii) Why cannot a single force balance the torque?
- To maintain a rotor at a uniform angular speed of 200 rad s^{-1} , an engine needs to transmit a torque of 180 Nm. What is the power required by the engine?
Note: Uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is 100 efficient.

8. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?
9. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?
10. What are the important features of angular momentum?

ANSWERS (2 MARS)

- 1 Two masses m_1 and m_2 are placed on the X-axis $m_1 = 10 \text{ kg}$, $m_2 = 20 \text{ kg}$

The first mass is displaced by a distance of 2 cm

$$\overline{X_{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{10 \times 2 + 20 x_2}{30} \Rightarrow 0 = \frac{20 + 20 x_2}{30} \Rightarrow x_2 = -1 \text{ cm}$$

$\therefore$ The 2nd mass should be displaced by a distance 1 cm towards left so as to kept the position of centre of mass unchanged.

- 2 Let F_1 and F_2 be the forces exerted by the level ground on front wheels and back wheels respectively. Considering rotational equilibrium about the front wheels, $F_2 \times 1.8 = mg \times 1.05$, $F_2 = (1.05/1.8) \times 1800 \times 9.8 \text{ N} = 10290 \text{ N}$, Force on each back wheel is $= 10290/2 \text{ N}$ or 5145 N .

Considering rotational equilibrium about the back wheels.

$$F_1 \times 1.8 = mg (1.8 - 1.05) = 0.75 \times 1800 \times 9.8, F_1 = 7350 \text{ N}$$

Force on each front wheel is $7350/2 \text{ N}$ or 3675 N .

- 3 We know, the turning effect of a force is $\tau = r \times F$

Now when the arm of the wrench is long, r is larger. Therefore, a smaller force will be required to produce the same turning effect. (ii) As we know, $\tau = r F \sin \theta$ Now, if $\theta = 0$ or 180 , Or, $r = 0$, then $\tau = 0$ Now, $r = 0$ refers to the applied force passing through the axis of rotation.

- 4 (i) C.M. and geometrical centre may coincide when the body in consideration has a uniform mass density. So, yes, they may coincide. For instance, C.M. and geometrical centre are the same in the case of a cube or a sphere or cylinder etc.

(ii) No, the C.M of the body in consideration does not necessarily be inside the body itself. I also may lie outside the body. In the case of the semi-circular ring, it is at the centre which is outside the ring.

- 5 (i) The M.I. of a body depends on the following factors- Position of the axis of the rotation, Mass of the body, Way of distribution of mass about the axis of rotation.

(ii) If the cat can land on its feet after a fall, she is using the principle of conservation of angular momentum.

- 6 (i) A rigid body refers to an object in which the distance between all the constituting particles remains fixed under any influence of external force. Therefore, a rigid body conserves its shape during motion.

(ii) The effect of torque can be seen to produce angular acceleration and its effect is entirely different from that of the force that causes linear acceleration. Therefore, a single force cannot balance the torque.

- 7 Here, $a = 200 \text{ rad s}^{-1}$; Torque, $\tau = 180 \text{ N-m}$

$$\text{Since, Power, } P = \text{Torque } (\tau) \times \text{angular speed } (\omega) = 180 \times 200 = 36000 \text{ W} = 36 \text{ KW.}$$

- 8 Use the **principle of moments** about the new balance point at 45 cm

Coins produce moment on one side: , Metre stick's weight acts at 50 cm

$M \times (50 - 45)$, Now equate clockwise moment = anticlockwise moment.

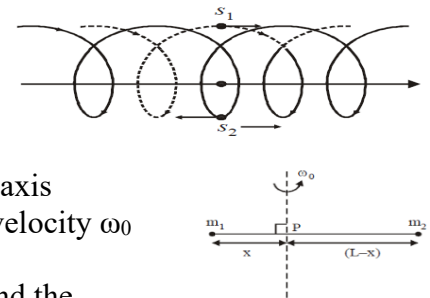
- 9 Let M be the mass and R the radius of the hollow cylinder, and also of the solid sphere. Their moments of inertia about the respective axes are $I_1 = MR^2$ and $I_2 = \frac{2}{5} MR^2$. Let τ be the magnitude of the torque applied to the cylinder and the sphere, producing angular accelerations α_1 and α_2 respectively. Then $\tau = I_1 \alpha_1 = I_2 \alpha_2$

The angular acceleration α produced in the sphere is larger. Hence, the sphere will acquire larger angular speed after a given time.

10. The following are the important features of angular momentum- The angular momentum of a particle with respect to a point provides an idea of the strength of its rotational tendency about the point. The magnitude of the angular momentum can be defined in terms of mass and velocity of the particle and its distance from the reference point i.e. $L = mvr$ The vector concept of angular momentum is important and useful. Its direction is the axial direction portrayed by the right-handed rule.

SHORT ANSWER TYPE 2 (3MARKS):

- A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1} . The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?
- Using expression of power & K.E. of rotational motion, derive the relation $\tau = I\alpha$.
- A circular ring having a diameter of 40cm and mass of 1 kg is rotating about an axis normal to its plane and passing through the center with a frequency of 10 rotations per second. Now, calculate the angular momentum about the axis of rotation.
- Three separate particles of masses of 1g, 2g, and 3g have their center of mass at the precise point (2,2,2). What should be the position of the fourth particle of mass 4g so that the combined center of mass may be at the point (0,0,0).
- A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minutes. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90 cm to 20 cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg m^2 .
(a) What is his new angular speed? (Neglect friction)(b) Is kinetic energy conserved in the process? If not, from where does the change come about?
- The motion of binary stars, S1 and S2 is the combination of X and Y . What are X and Y refer to?



- The rod is to be set rotating about an axis perpendicular to it. What is the position of point P on this rod through which the axis should pass so that the work required to set the rod rotating with angular velocity ω_0 is minimum?
(b) A force $F = \alpha \hat{i} + 3 \hat{j} + 6 \hat{k}$ is acting at a point $r = 2 \hat{i} - 6 \hat{j} - 12 \hat{k}$. Find the value of α for which angular momentum about origin is conserved.
- A solid cylinder of mass 50 kg and radius 0.5 m is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. What is the Tension in the string required to produce an angular acceleration of $2 \text{ revolutions s}^{-2}$
- (a) A particle moving in a circular path has an angular momentum of L . If the frequency of rotation is halved, then how its angular momentum changes.
(b) The instantaneous angular position of a point on a rotating wheel is given by the equation $\theta(t) = 2t^3 - 6t^2$. Then at what time torque on the wheel becomes zero?
- The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 seconds.
(i) What is its angular acceleration, assuming the acceleration to be uniform?
(ii) How many revolutions does the engine make during this time?

ANSWERS (3 MARKS)

- Answer: $M = 20 \text{ kg}$ Angular speed, $\omega = 100 \text{ rad s}^{-1}$; $R = 0.25 \text{ m}$
M.O.I of the cylinder about its axis $= \frac{1}{2} MR^2 = \frac{1}{2} \times 20 (0.25)^2 = 0.625 \text{ kg m}^2$
Rotational kinetic energy, $E_r = \frac{1}{2} I \omega^2 = \frac{1}{2} \times 0.625 \times (100)^2 \text{ J} = 3125 \text{ J}$
Angular momentum, $L = I \omega = 0.625 \times 100 \text{ Js} = 62.5 \text{ Js}$.
- Answer: As we know that power is given by

$$P = \tau(\omega) \dots \dots \dots (i)$$

$$\& K.E. = 1/2 I\omega^2 \dots \dots \dots (ii)$$

Now, the power of rotational motion is equal to time rate of work done during the rotational motion.

Since the work done is stored in the form of kinetic energy,

$P = d/dt$ (K.E. of rotational motion) Using the equations (i) and (ii), we have,

$$\tau(\omega) = d/dt(1/2 I(\omega)^2) = 1/2 I d/dt(\omega^2) = 1/2 \times I \times (2\omega d\omega/dt) = I\omega\alpha (\because d\omega/dt = \alpha) \dots \dots \dots (iii)$$

From (i) and (iii), we get: $\tau\omega = I\omega\alpha \Rightarrow \tau = I\alpha$.

3 Answer: From the question we can obtain, $R = 40/2 \text{ cm} = 20 \text{ cm} = 0.2 \text{ m}$

$M = 1 \text{ kg}$, $v = 10 \text{ rotations / sec}$ Now, we can calculate the moment of inertia as follows:

$$M.I = MR^2 = 1 \times (0.2)^2 = 0.04 \text{ kg sq.m} \quad \omega = 2\pi \times 10 = 20\pi \text{ rad / sec}$$

Thus, angular momentum can be calculated as,

$$\Rightarrow L = 0.04 \times 20\pi \Rightarrow L = 2.51 \text{ kg sq m / sec}$$

4 Answer: Given $m_1=1\text{g}$ $m_2=2\text{g}$ $m_3=3\text{g}$ $m_4=4\text{g}$

Centre of mass of three particle system $r_{cm}=(x_{cm}, y_{cm}, z_{cm})=(2,2,2)$

Position of 4th Particle $r_4=?$ If centre of mass of four particle is $(0,0,0)$

Centre of mass for 3 particle system is

$$X_{cm}=(m_1x_1+m_2x_2+m_3x_3)/(m_1+m_2+m_3)=(x_1+2x_2+3x_3)/(1+2+3)$$

$$2=(x_1+2x_2+3x_3)/6 \quad x_1+2x_2+3x_3=12 \dots \dots \dots (i)$$

$$\text{Similarly} \quad y_1+2y_2+3y_3=12 \dots \dots \dots (ii), \quad z_1+2z_2+3z_3=12 \dots \dots \dots (iii)$$

Centre of mass for 4 particle system is

$$X_{cm}=(m_1x_1+m_2x_2+m_3x_3+m_4x_4)/(m_1+m_2+m_3+m_4)=(x_1+2x_2+3x_3+4x_4)/(1+2+3+4)$$

$$\Rightarrow 0=(12+4x_4)/10 \Rightarrow 12+4x_4=0 \Rightarrow x_4=-3 \text{ similarly } y_4=-3 \quad z_4=-3$$

Thus the new particle should be at $(-3,-3,-3)$.

5 Answer: Here, $I_1 = 7.6 + 2 \times 5 (0.9)^2 = 15.7 \text{ kg m}^2$

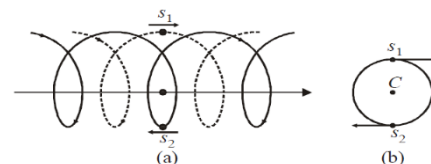
$$\omega_1 = 30 \text{ rpm} \quad I_2 = 7.6 + 2 \times 5 (0.2)^2 = 8.0 \text{ kg m}^2 \quad \omega_2 = ?$$

According to the principle of conservation of angular momentum, $I_1\omega_1 = I_2\omega_2$

$$\Rightarrow \omega_2 = (I_1 \omega_1) / I_2 = (15.7 \times 30) / 8.0 = 58.88 \text{ rpm}$$

No, kinetic energy is not conserved in the process. In fact, as moment of inertia decreases, K.E. of rotation increases. This change comes about as work is done by the man in bringing his arms closer to his body.

6 Answer: Here X and Y refer to motion of the CM and motion about the CM When no external force acts on the binary star, its CM will move like a free particle [Fig. (a)]. From the CM frame, the two stars will seem to move in a circle about the CM with diametrically opposite positions.



(a) Trajectories of two stars.

(b) The same binary system, with the centre of mass C at rest. So, to understand the motion of a complicated system, we can separate the motion of the system into two parts. So, the combination of the motion of the CM and motion about the CM could describe the motion of the system.

7 Answer: (a) work required to set the rod rotating with angular velocity ω_0 is $K.E = 1/2 I\omega^2$. Work is minimum when I is minimum, I is minimum about the centre of mass Therefore $m_1x = m_2(L-x) = m_2L - m_2x \Rightarrow x = m_2L/(m_1+m_2)$

(b) From Newtons second law for rotational mechanics

$$\tau = dL/dt, \text{ if } L \text{ is conserved then } dL = 0, \tau = 0 \Rightarrow \tau = r \times F = 0 \Rightarrow (2\hat{i} - 6\hat{j} - 12\hat{k}) \times$$

$$(\alpha\hat{i} + 3\hat{j} + 6\hat{k}) = 0, \text{ On solving we will get } \alpha = -1$$

8 Answer: Given mass $m = 50 \text{ kg}$, radius $r = 0.5 \text{ m}$

$$\text{Angular acceleration } \alpha = 2 \text{ revolutions s}^{-2} = 2 \times 2\pi \text{ rad s}^{-2} = 4\pi \text{ rad s}^{-2}, \text{ Tension } T = ?$$

$$\text{We know that } \tau = I\alpha \Rightarrow r \times F (\text{Tension}) = I\alpha \Rightarrow \text{Tension} = I\alpha / r \dots \dots \dots (i)$$

$$\text{Here } I_{\text{cylinder}} = \frac{1}{2}MR^2 = \frac{1}{2} (50) (0.5)^2 = 6.25 \text{ kg m}^2, \text{ Therefore from eq (i) } T = (6.25 \times 4\pi) / 0.5 = 50\pi$$

$$N = 157 \text{ N}$$

9 Answer: (a) Given angular momentum $L_1 = L$, Frequency $f_1 = f$

$$\text{Angular momentum } L_2 = ? \quad \text{If Frequency } f_2 = f/2$$

W.K.T angular velocity $\omega_1 = 2\pi f$, angular velocity $\omega_2 = 2\pi f_2 = 2\pi f/2 = \pi f = \omega_1/2$
 and angular momentum $L_1 = I\omega_1$ angular momentum $L_2 = I\omega_2 = I\omega_1/2 = L_1/2$
 (b) When angular acceleration (α) is zero then torque on the wheel becomes zero.
 $\theta(t) = 2t^3 - 6t^2$, $\omega = d\theta(t)/dt = 6t^2 - 12t$, $\alpha = d\omega/dt = 12t - 12$, Here $\alpha = 0$
 Therefore $12t - 12 = 0$, $12t = 12$, $t = 1$ sec

- 10** Answer (i) We know $\omega = \omega_0 + \alpha t$
 $\omega_0 =$ initial angular speed in rad/s $= 2\pi \times$ angular speed in rev/s $= 2\pi \times 20$ rad/s $= 40\pi$ rad/s
 Similarly $\omega =$ final angular speed in rad/s $= 2\pi \times 52$ rad/s $= 104\pi$ rad/s
 $\therefore$ Angular acceleration $\alpha = (\omega - \omega_0)/t = (104\pi - 40\pi)/16 = 64\pi/16 = 4\pi$ rad/s²
 The angular acceleration of the engine $= 4\pi$ rad/s²
 (ii) The angular displacement in time t is given by
 $\theta = \omega_0 t + 1/2 \alpha t^2 = 40\pi \times 16 + 1/2 (4\pi \times 16 \times 16) = (640\pi + 512\pi)$ rad $= 1152\pi$ rad
 Number of revolutions $= 1152\pi/2\pi = 576$

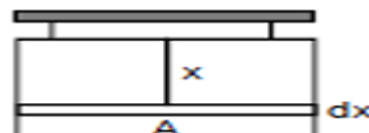
LONG ANSWER TYPE QUESTIONS:

- What are the differences between the centre of gravity and the centre of mass?
 - A square plate of mass 120 g and edge 5.0 cm rotates about one of edges. If it has a uniform angular acceleration of 0.2 rad/s^2 , what torque acts on the plate?
- A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to $2/5$ times the initial value? Assume that the turntable rotates without friction.
 - Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?
- A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.
 - Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time.
- Why there are two propellers in a helicopter?
 - Can a body in translatory motion have angular momentum? Explain.
 - A particle performs uniform circular motion with an angular momentum L . If the frequency of particle's motion is doubled and its K.E is halved, what happens to the angular momentum?

ANSWERS (5M)

1 Answer: a) (i) The centre of gravity is a point in a body where weight distribution is equal in all directions. It depends on the gravitational field. (ii) The centre of mass is a point where mass distribution is uniform in all directions. It does not rely on the gravitational field. However, a body's centre of gravity and centre of mass is positioned at the same point in a homogeneous gravitational field.

(b) A square plate of mass 120 gm and edge 5 cm rotates about one of the edge. Let take a small area of the square of width dx and length a which is at a distance x from the axis of rotation.



Therefore mass of that small area

$$\frac{m}{a^2} \times adx \quad (m = \text{mass}, x = \text{side})$$

$$I = \int_0^a (m/a^2) \times ax^2 dx = (m/a)(x^3/3) \Big|_0^a = ma^2/3$$

$$\text{torque produced} = I \times \alpha = (ma^2/3) \times \alpha = 0.2 \times 10^{-4} = 2 \times 10^{-5} \text{ N-m.}$$

$$n \text{ final moment of inertia } I_2 = (2/5) I_1 \text{ Angular speed } n_1 = 40 \text{ rev/min Angular speed } n_2 = 40 \text{ rev/min}$$

By principle of conservation of angular momentum $I_1\omega_1 = I_2\omega_2$

$$I_1 (2 \pi n_1) = I_2 (2 \pi n_2) \rightarrow n_2 = (I_1 n_1) / I_2 = (I_1 40) / (2/5) I_1 = 100 \text{ rev/min}$$

$$(b) \text{Final K.E}_{\text{Rotation}} / \text{initial K.E}_{\text{Rotation}} = \frac{1}{2} I_2 \omega_2^2 / \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} I_2 (2 \pi n_2)^2 / \frac{1}{2} I_1 (2 \pi n_1)^2 \\ = I_2 n_2^2 / I_1 n_1^2 = (2/5) I_1 (100)^2 / I_1 (40)^2 \quad \text{Final K.E}_{\text{Rotation}} / \text{initial K.E}_{\text{Rotation}} = 2.5$$

From here we conclude that K.E becomes more because the child uses his internal energy when he folds his hands to increase the kinetic energy.

3 Answer: (a) Given Force applied $F = 30 \text{ N}$, mass $m = 3 \text{ kg}$, radius $r = 40 \text{ cm} = 0.4 \text{ m}$

Moment of inertia of the hollow cylinder about its axis.

$$I = MR^2 = 3(0.4)^2 = 0.48 \text{ kg m}^2 \quad \text{Torque } \tau = r \times F = 0.4 \times 30 = 12 \text{ Nm}$$

$$\text{If } \alpha \text{ is angular acceleration produced, then from } \tau = I\alpha \rightarrow \alpha = \tau/I = 12/0.48 = 25 \text{ rad s}^{-2}$$

$$\text{Linear acceleration } a = R\alpha = 0.4 \times 25 = 10 \text{ ms}^{-2}$$

Let M be mass and R the radius of the hollow cylinder, and also of the solid sphere. Their moments of inertia about respective axes are $I_1 = MR^2$ & $I_2 = \frac{2}{5} MR^2$

Let τ be the magnitude of the torque applied to the cylinder and the sphere, producing angular accelerations α_1 and α_2 respectively. Then $\tau = I_1 \alpha_1 = I_2 \alpha_2$

The angular acceleration α_2 produced in the sphere is larger. Hence, the sphere will acquire larger angular speed after a given time.

4 Answer: (a) If there were only one propeller in a helicopter then, due to conservation of angular momentum, the helicopter itself would have turned in the opposite direction.

(b) Yes, a body in translatory motion shall have angular momentum, the fixed point about which angular momentum is taken lies on the line of motion of the body. This follows from $|L| = r p \sin \Theta$, $L=0$, only when $\Theta = 0^\circ$ or $\Theta = 180^\circ$.

$$(c) L = mvr \text{ and } v = r \omega = r(2\pi n) \rightarrow r = v/(2\pi n)$$

$$L = mv[v/(2\pi n)] = mv^2/(2\pi n) \dots\dots\dots (i) \quad \text{As K.E} = \frac{1}{2} mv^2 \dots\dots\dots (ii) \quad \text{From (i) \& (ii)}$$

$$L = \text{K.E} / \pi n \text{ when K.E is halved and frequency } n \text{ is doubled then } L_2 = (\text{K.E}/2)/(\pi 2n), L_2 = \text{K.E} / 4 \pi n \\ = L/4, \text{ Therefore The angular momentum becomes } \frac{1}{4}$$

COMPETENCY-BASED QUESTION 1: CENTRE OF MASS

Two students are carrying a long wooden plank of mass 40 kg. One student holds it near one end while the other supports it at the opposite end. Their teacher explains that the plank remains balanced because its centre of mass lies at the middle of the plank. If one student moves closer to the centre, the other student experiences more load. This concept is useful in carrying goods, balancing objects and designing vehicles.

1. The point where the entire mass of the body appears to be concentrated is called:

- A) Axis of rotation B) Torque C) Centre of mass D) Momentum

2. For a uniform rod, the centre of mass lies:

- A) At one end B) At the middle C) Outside the rod D) Near the heavier side

3. If one student moves closer to the centre of the plank, the other student:

- A) Experiences more load B) Experiences less load
C) Experiences no load D) Stops balancing

4. A uniform rod is 4 m long. The centre of mass of the rod is located at:

- A) 1 m B) 2 m C) 3 m D) 4 m

COMPETENCY-BASED QUESTION 2: TORQUE AND ROTATIONAL MOTION

A mechanic uses a long spanner to loosen a tight nut of a machine. He finds that applying force at the far end of the spanner makes the nut rotate more easily. His assistant learns that the turning effect of force is called torque. The magnitude of torque depends on the force applied and the perpendicular distance from the axis of rotation.

1. The turning effect of force is called:

- A) Momentum B) Torque C) Energy D) Pressure

2. Torque depends on:

- A) Mass only B) Speed only C) Force and perpendicular distance D) Density only

3. A longer spanner helps because it:
 A) Reduces force B) Increases torque C) Reduces mass D) Stops rotation
4. A force of 20 N is applied perpendicular to a spanner of length 0.5 m. The torque produced is $\tau = r \times F$
 A) 5 Nm B) 10 Nm C) 15 Nm D) 20 Nm

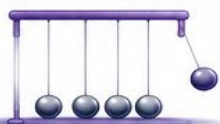
CBQ 3: CONSERVATION OF ANGULAR MOMENTUM

A dancer performing on a stage starts spinning with her arms stretched outward. As she pulls her arms closer to her body, her spinning speed increases. The audience observes that no external torque acts on the dancer during the motion. Her physics teacher explains that this happens due to the conservation of angular momentum, which is also observed in planets, spinning tops and athletes.

- The spinning speed of the dancer increases when:
 A) She stretches her arms outward B) She stops rotating
 C) She pulls her arms inward D) She bends down
- The principle involved is:
 A) Conservation of energy B) Conservation of angular momentum
 C) Newton's first law D) Archimedes principle
- Angular momentum remains constant when:
 A) External torque acts B) Mass becomes zero
 C) No external torque acts D) Velocity becomes zero
- A wheel rotates at 2 rad/s. If its angular velocity becomes 6 rad/s due to decrease in moment of inertia, the angular velocity increases by:
 A) 2 rad/s B) 3 rad/s C) 4 rad/s D) 8 rad/s

SOLUTIONS OF CBQ

CBQ 1 : ANSWERS 1Q C 2Q B 3Q A 4Q B
 CBQ 2 : ANSWERS 1Q B 2Q C 3Q C 4Q C
 CBQ 3 : ANSWERS 1Q C 2Q B 3Q C 4Q C



SYSTEM OF PARTICLES & ROTATIONAL MOTION



CBSE CLASS XI PHYSICS

1. SYSTEM OF PARTICLES

A number of particles forming a system may interact with one another through forces. The motion of each particle is affected by external forces as well as forces due to other particles of the system.

Centre of Mass (COM)

For a system of n particles with masses $m_1, m_2, \dots, m_n$ located at position vectors $\vec{r}_1, \vec{r}_2, \dots, \vec{r}_n$ with respect to an origin O .

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots + m_n\vec{r}_n}{m_1 + m_2 + \dots + m_n} = \frac{1}{M} \sum_{i=1}^n m_i\vec{r}_i$$

where $M = m_1 + m_2 + \dots + m_n$ (Total mass)

For a two particle system



$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

Properties of Centre of Mass

- ✓ The COM of a system always lies on the line joining the particles.
- ✓ If masses are equal, COM lies at the midpoint.
- ✓ In absence of external force, COM moves with constant velocity.
- ✓ If external force acts, COM accelerates as a single particle of mass M .

PYQ A bullet of mass m is fired into a block of mass M and gets embedded in it. Find the velocity of the combined system. (No external horizontal force acts on the system.) (Ans. $\frac{mu}{M+m}$)

Motion of Centre of Mass

Let $\vec{F}_i^{ext}$ be the external force acting on i^{th} particle.

Total external force on system,

$$\vec{F}_{ext} = \sum_{i=1}^n \vec{F}_i^{ext}$$

From Newton's second law for system,

$$\vec{F}_{ext} = M\vec{a}_{CM}$$

$$\vec{a}_{CM} = \frac{\vec{F}_{ext}}{M}$$

Special Cases

- If $\vec{F}_{ext} = \vec{0} \Rightarrow \vec{a}_{CM} = \vec{0}$
COM moves with constant velocity.
- If $\vec{F}_{ext} \neq \vec{0} \Rightarrow \vec{a}_{CM}$ is in direction of $\vec{F}_{ext}$.

2. MOMENTUM CONSERVATION

Linear Momentum of a particle

$$\vec{p} = m\vec{v}$$

For a System of Particles

$$\vec{P} = \sum_{i=1}^n \vec{p}_i$$

Law of Conservation of Linear Momentum

If the resultant external force on a system of particles is zero, the total linear momentum of the system remains constant.

$$\vec{F}_{ext} = \vec{0} \Rightarrow \vec{P} = \text{constant}$$

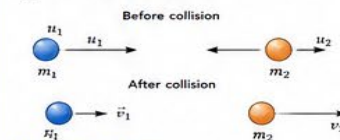
Applications

- ✓ Recoil of a gun
- ✓ Explosion
- ✓ Collision problems

Types of Collision

(a) Elastic Collision

- ✓ Linear momentum conserved.
- ✓ Kinetic energy conserved.
- e.g., Collision of billiard balls.

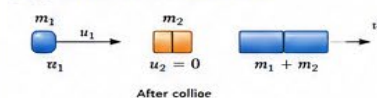


(b) Inelastic Collision

- ✓ Linear momentum conserved.
- ✓ Kinetic energy not conserved.
- e.g., Collision of clay with wall.

(c) Perfectly Inelastic Collision

- ✓ Bodies stick together after collision.
- e.g., Bullet fired into a wooden block.



PYQ Two trolleys of masses m_1 and m_2 moving with velocities u_1 and u_2 collide and move with velocities v_1 and v_2 respectively. Write the expression for conservation of linear momentum.

3. ROTATIONAL MOTION

Angular Displacement (θ)

Angle through which a body rotates about an axis.

SI Unit: radian (rad)

Angular Velocity (ω)

Rate of change of angular displacement.

$$\omega = \frac{d\theta}{dt}$$

SI Unit: rad s^{-1}

Angular Acceleration (α)

Rate of change of angular velocity.

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

SI Unit: rad s^{-2}

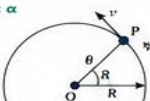
Kinematic Equations for Constant α

$$\omega = \omega_0 + \alpha t \quad \dots\dots(1)$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \quad \dots\dots(2)$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad \dots\dots(3)$$

$$\theta = \frac{(\omega + \omega_0)}{2} t \quad \dots\dots(4)$$



Relations

- ✓ Arc length, $s = R\theta$
- ✓ Tangential velocity, $v = R\omega$
- ✓ Tangential acceleration, $a_t = R\alpha$
- ✓ Centripetal acceleration, $a_c = \frac{v^2}{R} = R\omega^2$

PYQ A wheel starts rotating with angular acceleration α from rest. Find the angle turned by the wheel in time t . (Ans. $\frac{1}{2} \alpha t^2$)

4. MOMENT OF A FORCE (TORQUE)

Moment of a force $\vec{F}$ about a point O

$$\vec{\tau} = \vec{r} \times \vec{F}$$

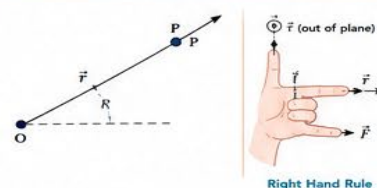
Magnitude

$$\tau = rF \sin \theta$$

where θ = angle between $\vec{r}$ and $\vec{F}$

Direction

Perpendicular to the plane of $\vec{r}$ and $\vec{F}$ (Right hand rule)



Properties

- ✓ τ is maximum when $\theta = 90^\circ$.
- ✓ If line of action of $\vec{F}$ passes through O , $\tau = 0$.

Conditions for Rotational Equilibrium

- (a) $\sum \vec{F} = \vec{0}$ (No translational motion)
- (b) $\sum \vec{\tau} = 0$ (No rotational motion)

PYQ A force of 10 N is applied on a rod of length 50 cm at an angle of 60° with the rod. Calculate the torque about the fixed end. (Ans. 2.5 N m)

5. MOMENT OF INERTIA (I)

It is the property of a body by virtue of which it resists any change in its state of rotation.

Moment of Inertia of Some Common Bodies

Thin Rod (length L) about center	Thin Rod about one end	Solid Disc (radius R) about center	Ring (radius R) about center	Solid Sphere (radius R) about center
				
$I = \frac{1}{12} ML^2$	$I = \frac{1}{3} ML^2$	$I = \frac{1}{2} MR^2$	$I = MR^2$	$I = \frac{2}{5} MR^2$

SI Unit: kg m^2

PYQ A thin rod of length L and mass M is rotated about an axis perpendicular to the rod and passing through one end. Find its moment of inertia. (Ans. $\frac{1}{3} ML^2$)

6. ROTATIONAL DYNAMICS

Rotational Analogue of Newton's 2nd Law

Net external torque on a rigid body is equal to the product of moment of inertia and angular acceleration.

$$\sum \tau_{ext} = I\alpha$$

Angular Momentum ($\vec{L}$)

$$\vec{L} = I\vec{\omega}$$

For a System of Particles

$$\vec{L} = \sum \vec{r}_i \times m_i \vec{v}_i$$

Conservation of Angular Momentum

If $\sum \tau_{ext} = 0$, then

$$\vec{L} = \text{constant}$$

PYQ A disc is rotating with angular velocity ω . If no external torque acts on it, what will happen to its angular momentum? (Ans. It will remain constant.)

KEY TAKEAWAYS

- ✓ COM behaves as a single particle of mass M .
- ✓ Linear momentum is conserved in an isolated system.
- ✓ Rotational motion involves angular quantities (θ, ω, α).
- ✓ Torque causes rotation.
- ✓ Moment of inertia depends on mass distribution.
- ✓ $\sum \tau = I\alpha$ is rotational form of $\sum F = ma$.

FORMULA SUMMARY

- $\vec{R} = \frac{1}{M} \sum m_i \vec{r}_i$
- $\vec{F}_{ext} = M\vec{a}_{CM}$
- $\vec{P} = \sum \vec{p}_i = \text{constant (if } \vec{F}_{ext} = \vec{0})$
- $\omega = \frac{d\theta}{dt}, \alpha = \frac{d\omega}{dt}$
- $v = R\omega, a_t = R\alpha, a_c = R\omega^2$
- $\vec{\tau} = \vec{r} \times \vec{F}, |\tau| = rF \sin \theta$
- $I = \sum m_i r_i^2$
- $\sum \tau_{ext} = I\alpha$
- $\vec{L} = I\vec{\omega}$ (for rigid body)

COMMON MISTAKES

- ✗ Confusing linear and angular quantities.
- ✗ Forgetting vector nature of τ and $\vec{L}$.
- ✗ Wrong axis of rotation while using I formulas.
- ✗ Assuming energy conserved in inelastic collisions.
- ✗ Ignoring direction in cross product ($\vec{r} \times \vec{F}$).

BOARD EXAM TIPS

- ✓ Draw clear diagrams.
- ✓ Write all formulas before solving.
- ✓ Use sign convention carefully.
- ✓ State laws and conditions explicitly.
- ✓ In numerical problems, write units with final answer.
- ✓ Practice PYQs of last 10 years.

SYMBOLS & UNITS

Symbol	Quantity	SI Unit
$\vec{r}$	Position vector	m
$\vec{R}$	Centre of mass vector	m
M	Mass	kg
$\vec{p}$	Linear momentum	kg m s^{-1}
θ	Angular displacement	rad
ω	Angular velocity	rad s^{-1}
α	Angular acceleration	rad s^{-2}
τ	Torque	N m
I	Moment of inertia	kg m^2
$\vec{L}$	Angular momentum	$\text{kg m}^2 \text{ s}^{-1}$



REMEMBER THIS

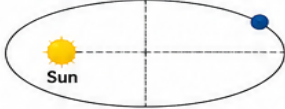
"Translation of COM and rotation about COM completely describe the motion of any rigid system."



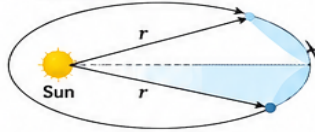
GRAVITATION

1 KEPLER'S LAWS OF PLANETARY MOTION

(a) **Kepler's First Law (Law of Orbit):**
Every planet revolves around the sun in an elliptical orbit with the sun situated at one focus of the ellipse.



(b) **Kepler's Second Law (Law of Area):**
The radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time.



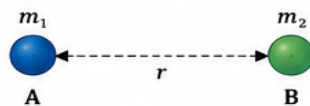
$$\text{Areal velocity} = \frac{dA}{dt} \Rightarrow \text{constant.}$$

(c) **Kepler's Third Law (Law of Period):**
The square of the time period of revolution of a planet around the sun is directly proportional to the cube of the semi-major axis of the elliptical orbit.

$$T^2 \propto r^3$$

2 UNIVERSAL LAW OF GRAVITATION (NEWTON'S LAW)

It states that the gravitational force of attraction acting between two-point mass bodies of the universe is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.



$$F = G \frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant and in SI unit,

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

WEIGHT OF A BODY

It is the gravitational force of attraction exerted on a body by the earth towards its centre lying on or near the surface of the earth.



$$W = mg$$

Unit of weight: Newton (N)

3 ACCELERATION DUE TO GRAVITY (g)

It is an acceleration set up in a body while falling freely under the effect of the gravitational force of the earth alone.

$$g = \frac{GM}{R^2}$$

where M and R are the mass and radius of the earth.

The value of g changes with the height, depth, rotation and shape of the earth.

On the surface of the earth,
 $g = 9.8 \text{ m s}^{-2}$

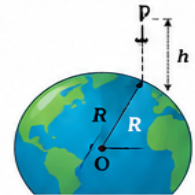


4 VARIATION OF g WITH ALTITUDE

Acceleration due to gravity decreases with altitude.

Acceleration due to gravity at a height ' h ' from the earth's surface is given by

$$g' = g \left(\frac{R}{R+h} \right)^2$$



- (i) As we go above the surface of the earth, the value of g decreases because.
- (ii) If $h \rightarrow \infty$ then $g' \rightarrow 0$, i.e., at infinite distance from the earth, the value of g becomes zero.
- (iii) If $h \ll R$ i.e., height is negligible in comparison to the radius then

$$g' \approx g$$

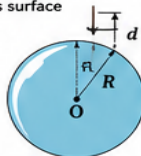
[Valid only for $h \ll R$]

5 VARIATION OF g WITH DEPTH

Acceleration due to gravity decreases with the increase in depth d from the earth's surface and it becomes zero at the centre of the earth.

Acceleration due to gravity at a depth ' d ' from the earth's surface is given by

$$g' = g \left(1 - \frac{d}{R} \right)$$



At centre ($d = R$) $\Rightarrow g' = 0$

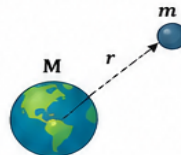
6 GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy of a body at a point in the gravitational field of the earth is the amount of work done in bringing the body from infinity to that point without acceleration.

$$U = - \frac{GMm}{r}$$

where M = mass of the earth
 m = mass of the body

r = distance between the centre of the body and the centre of the earth.



7 GRAVITATIONAL POTENTIAL

The gravitational potential at a point is the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.

$$V = - \frac{GM}{r}$$

- It is a scalar quantity.
- SI unit: Joule/kg (J/kg)

At a point in earth's gravitational field:

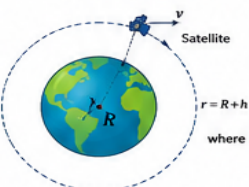
$$V = - \frac{GM}{r}$$

where M = mass of the earth
 r = distance of the point from the centre of the earth.

$$\text{Gravitational Potential Energy, } U = V \times m$$

8 ORBITAL VELOCITY OF A SATELLITE

The orbital speed of the satellite is the minimum velocity required tangentially to the circular orbit to put the satellite into an orbit around earth.



$$v_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{GM}{r}}$$

where M = mass of the earth
 R = radius of the earth
 h = height of satellite from earth's surface
 $r = R + h$ [As $r > R$ and $h > 0$]

9 ESCAPE VELOCITY

The escape velocity is the minimum velocity required to project a body vertically upwards from the surface of the earth so that it just crosses the gravitational field of earth and never returns on its own.

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

$$\Rightarrow v_e = \sqrt{2} v_0 \quad \left[\text{As } g = \frac{GM}{R^2} \right]$$

On the earth surface, $g = 9.8 \text{ m s}^{-2}$
 $R = 6.4 \times 10^6 \text{ m}$

Therefore, $v_e = 11.2 \text{ km s}^{-1}$

ENERGY OF SATELLITE

When a satellite revolves around a planet in its orbit, it possesses both potential energy (due to its position against gravitational pull of earth) and kinetic energy (due to orbital motion).

1) Potential Energy : $U = - \frac{GMm}{r}$

2) Kinetic Energy : $K = \frac{1}{2} m v^2$

3) Total Energy : $E = K + U = - \frac{GMm}{2r}$

SYMBOLS & CONSTANTS

G Universal gravitational constant
 $6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$

M Mass of the earth
 m Mass of the body (or satellite)

R Radius of the earth
 $6.4 \times 10^6 \text{ m}$

v_0 Orbital velocity
 v_e Escape velocity

r Distance from centre of the earth
 h Height above earth's surface

d Depth below earth's surface



MULTIPLE CHOICE QUESTIONS

1. The gravitational force of attraction between two mass m_1 and m_2 kept at a Distance r from each other is F . What will be the gravitational force if m_1 is Doubled, m_2 is halved and the distance between them is made thrice?
a. F b. $F/9$ c. $4F/9$ d. None of these
2. The mass and radius of earth is reduced to half the acceleration due to gravity would
(a) Remains same (b) decreases by half
(c) Decreases by same amount (d) increases by same amount
3. At what point regarding earth the value of Force of gravitational attraction of earth is
a. At the equator b. At the poles
c. At a point in between equator and any pole d. At surface of earth.
4. Which of the following relation is correct?
a. $GM = g R^2$ b. $GM = gR^3$ c. $gM = GR^2$ d. $GM = gR$
5. What will be the effect on acceleration due to gravity? When the average density of Earth is doubled,
a)Remain constant b)Will become doubled c)Will become four times d)None of these
6. We have a planet whose mass is doubled and radius is halved to that of the earth Then What will be the value of gravity on its surface If acceleration due to gravity is g on the Surface of the Earth,
a . $8g$ b. $4g$ c. g d. $6g$
7. If the law of gravitation, instead of being inverse-square law, becomes an inverse-cube Law
(a) Planets will not have elliptic orbits.
(b) Circular orbits of planets is not possible.
(c) Projectile motion of a stone thrown by hand on the surface of the earth will not be Parabolic.
(d) There will be no gravitational force inside a spherical shell of uniform density.
8. for earth, Acceleration due to gravity
a. Decreases with depth b. Increases with depth
c. Remain constant with depth d. May increase or decrease
9. There have been suggestions that the value of the gravitational constant G becomes Smaller when considered over very large time period (in billions of years) in the future. If that happens, for our earth,
(a) Nothing will change.
(b) We will become hotter after billions of years.
(c) We will be going around in closed orbits.
(d) After sufficiently long time we will leave the solar system.
10. At what height from the surface of the Earth, acceleration due to gravity is reduced by 36% to that of its value from the surface of the Earth? Given that radius of the Earth is R
a. $R/4$ b. R c. $R/2$ d. None of these
11. Escape velocity of an object of mass m is proportional to
(a) m^2 (b) m (c) m^{-1} (d) m^0
12. An object of mass m is lifted at a height equal to the radius (R) of the earth. If the mass of the earth is M and acceleration due to gravity is g , then the change in gravitational potential energy of the object is
(a) mgR (b) $2MgR$ (c) $MRg/2$ (d) $mRg/2$
13. Two particles of equal mass m are moving along a circle of radius r under the action of Their mutual gravitational attraction. The speed of each particle is
(a) $\sqrt{(Gm/r^4)}$ (b) $\sqrt{(Gm/r^2)}$ (c) $\sqrt{(Gm/r)}$ (d) $\sqrt{(Gmr)}$
14. If R is the radius of earth, then the acceleration due to gravity will be $g/4$ at a height from the Earth surface.
(a) R (b) $2R$ (c) $3R$ (d) $R/2$
15. The time period of an earth satellite in circular orbit is independent of (JEE 2004)

- (a) The mass of the satellite (b) radius of its orbit
 (c) both the mass and radius of the orbit
 (d) neither the mass of the satellite nor the radius of its orbit.
16. The escape velocity of a body depends upon mass as (AIEEE 2002)
 (a) m (b) m^1 (c) m^2 (d) m^3 .
- 17 A satellite is revolving close to the surface of the Earth with speed v . If the radius of the earth is suddenly reduced to half while its mass remains unchanged, the new orbital speed will be:
 (a) v (b) $\sqrt{2}v$ (c) $2v$ (d) $v/\sqrt{2}$
18. The escape velocity from the surface of Earth is 11.2 km/s. The escape velocity from a planet having twice the radius and same mean density as Earth is:
 (a) 11.2 km/s (b) 22.4 km/s (c) 15.8 km/s (d) 7.9 km/s
19. Two bodies of masses m and $4m$ are separated by a distance d . The point where the net gravitational field is zero lies:
 (a) At midpoint (b) Closer to mass m
 (c) Closer to mass $4m$ (d) outside the line joining them
20. A body is projected vertically upward from the surface of Earth with a speed of 8 km/s. Given that the escape velocity from Earth is 11.2 km/s, the maximum height attained by The body is approximately:
 (a) 1.04×10^7 m (b) 2.5×10^6 m (c) 6.4×10^6 m (d) 3.2×10^7 m

ANSWERS:-

- 1 b. $F/9$ (Hint: $F = \frac{GMm}{r^2}$)
- 2 d. (Hint: $g = \frac{GM}{R^2}$ and $g' = \frac{GM}{(\frac{R}{2})^2} = 4g$, % increase $g = \frac{\Delta g}{g} \times 100$)
3. a., (Hint: Radius of earth is less at equator $g = \frac{GM}{R^2}$)
4. a. (Hint: $GM = g R^2$)
5. b (Hint - $g = \frac{4}{3}\pi\rho GR$)
6. a. (Hint - $g = GM/R^2$)
7. a.
8. a. (Hint - $g' = g\left[1 - \frac{d}{R}\right]$)
9. d
- 10.a. $R/4$ Hint - $g' = g\left(\frac{R}{R+h}\right)^2$
- 11.d. Explanation: Escape velocity $= \sqrt{2gR}$, It is independent of mass.
- 12.d. Use gravitational potential energy formula: $U = -GMm/r$
 Initial distance $= R$, Final distance $= 2R$, Find: $\Delta U = U_f - U_i$, Also use: $g = GM/R^2$
- 13.a Gravitational force provides centripetal force. $Gm^2/(2r)^2 = mv^2/r$ Solve for v .
14. a.. $g_h = g(R/(R+h))^2$, Given: $g_h = g/4$, Solve for h
- 15.a. $T = 2\pi \sqrt{r^3/GM}$, Time period depends on orbital radius r , not on satellite mass.
- 16 a. Hint: Escape velocity: $v_e = \sqrt{2GM/R}$, Mass of the body does not appear in the formula.
- 17.b..Hint: Orbital speed: $v = \sqrt{GM/R}$, If radius becomes $(R/2)$, $v' = \sqrt{(GM)/(R/2)} = \sqrt{2}v$
18. c. 15.8 km/s, Hint: Escape velocity: $v_e = \sqrt{2GM/R}$, For same density: $M \propto R^3$,
 Substitute in the formula and compare with Earth's escape velocity.
- 19.b. Closer to mass m

20.a. 1.04×10^7 m use this simpler formula: $h = Rv^2 / (v_e^2 - v^2)$

EXERCISE MULTIPLE CHOICE QUESTIONS

1. at what height above the earth's surface weight becomes 30 kg if he weighs 60 kg at Earth's surface. Given radius of earth is 6400 km.
(a) 2624 km (b) 3000 km (c) 2020 km (d) None the these
2. There are two bodies of masses 1 kg and 100 kg separated by a distance 1 m. At what Distance from the smaller body, the intensity of gravitational field will be zero
(a) 1/9 m (b) 1/10 m (c) 1/11 m (d) 10/11 m
3. A particle falls towards earth from infinity. It's velocity on reaching the earth would be
(a) Infinity (b) $\sqrt{2gR}$ (c) $2\sqrt{gR}$ (d) zero
4. If g is the acceleration due to gravity on the earth's surface, the gain in potential energy Of on object of mass m raised from the surface of earth to a height equal to radius R of the Earth is
(a) $1/4 mgR$ (b) $1/2 mgR$ (c) $2mgR$ (d) mgR
5. Assuming the earth to be a sphere of uniform mass density, how much would a body Weigh half way down to the centre of earth, if it is weighed 250N on the surface.
(a) 240 N (b) 210 N (c) 195 N (d) 125 N

ANSWERS OF EXERCISE QUESTIONS:

1. (d) 2. (c) 3. (b) 4. (b) 5. (d)

ASSERSSION – REASON QUESTIONS

1. Assertion (A): Two satellites A and B are in the same orbit around the earth, such that A Leads B. Satellite B can overtake satellite A by increasing its speed.

Reason (R): Orbital speeds of two satellites in same orbit may different

2. Assertion (A): For a system of masses at some finite distance, gravitational field can be zero but gravitational potential cannot be zero.

Reason (R): Gravitational field is a scalar quantity while gravitational potential is a vector quantity.

3. Assertion (A): Period of revolution of satellite in circular orbit around earth is inversely proportional to cube of its orbital speed.

Reason (R): Period of revolution in uniform circular motion is given by $T = 2\pi r/v$, where r is radius of orbit and v is speed.

4. Assertion (A): Assuming zero potential at infinity, the gravitational potential at a point can never be positive.

Reason (R): The magnitude of gravitational force between two particles has inverse square Dependence on the distance between two particles.

5. Assertion (A): Gravitational field of a uniform spherical shell outside it is same as that of Particle of same mass placed at its centre of mass.

Reason (R): For the calculation of gravitational force between any two-uniform spherical shells, they can always be replaced by particles of same mass placed at respective centres.

6. Assertion (A): The gravitational force between two finite bodies is necessarily along the Line joining their centre of mass.

Reason (R): The gravitational force between two particles is not central.

7. Assertion (A): Moon revolving around earth does not come closer despite earths gravitational attraction.

Reason (R): A radially outward force balances earth's force of attraction during revolution of moon.

8. Assertion (A): Charged particles experience both electrical and gravitational force. But gravitational force is ignored.

Reason (R): Gravitational force is due to mass of particles while electrical force is due to charge of particles

9. Assertion (A): An artificial satellite of earth releases a packet. It will hit the earth exactly Below the satellite.

Reason (R): Packet will move along a straight line towards earth's centre with respect to satellite.

10. **Assertion (A):** If earth stops rotating about its axis, then the value of acceleration due to Gravity increases everywhere, except at the poles.

Reason (R): The value of acceleration due to gravity is maximum at the poles

ANSWERS:

1 – D 2- C 3- A 4- B 5- C 6- D 7- C 8- D 9- B 10- B

EXERCISE QUESTIONS

1. **Assertion (A):** At the centre of the earth, a body has centre of mass, but no centre of gravity.

Reason (R): Acceleration due to gravity is zero at the centre of the earth.

2. **Assertion (A):** The mechanical energy of earth–moon system remains same when another Heavenly body passes nearby the earth–moon system.

Reason (R): Force exerted by heavenly body on the earth–moon system is non-conservative.

3. **Assertion (A):** Gravitational potential of earth at every place upon it is negative.

Reason (R): Everybody on earth is bound by the attraction of earth.

4. **Assertion (A):** Gravitational potential energy of any mass particle may not be zero at earth Centre.

Reason (R): Gravitational field intensity at earth centre is zero.

5. **Assertion (A):** When planet moves in elliptical orbit around Sun. Its angular momentum About sun remains conserved.

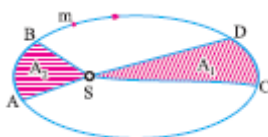
Reason (R): Total mechanical energy of planet – sun system remains conserved.

Reason (R): There is no gravitational force on the satellite.

ANSWERS: 1 – A 2- D 3-A 4-B 5-B

2 MARKS QUESTIONS

1. If existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?
2. The gravitational force on the Earth due to the Sun is greater than that of the moon, However, the tidal effect of the moon's pull is greater than the tidal effect of the Sun. Why?
3. Derive the expression for acceleration due to gravity at the earth's surface. Given that M is Mass and R is the radius of the earth.
4. The figure shows elliptical orbit of a planet m about the sun S . The shaded area of SCD is Twice the shaded area SAB . If t_1 is the time for the planet to move from D to C and t_2 is Time to move from A to B , what is the relation between t_1 and t_2 ?



5. Prove that the weight of a body at the centre of the Earth is zero.
6. Hydrogen escapes faster from the Earth than oxygen. Why?
7. We cannot move even our little fingers without disturbing the whole universe. Explain why.
8. Explain why tennis ball bounces higher on hills than on plains.
9. If the planet Earth is considered to have mass M and radius R . calculate the average Density of Earth.
10. Calculate the energy required to move a body of mass m from an orbit of radius $2R$ to $3R$.

ANSWERS

1. **Answer :** According to Kepler's third law

If planets move faster with double speed then it will take half time period of revolution

Around sun as that of earth. $T_e = 1 \text{ year}$ $T_p = 0.5 \text{ year}$ $r_e = 1 \text{ A.U.}$ $r_p = r_e (T_p/T_e)^{2/3} =$

$$1\text{AU} (0.5\text{year}/1\text{year})^{2/3} = 0.63 \text{ AU}$$

2. **Answer:** The tidal effect depends inversely on the cube of the distance, unlike the force which depends inversely on the square of the distance. The distance of the moon from the Earth is very small compared to the distance of the sun from the Earth. Therefore, the tidal effect of the Moon's pull is greater than the tidal effect of the sun.

3. **Answer: Derivation**

4. **Answer:** Using Kepler's II Law $t_1 = 2t_2$

5. **Answer:** An atmosphere means the presence of a mixture of a number of gases. The molecules of these gases are in a state of continuous random thermal motion moving at different velocities. As the value of escape velocity on the surface of the moon is small (only 2.5 km s^{-1}), the molecules of gases with velocities greater than the escape velocity moved out of the atmosphere. As time passed, nearly all the molecules escaped from the moon's atmosphere.

6. The thermal speed of hydrogen is much larger than oxygen. Therefore, a large number of hydrogen molecules are able to acquire escape velocity than that of oxygen molecules. Hence hydrogen escapes faster from the Earth than oxygen.

7. According to Newton's law of gravitation, every particle of this universe attracts every other particle with a force that is inversely proportional to the square of the distance between them. When we move our fingers, the distance between the particle's changes, and hence the force of attraction changes which in turn disturbs the whole universe

8. The value of acceleration due to gravity is lesser on hills than on the plains, so the weight of the tennis ball at hills is lesser than that on the plains and hence it re-bounces more. In other words, the force with which the Earth attracts the ball on hills will be lesser than that on the plains.

9. Newton's law of gravitation can be used to estimate the mass and density of the earth.

As we know $g = \frac{GM}{R^2}$, so we have $M = \frac{gR^2}{G}$

$$\therefore M = \frac{9.8 \times (6.4 \times 10^6)^2}{6.67 \times 10^{-11}} = 5.98 \times 10^{24} \text{ kg} \approx 10^{25} \text{ kg}$$

and as we know $g = \frac{4}{3} \pi \rho GR$, so we have $\rho = \frac{3g}{4\pi GR}$

$$\therefore \rho = \frac{3 \times 9.8}{4 \times 3.14 \times 6.67 \times 10^{-11} \times 6.4 \times 10^6} = 5478.4 \text{ kg / m}^3$$

10 **Answer:**

$$\text{Gravitational P.E. of mass } m \text{ in orbit of radius } R = U = -\frac{GMm}{R}$$

$$\therefore U_i = -\frac{GMm}{2R}$$

$$U_f = -\frac{GMm}{3R}$$

$$\begin{aligned} \Delta U &= U_f - U_i = GMm \left[\frac{1}{2} - \frac{1}{3} \right] \\ &= \frac{GMm}{6R} \end{aligned}$$

EXERCISE QUESTIONS

- 1 The radii of two planets are R and $2R$ respectively and their densities ρ and $\rho/2$ respectively. What is the ratio of acceleration due to gravity at their surfaces ?
- 2 If earth has a mass 9 times and radius 4 times than that of a planet 'P'. Calculate the escape velocity at the planet 'P' if its value on earth is 11.2 km s^{-1} .
- 3 At what height from the surface of the earth will the value of 'g' be reduced by 36% of its value at the surface of earth.

- 4 The mean orbital radius of the earth around the sun is 1.5×10^8 km. Calculate mass of the sun. if $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^{-2}$?
- 5 Why the space rockets are generally launched west to east?

ANSWERS

1.1:1, 2.7.47 km/s, 3.1600 km, $4.2.01 \times 10^{30}$ kg.

5. Since the earth revolves from west to east, so when the rocket is launched from west to east the relative velocity of the rocket increases which helps it to rise without much consumption of fuel.

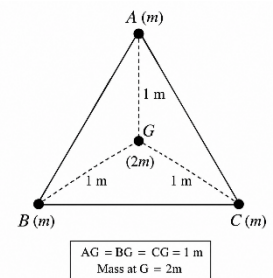
SHORT ANSWER QUESTION (3 MARKS)

1. Three equal point masses, each of mass m , are fixed at the vertices **A, B, and C** of an equilateral triangle **ABC**. The centroid of the triangle is **G**, and the distance from the centroid to each vertex is **1 m**, i.e.,

$$AG = BG = CG = 1 \text{ m}$$

A particle of mass $2m$ is placed at the centroid **G**.

Determine the net gravitational force acting on the particle of mass $2m$ due to the three masses placed at the vertices of the triangle.



2. State the Kepler laws of the planetary motions.

3. In a two stage launch of a satellite, the first stage brings the satellite to a height of 150 km and the 2nd stage gives it the necessary critical speed to put it in circular orbit. Which stage requires more expenditure of fuel? Given mass of earth = 6.0×10^{24} kg, radius of Earth = 400 km.

4. Derive the expression for the acceleration due to gravity at a depth's' from the surface of the Earth.

5. Three equal masses of each mass m are placed at the three vertices of an equilateral triangle of each side
a. Find the gravitational potential at the centroid of the triangle.

6. Derive the relation between escape and orbital velocities for a body revolving very close to the earth's surface. Find the percentage increase in the orbital velocity by which the body escapes from its orbit forever.

7. The gravitational field intensity at a point 10,000 km from the centre of the earth is 4.8 N kg^{-1} . Calculate gravitational potential at that point.

8. Assuming the globe is a sphere with constant mass density, how much would a body weigh halfway down to the planet's centre? Given that weight of the body on the surface of the planet is 250N.

9. Write the expression for the potential energy of a body on the surface of the earth. A body of mass m has been elevated from the earth's surface to a height equal to the radius of the earth R . What will be the increase in the potential energy of the body?

10. State and prove Kepler's third law of planetary motion.

ANSWERS

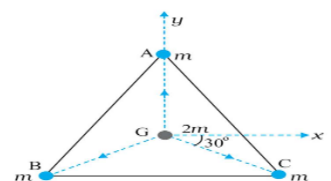
1. Three equal masses of m kg each are fixed at the vertices of an equilateral triangle **ABC**. What is the force acting on a mass $2m$ placed at the centroid **G** of the triangle

$$\text{Net force } F = F_{GA} + F_{GB} + F_{GC} = 0$$

$$F_{GA} = \frac{Gm(2m)}{1^2} \hat{j}$$

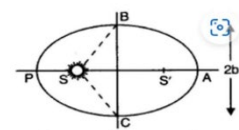
$$F_{GB} = \frac{Gm(2m)}{1^2} (-\hat{i} \cos 30^\circ - \hat{j} \sin 30^\circ)$$

$$F_{GC} = \frac{Gm(2m)}{1^2} (\hat{i} \cos 30^\circ - \hat{j} \sin 30^\circ)$$



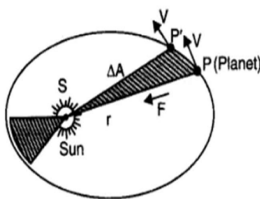
2. Kepler's 1st Law of Orbits: This law is popularly known as the law of orbits.

The orbit of any planet is an ellipse around the Sun with Sun at one of the two foci of an ellipse. In an ellipse, there are two foci and Sun is situated at one of the foci of the ellipse.



Kepler's 2nd Law of Areas: This law is known as the law of areas. The line

joining a planet to the Sun sweeps out equal areas in equal interval of time. The rate of change of area with time will be constant.



$$\text{Areal Velocity} = V = \frac{\Delta A}{\Delta t}$$

$$\text{Areal Velocity} = \frac{L}{2m} \quad \text{where } L = \text{Angular momentum and } m \text{ are mass of the planet}$$

Kepler's 3rd Law of Periods: This law is known as the law of Periods.

The square of the time period (T) of the planet is directly proportional to the cube of the semimajor axis(a) of its orbit. $T^2 \propto a^3$

3. Work done on satellite in first stage = W_1 = PE at 150 km – PE at the surface Ratio w_1/w_2

= 3/64 (less than one) $W_2 > W_1$, so second stage requires more energy.

4. Acceleration due to gravity at the surface of the earth

$$g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho GR \quad \dots (i)$$

Acceleration due to gravity at depth d from the surface of the earth

$$g' = \frac{4}{3}\pi\rho G(R-d) \quad \dots (ii) \quad \text{From (i) and (ii)} \quad g' = g\left[1 - \frac{d}{R}\right]$$

5. Gravitational Potential at point is given by $V = -\frac{GM}{r}$

Gravitational potential at centroid due all three masses will be equal and will be given by

$$V_1 = V_2 = V_3 = -\frac{GM}{x}$$

$$V = -3\frac{GM}{x}$$

Now total gravitational potential at the centroid of the triangle. x (Put x)

6 Derivation

$$7. E = \frac{GM}{R^2}, \quad V = -\frac{GM}{r} \quad V = -4.8 \times 10000 \times 10^3 = -4.8 \times 10^7 \text{ Jkg}^{-1}$$

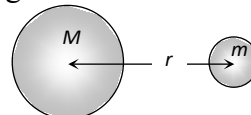
8. Acceleration due to gravity at depth d is given by: g'

$$= g\left[1 - \frac{d}{R}\right], \quad = g\left[1 - \frac{d}{R}\right]_m = 125\text{N}$$

9. The gravitational potential energy of a body at a point is defined as the amount of work Done in bringing the body from infinity to that point against the gravitational force.

$$W = \int_{\infty}^r \frac{GMm}{x^2} dx = -GMm \left[\frac{1}{x} \right]_{\infty}^r$$

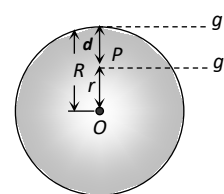
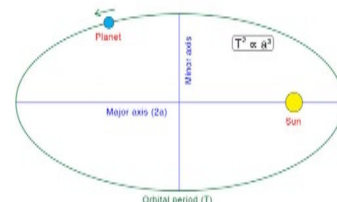
$$W = -\frac{GMm}{r}$$



This work done is stored inside the body as its gravitational potential energy

$$\therefore U = -\frac{GMm}{r}$$

10. Derivation



EXERCISE QUESTIONS

1. A body released at the distance r ($r > R$) from the centre of the earth. What is the velocity of the body when it strikes the surface of the earth ?
2. Derive the expression for the acceleration due to gravity at height 'h' from the surface of the earth.
3. Jupiter has a mass 318 times that of the earth, and its radius is 11.2 times the earth's radius. Estimate the escape velocity of a body from Jupiter's surface, given that the escape velocity from the earth's surface is 11.2 km s^{-1} .
4. The distance of Neptune and Saturn from the sun is nearly 10^{13} m and 10^{12} m respectively. Assuming that they move in circular orbits, then what will be the ratio of their periods.
5. If the radius of earth shrinks by 2%, mass remaining constant. How would the value of acceleration due to gravity change?

HINTS AND ANSWERS

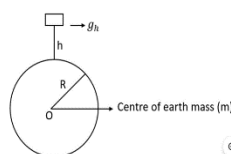
$$1. \quad v = R \sqrt{2g \left(\frac{1}{R} - \frac{1}{r} \right)}$$

$$2. \quad g' = g \left(\frac{R}{R+h} \right)^2 = g \left(1 + \frac{h}{R} \right)^{-2} = g \left[1 - \frac{2h}{R} \right]$$

$$3. 59.7 \text{ km s}^{-1}$$

$$4. \text{By Kepler's III law } T_n : T_s = 36.6 : 1.$$

$$5. \text{Taking logarithm } \log g = \log G + 2 \log m - 2 \log R, \text{ differentiating it we get it } 4\%$$



[As $h \ll R$]

5-MARKS QUESTIONS

1. Define the gravitational potential energy of a body on the surface of the Earth. A body of mass m is at a distance of r from the centre of the earth. If M is the mass of the earth and R is the radius of the earth, then find the gravitational potential energy of the body.
2. Define escape velocity. Does the escape velocity of a body from the Earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, and (d) the height of the location from where the body is launched? Why the atmosphere of Jupiter contains light gases (generally hydrogen) whereas the Earth's atmosphere has little hydrogen gas?
3. (i) State Kepler's second law of planetary motion.
(ii) Show that Kepler's Second law is the law of conservation of angular momentum.
(iii) A Saturn year is 29.5 times the Earth year. How far is Saturn from the Sun if the Earth is $1.50 \times 10^8 \text{ km}$ away from the Sun?
4. Derive the expression for escape velocity for a body projected from the surface of the earth. Calculate escape velocity on the surface of the earth if the radius of the earth is 6400 Km and acceleration due to gravity is 9.8 m/s^2 on the surface of the earth. A body is projected out thrice the escape velocity. What will be the velocity of the body far away from the earth? Ignore the presence of the sun and other planets.
5. Define orbital velocity. Derive the expression for the orbital velocity of a satellite orbiting in an orbit at a height h from earth surface.
(ii) A rocket is fired from the earth towards the sun. At what distance from the earth's Centre is the gravitational force on the rocket zero? Mass of the sun = $2 \times 10^{30} \text{ kg}$, mass of the earth = $6 \times 10^{24} \text{ kg}$. Neglect the effect of other planets etc. (orbital radius = $1.5 \times 10^{11} \text{ m}$).

ANSWERS

1. Define the term and then derive the expression
2. Escape Velocity is the minimum velocity an object must have to escape a celestial body's gravitational field permanently, or without ever falling back again. Escape velocity of the body is given by

$$h \Rightarrow v_e = \sqrt{2gR} \quad [\text{As } GM = gR^2]$$

Where M is mass of the earth and R is radius of the earth and g is acceleration due to gravity on the earth surface.

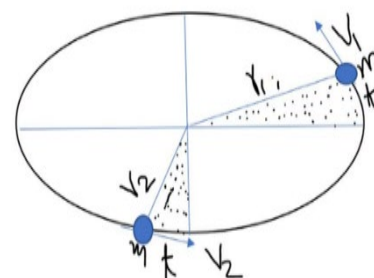
- (a) Escape velocity does not depend upon mass of the body to be escaped
- (b) Escape velocity does not depend the location
- (c) Escape velocity does not depend the direction of the projection
- (d) Yes, Escape velocity depends on the height of the location.

The escape velocity of Jupiter is much larger than the escape velocity of Earth. So in order to escape from the surface of Jupiter, a very large velocity is required. Since the thermal

Velocity of hydrogen gas molecules is lesser than the escape velocity of Jupiter, therefore Hydrogen can't escape from the surface of Jupiter.

3. Kepler's 2nd Law of Areas: The second law states that

The areal velocity is constant i.e. area covered by the Radius vector is the same in equal intervals of time. If the velocity and radius at the time, t_1 is v_1 and r_1 while at another place these are v_2 and r_2 in the same time, then the area covered by the planet in these intervals are



- (ii) using $\frac{1}{2} (v_1 t) r_1 = \frac{1}{2} (v_2 t) r_2$ (area of triangle = $\frac{1}{2}$ base $\times$ height), $mv_1 r_1 = mv_2 r_2$, $L_1 = L_2$

- (iii) A Saturn year is 29.5 times the Earth year. Earth is 1.50×10^8 km away from the Sun
Using Kepler's III law distance = 14.32×10^{11} m

4. Escape velocity is the minimum velocity with which a body must be projected vertically Upward so that it can just escape the gravitational field of the Earth.

Expression for escape velocity:

$$v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e = \sqrt{2gR} \quad [As \quad GM = gR^2]$$

Total Kinetic energy by which body is projected

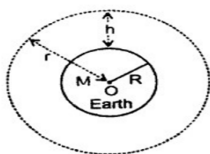
$K_1 = \frac{1}{2} m V_1^2$, Minimum Kinetic energy required by body to overcome earth

Gravitational field, $K_2 = \frac{1}{2} m V_2^2$, Kinetic Energy of body just after crossing earth Gravitational field

$K_3 = \frac{1}{2} m V_3^2$, From conservation of energy, $K_1 = K_2 + K_3$

$\frac{1}{2} m V_1^2 = \frac{1}{2} m V_2^2 + \frac{1}{2} m V_3^2$, $V_3^2 = V_1^2 - V_2^2$, Here $V = 11.2$ km/s and $V_1 = 3 \times 11.2$ km/s $V_3 = 31.68$ km/s

5. The centripetal force mv^2/r required by the satellite to move in a circular orbit is provided by the gravitational force between satellite and the Earth.



$$\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{gR^2}{R+h}} = R \sqrt{\frac{g}{R+h}} \quad [As \quad GM = gR^2 \text{ and } r = R+h]$$

- (ii) Using universal law of gravitation, $X = 2.59 \times 10^8$ m

EXERCISE QUESTIONS

1. (a) A satellite moving with an angular speed equal to the angular speed of earth's rotation. Find the radius of its circular orbit
- (b) find the time it takes to come over equatorial plane. If the satellite is directly above the north pole at some instant, Mass of the earth = 6×10^{24} kg
2. A satellite of mass 500kg is orbiting the Earth at a height of 300 km above the Earth's

Surface. The radius of the Earth is 6400 km. Calculate:

- (i) The gravitational potential energy of the satellite in orbit.
- (ii) The kinetic energy of the satellite in orbit.
- (iii) The total mechanical energy of the satellite.

ANSWERS:

- 1 (i) radius = 42.3km (ii) time = 6 hours
2 (i) $-2.97 \times 10^9 \text{ J}$ (ii) $1.27 \times 10^{10} \text{ J}$ (iii) $-1.7 \times 10^{10} \text{ J}$

CASE STUDY-BASED QUESTIONS

1. Origin of Gravitation

The origin of gravity evolved from Newton's description of a force between masses to Einstein's conceptualization of gravity as the curvature of space time. While Newton's laws work well for everyday situations, Einstein's theory provides a deeper understanding of gravity, explaining phenomena that Newton's theory could not, such as the bending of light and the behaviour of objects near black holes. Today, gravity is understood as a result of the interaction between mass and the fabric of space time, a concept that has been validated through both theory and experiment.

- (i) Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will
 - (a) move towards each other
 - (b) move away from each other
 - (c) will become stationary
 - (d) keep floating at the same distance between them
- (ii) Two spheres of masses m and M are situated in air and the gravitational force between them is $2F$. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be
 - (a) $6F$
 - (b) $2F$
 - (c) $F/6$
 - (d) $F/2$
- (iii) Which of the following best describes gravity according to Newton's Law of universal Gravitation?
 - a) It is a force that attracts two objects based on their mass and the distance between them
 - b) It is a force that repels objects based on their mass and the distance between them
 - c) It is caused by the curvature of space time around objects
 - d) It is a quantum mechanical effect between subatomic particles
- (iv) Why is gravity so much weaker than the other fundamental forces (strong nuclear force, electromagnetic force, and weak nuclear force)?
 - (a) it acts specifically
 - (b) it acts universally
 - (c) its value decrease with height
 - (d) it obeys inverse square law

2. 'g' IN OUR LIFE

Early in our lives, we become aware of the tendency of all material objects to be attracted to the earth. Anything thrown up falls down towards the earth, going uphill is lot more tiring than going downhill, raindrops from the clouds above fall towards the earth and there are many other such phenomena. Historically it was the Italian Physicist Galileo (1564-1642) who recognized the fact that all bodies, irrespective of their masses, are accelerated towards the earth with a constant acceleration. It is said that he made a public demonstration of this fact. To find the truth, he certainly did experiments with bodies rolling down inclined planes and arrived at a value of the acceleration due to gravity which is close to the more accurate value obtained later.

- (i). Which of the following factors causes the acceleration due to gravity to be slightly weaker at the equator compared to the poles?
 - A) Earth's shape (oblate spheroid)
 - B) Earth's rotation
 - C) Altitude above sea level
 - D) Both A and B
- (ii). at which location will the acceleration due to gravity be the least on Earth?
 - a) At the North Pole
 - b) At the South Pole
 - c) At sea level at the equator
 - d) At the summit of Mount Everest
- (iii). which of the following is NOT a factor that directly influences the acceleration due to

Gravity?

- a) The mass of the object
- b) The altitude above Earth's surface
- c) The Earth's radius
- d) the latitude of the location

(iv). Does the gravitational force between two particles depends upon the medium between them?

3. Case Study (ENERGY OF A SATELLITE)

we will explore the energy dynamics of a satellite in orbit around the Earth. The satellite is assumed to be in a circular orbit, and we will examine the key aspects of its energy—kinetic energy, potential energy, and total mechanical energy—over time. The total mechanical energy is negative, which indicates that the satellite is in a stable bound orbit. This energy remains constant as long as no external forces (e.g., atmospheric drag or orbital adjustments) affect the satellite's motion.

(i) What type of energy is primarily responsible for keeping a satellite in its orbit around the Earth?

- a) Kinetic energy
- b) Gravitational potential energy
- c) Elastic potential energy
- d) Electrical energy

(ii) If a satellite in orbit loses energy due to atmospheric drag, which of the following is likely to happen?

- a) Its orbit will remain unchanged
- b) Its orbital altitude will increase
- c) Its orbital altitude will decrease
- d) Its velocity will increase

(iii) At what point in a satellite's orbit is its kinetic energy the greatest?

- a) At the apogee (farthest point from Earth)
- b) At the perigee (closest point to Earth)
- c) At the center of the orbit
- d) At the equilibrium point

(iv) Which of following is true about the gravitational potential energy of a satellite in orbit?

- a) It is always positive and increases with altitude
- b) It is always negative and decreases with altitude
- c) It is always zero
- d) It is positive and decreases with altitude

ANSWER -CASE STUDY BASED QUESTIONS

1. (ORIGIN OF GRAVITY) i.(d) ii (b) iii (a) iv (b)

2. ('g' IN OUR LIFE (i) d)Both A and B (ii). c (iii).(a) (iv). No

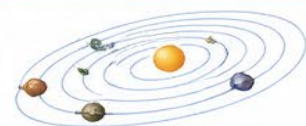
3(i) b) (ii). c (iii).(b) (iv). b



Gravitation is a fundamental force that exists between any two masses in the universe.

GRAVITATION

CBSE CLASS XI PHYSICS



1. UNIVERSAL LAW OF GRAVITATION

Every particle in the universe attracts every other particle with a force which is:

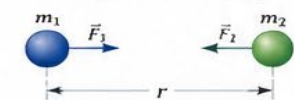
- Directly proportional to the product of their masses.
- Inversely proportional to the square of the distance between them.

Mathematically,

$$F = G \frac{m_1 m_2}{r^2}$$

where,

F = gravitational force (N)
 m_1, m_2 = masses (kg)
 r = distance between centres (m)
 G = universal gravitational constant
 $= 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$



$\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude and opposite in direction (Newton's 3rd Law).

2. GRAVITATIONAL FIELD AND INTENSITY

Gravitational field at a point: The region around a mass within which another mass experiences gravitational force.

Gravitational field intensity ($\vec{g}$): Gravitational force experienced per unit mass placed at that point.

$$\vec{g} = \frac{\vec{F}}{m}$$

Due to a point mass M at distance r :

$$\vec{g} = -\frac{GM}{r^2} \hat{r}$$

(Radially inward)

Gravitational field lines:

- Always directed towards the mass.
- Never intersect each other.
- Closer lines $\Rightarrow$ stronger field.



3. GRAVITATIONAL POTENTIAL AND POTENTIAL ENERGY

Gravitational potential (V): Potential energy per unit mass.

$$V = \frac{U}{m}$$

Due to a point mass M at distance r (from infinity):

$$V = -\frac{GM}{r}$$

Gravitational potential energy (U): Energy possessed by a body due to its position in a gravitational field.

$$U = -\frac{GMm}{r}$$

Properties:

- V is negative (zero at infinity).
- Work done in bringing mass m from infinity to r is equal to U .

4. ACCELERATION DUE TO GRAVITY (g)

Acceleration produced in a body due to gravitational pull of a planet.

At a distance r from the centre of a spherical planet of mass M :

$$g = \frac{GM}{r^2}$$

At the surface ($r = R$):

$$g = \frac{GM}{R^2}$$

Variation of g :

- Inside the Earth (at depth d):

$$g' = g \left(1 - \frac{d}{R}\right)$$

- Above the surface (at height h):

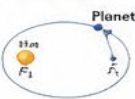
$$g' = g \left(\frac{R}{R+h}\right)^2$$

where R = radius of Earth

5. MOTION OF PLANETS (KEPLER'S LAWS)

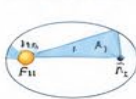
(i) Law of Orbits

Each planet moves in an elliptical orbit with the Sun at one focus.



(ii) Law of Areas

The line joining the planet and the Sun sweeps out equal areas in equal intervals of time.



(iii) Law of Periods

The square of the time period (T) of revolution is directly proportional to the cube of the semi-major axis (a) of the orbit.

$$T^2 \propto a^3$$

$$\frac{T^2}{a^3} = \text{constant}$$

6. SATELLITE ORBITS

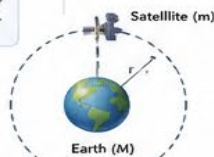
Orbital speed (v):

For a satellite of mass m revolving in a circular orbit of radius r around Earth (mass M):

$$v = \sqrt{\frac{GM}{r}}$$

Time period (T):

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$



7. ESCAPE VELOCITY

Minimum speed required by a body to escape from the gravitational field of a planet without further propulsion.

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

For Earth: $v_e \approx 11.2 \text{ km s}^{-1}$



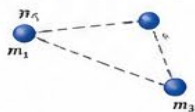
8. GRAVITATIONAL POTENTIAL ENERGY OF A SYSTEM

For a system of two masses m_1 and m_2 separated by distance r :

$$U = -\frac{Gm_1 m_2}{r}$$

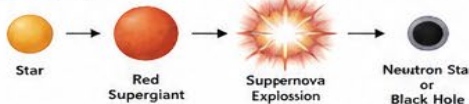
For a system of n particles:

Total potential energy of the system is the sum of potential energies of all pairs.



9. GRAVITATIONAL COLLAPSE

When a massive star exhausts its nuclear fuel, its core collapses under its own gravity.



Neutron Star

- Extremely dense
- Mass $\sim 1-2 M_\odot$
- Radius $\sim 10 \text{ km}$
- Supported by neutron degeneracy pressure

Black Hole

- Extremely dense
- Mass $> 3 M_\odot$ (approx.)
- Escape velocity $> c$
- Formed when core collapses beyond a limit

10. WEIGHTLESSNESS

A body appears to have zero weight when the normal reaction acting on it is zero.

Occurs when:

- Body is in free fall under gravity only.
- Body and support both fall with same acceleration.

Examples:

- Astronauts in orbiting.
- Lift falling freely



QUICK REVISION

- ✓ Gravitation is universal and always attractive.
- ✓ Field lines are radially inward.
- ✓ g decreases with height and depth.
- ✓ Orbits follow Kepler's laws.
- ✓ Escape velocity depends only on mass and radius of planet.
- ✓ Energy of bound system is negative.

FORMULA SUMMARY

- $F = G \frac{m_1 m_2}{r^2}$
- $\vec{g} = -\frac{GM}{r^2} \hat{r}$
- $V = -\frac{GM}{r}$
- $U = -\frac{GMm}{r}$
- $g = \frac{GM}{R^2}$
- $v = \sqrt{\frac{GM}{r}}$
- $T = 2\pi\sqrt{\frac{r^3}{GM}}$
- $v_e = \sqrt{\frac{2GM}{R}}$
- $U = -\frac{Gm_1 m_2}{r}$

CONSTANTS & VALUES

$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
 Mass of Earth (M)
 $= 5.97 \times 10^{24} \text{ kg}$
 Radius of Earth (R)
 $= 6.37 \times 10^6 \text{ m}$
 g at Earth's surface
 $= 9.8 \text{ m s}^{-2}$
 Escape velocity (Earth)
 $= 11.2 \text{ km s}^{-1}$

PYQ TIPS

- ★ Derivations are important.
- ★ Draw diagrams neatly.
- ★ Remember sign conventions for potential energy.
- ★ Understand physical meaning of each formula.
- ★ Practice numericals regularly.

COMMON MISTAKES

- ✗ Forgetting negative sign in V and U .
- ✗ Using wrong distance r .
- ✗ Confusing mass and weight.
- ✗ Ignoring vector nature of $\vec{g}$.
- ✗ Incorrect unit conversions.

INTERESTING FACT

Inside a hollow spherical shell, the gravitational field is zero at every point.

REAL LIFE APPLICATIONS

Satellite communication

GPS technology

Space missions

Study of stars and galaxies

REMEMBER THIS

Gravity is the weakest force, but it is responsible for the largest structures in the universe!



SCAN TO PRACTICE MORE!

CHAPTER - 8: MECHANICAL PROPERTIES OF SOLIDS

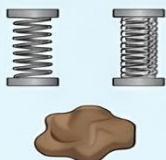
A comprehensive, exam-oriented summary for **CBSE, JEE/NEET** aspirants.

1. Fundamentals of Elasticity

Elasticity vs. Plasticity

Elasticity: Property to regain original shape/size after deforming force removal (e.g., Quartz fiber).

Plasticity: Inability to regain shape (e.g., Putty, Mud).



The Elastic Limit

The maximum deforming force up to which a body retains its elasticity, exceeding this causes permanent deformation.

Perfect Materials (Idealized)

Phosphor bronze is the nearest approach to a perfectly elastic body, while **paraffin wax** is nearly perfectly plastic.

4. Moduli of Elasticity

Ratios of **Stress** to **Strain** for different types of deformation.

Young's Modulus (Y)

Ratio of **Normal Stress** to **Longitudinal Strain**; applies only to solids.

$$Y = \frac{F \cdot L}{A \cdot \Delta L}$$

Bulk Modulus (B or K)

Ratio of **Hydraulic Stress** to **Volumetric Strain**; negative sign indicates volume decrease.

$$B = \frac{-P}{\Delta V/V}$$

Shear Modulus (G or η)

Ratio of **Shearing Stress** to **Shearing Strain**; represents the material's rigidity.

$$G = \frac{F/A}{\theta}$$

Quick Moduli Reference

Modulus	Definition	Formula	Applicable to
Young's (Y)	Longitudinal Stress / Longitudinal Strain	$Y = \frac{MgL}{\pi r^2 \Delta l}$	Solids only
Bulk (B)	Hydraulic Stress / Volume Strain	$B = \frac{-P \Delta V}{\Delta V}$	Solids, Liquids, Gases
Shear (G)	Shearing Stress / Shearing Strain	$G = \frac{F/A}{\theta}$	Solids only

2. Quantifying Deformation: Stress & Strain

Stress: The Internal Restoring Force

Defined as **Restoring Force per unit Area**.

$$\text{Stress} = \frac{F}{A}$$

SI Unit: **N/m²** or **Pascal (Pa)**.
Dimensions: **[ML⁻¹T⁻²]**.

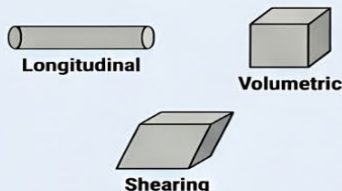
Strain: The Relative Deformation

The ratio of change in configuration to the original configuration; it is dimensionless and unitless.



Types of Stress & Strain

Longitudinal (Length change), **Volumetric** (Volume change), and **Shearing** (Shape change via angular displacement θ).



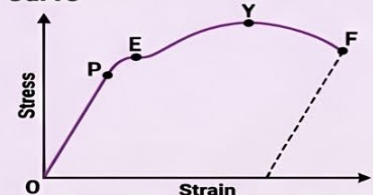
3. Hooke's Law & The Stress-Strain Curve

Hooke's Law

Within the elastic limit, **Stress** is directly proportional to **Strain**.

$$\text{Stress} \propto \text{Strain}$$

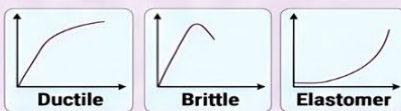
Regions of the Stress-Strain Curve



Includes the **Proportional Limit** (linear), **Elastic Limit** (Yield Point), **Permanent Set** (plastic deformation), and **Fracture Point** (breaking).

Material Classification via Graph

Ductile materials have a large plastic range (wire). **Brittle** materials have a small plastic range (glass). **Elastomers** (rubber) don't obey Hooke's Law but return to shape.



5. Advanced Concepts: Poisson's Ratio & Energy

Poisson's Ratio (σ)

The ratio of **Lateral Strain** (change in diameter) to **Longitudinal Strain** (change in length).

$$\sigma = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

Practical values range between 0 and 0.5.

Elastic Potential Energy

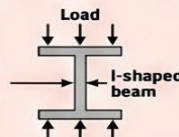
Work done in stretching a wire is stored as energy.

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

6. Real-World Applications (NCERT Case Studies)

Crane Ropes & Bridge Beams

Ropes are designed so load stress never exceeds the elastic limit.



Beams (I-shape) are used to reduce bending (δ) which depends on 1/Y.

Why Mountains have a Height Limit

The maximum height (~10km on Earth) is limited by the elastic limit of the rock at the mountain's base under its own weight.

7. Exam Corner: Vital Formulae Box

$$\text{Compressibility (C)} = \frac{1}{B}$$

Reciprocal of Bulk Modulus.

$$\text{Force Constant (k)} = \frac{YA}{L}$$

From $F = k\Delta L$.

$$\text{Energy Density} = \frac{1}{2} \times \text{Stress} \times \text{Strain} \quad \text{or} \quad \frac{1}{2} Y \times (\text{Strain})^2$$

Energy per unit volume.

MULTIPLE CHOICE QUESTIONS

1. Modulus of rigidity of ideal liquid is
(a) infinity (b) Zero (c) Unity (d) Finite small non-Zero constant
2. With rise in temperature, the Young's modulus of elasticity: [JEE Main 2024]
(a) Changes erratically (b) Decreases (c) Increases (d) Remains unchanged
3. One end of a metal wire is fixed to a ceiling and a load of 2 kg hangs from it. Another identical wire is attached below carrying a 1 kg load. The ratio of longitudinal strain in upper wire to lower wire is: [JEE Main 2024]
(a) 1 : 1 (b) 2 : 1 (c) 3 : 1 (d) 1 : 2
4. The stress-strain curve for a ductile material shows: [NEET 2024]
(a) Sudden fracture (b) Large plastic deformation (c) No elastic region (d) Constant stress throughout
5. The property of a material by virtue of which it can be drawn into wires is:
(a) Elasticity (b) Ductility (c) Malleability (d) Brittleness
6. Assertion (A): Elasticity is the property by virtue of which a body regains its original shape. Reason (R): Plastic bodies regain original shape completely after deformation.
(a) Both A and R are true (b) Both A and R are false
(c) A is true, R is false (d) A is false, R is true
7. The dimensional formula of stress is:
(a) $[MLT^{-2}]$ (b) $[ML^{-1}T^{-2}]$ (c) $[ML^2T^{-2}]$ (d) $[M^0LT^{-2}]$
8. The SI unit of Young's modulus is:
(a) N (b) J (c) Pa (d) Watt
9. The ratio of lateral strain to longitudinal strain is called: [JEE Main 2022]
(a) Shear modulus (b) Poisson's ratio (c) Bulk modulus (d) Elastic limit
10. A body returns to its original shape after removal of deforming force. The body is said to possess: [NEET 2024]
(a) Plasticity (b) Viscosity (c) Elasticity (d) Rigidity
11. Two wires A and B of the same material have radii in the ratio 1 : 2. If equal forces are applied, the ratio of their extensions will be:
(a) 1 : 2 (b) 2 : 1 (c) 4 : 1 (d) 1 : 4
12. Two wires A and B are of same materials. Their lengths are in the ratio 1 : 2 and diameters are in the ratio 2 : 1 when stretched by force and respectively they get equal increase in their lengths. Then the ratio should be
(a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) 8 : 1
13. An Indian rubber cord L metre long and area of cross-section A meter² is suspended vertically. Density of rubber is D kg/m³ and Young's modulus of rubber is E N/m². If the wire extends by l metre under its own weight, then extension l is
(a) L^2Dg/E (b) $L^2Dg/2E$ (c) $L^2Dg/4E$ (d) L
14. To break a wire, a force of 10^6 N/m² is required. If the density of the material is 3×10^3 kg/m³, then the length of the wire which will break by its own weight will be

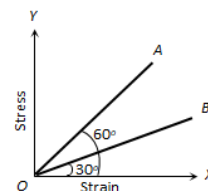
- (a) 34 m (b) 30 m (c) 300 m (d) 3 m

15. An elastic material of Young's modulus Y is subjected to a stress S . The elastic energy stored per unit volume of the material is

- (a) $\frac{2Y}{S^2}$ (b) $\frac{S^2}{2Y}$ (c) $\frac{S}{2Y}$ (d) $\frac{S^2}{Y}$

16. The stress versus strain graphs for wires of two materials A and B are as shown in the figure. If γ_A and γ_B are the Young's moduli of the materials, then

- (a) $\gamma_B = 2\gamma_A$ (b) $\gamma_A = \gamma_B$ (c) $\gamma_B = 3\gamma_A$ (d) $\gamma_A = 3\gamma_B$



17. The diameter of a brass rod is 4 mm and Young's modulus of brass is $9 \times 10^{10} \text{ N/m}^2$. The force required to stretch by 0.1% of its length is

- (a) $360\pi \text{ N}$ (b) 36 N (c) $144\pi \times 10^3 \text{ N}$ (d) $36\pi \times 10^5 \text{ N}$

18. Two wires of equal lengths are made of the same material. Wire A has a diameter that is twice as that of wire B . If identical weights are suspended from the ends of these wires, the increase in length is

- (a) Four times for wire A as for wire B (b) Twice for wire A as for wire B
(c) Half for wire A as for wire B (d) One-fourth for wire A as for wire B

19. A 5 metre long wire is fixed to the ceiling. A weight of 10 kg is hung at the lower end and is 1 metre above the floor. The wire was elongated by 1 mm. The energy stored in the wire due to stretching is

- (a) Zero (b) 0.05 joule (c) 100 joule (d) 500 joule

20. A wire undergoes longitudinal strain of 0.004 and lateral strain of 0.001. Poisson's ratio is:

- (a) 0.25 (b) 0.5 (c) 0.75 (d) 1

ANSWERS OF MULTIPLE-CHOICE QUESTIONS

Q No.	OPTION	Q. No	OPTION	Q No.	OPTION	Q. No	OPTION
1	b	6	c	11	c	16	d
2	b	7	b	12	d	17	b
3	c	8	c	13	a	18	d
4	b	9	b	14	a	19	b
5	b	10	c	15	b	20	a

HINTS OF MULTIPLE-CHOICE QUESTIONS

1. Modulus of rigidity of ideal liquid is

Answer: (b) Zero

Solution: Liquids cannot resist shear stress, so modulus of rigidity = 0.

2. With rise in temperature, Young's modulus

Answer: (b) Decreases

Solution: Increase in temperature weakens intermolecular forces, reducing elasticity.

3. Ratio of longitudinal strain in upper wire to lower wire

Answer: (c) 3 : 1

Sol: Upper wire supports 3 kg while lower wire supports 1 kg. Since strain $\propto$ force, ratio = 3:1.

4. Stress-strain curve for ductile material shows

Answer: (b) Large plastic deformation

Solution: Ductile materials elongate considerably before breaking.

5. Property by which material can be drawn into wires
Solution: Ductility is the property of forming thin wires.

Answer: (b) Ductility

6. Assertion–Reason

Answer: (c) A is true, R is false

Solution: Elastic bodies regain original shape. Plastic bodies do not regain original shape completely.

7. Dimensional formula of stress

Answer: (b) $[ML^{-1}T^{-2}]$

Solution: Stress = Force/Area = $[MLT^{-2}]/[L^2] = [ML^{-1}T^{-2}]$.

8. SI unit of Young's modulus

Answer: (c) Pa

Solution: Young's modulus has the same unit as stress, i.e., Pascal.

9. Ratio of lateral strain to longitudinal strain

Answer: (b) Poisson's ratio

Solution: Poisson's ratio = lateral strain / longitudinal strain.

10. Body regains original shape after removal of force

Answer: (c) Elasticity

Solution: This property is called elasticity.

11. Ratio of extensions of wires

Answer: (c) 4 : 1

Solution: Extension is inversely proportional to area and $\text{area} \propto r^2$.

12. Ratio of forces applied

Answer: (d) 8 : 1

Solution: Using $\Delta L = FL/AY$ and equal extensions, the ratio becomes 8:1.

13. Extension of rubber cord under own weight

Answer: (a)

Solution: Extension under self-weight is $l = DL^2g / 2E$.

14. Length of wire breaking under own weight

Answer: (a) 34 m

Solution: Using $\sigma = \rho gL$, the calculated length is about 34 m.

15. Elastic energy stored per unit volume

Answer: $S^2 / 2Y$

Solution: Energy density = $\frac{1}{2} \times \text{stress} \times \text{strain} = S^2 / 2Y$.

16. Relation between Young's moduli

Answer: Depends on graph slope

Solution: Young's modulus is proportional to slope of stress–strain graph.

17. Force required to produce 0.1% strain

Answer: (b) 36 N

Solution: Using $Y = (F/A)/\text{strain}$ gives approximately 36 N.

18. Increase in length of wires

Answer: (d) One-fourth for wire A as for wire B

Solution: Extension is inversely proportional to area.

19. Energy stored in stretched wire

Answer: (b) 0.05 J

Solution: Using $U = \frac{1}{2}F\Delta L$ gives 0.05 J.

20. Poisson's ratio

Answer: (a) 0.25

Solution: Poisson's ratio = $0.001 / 0.004 = 0.25$.

ANSWERS OF ASSERTION-REASON TYPE QUESTIONS

Choose the correct option:

- (A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(B) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(C) Assertion is true but Reason is false. (D) Assertion is false but Reason is true.

1. Assertion: Stress is directly proportional to strain within elastic limit.

Reason: Hooke's law is valid only within elastic limit.

2. Assertion: Steel is more elastic than rubber.

Reason: Steel has greater Young's modulus than rubber.

3. Assertion: Bulk modulus is applicable to liquids.

Reason: Liquids can easily change their shape but not volume.

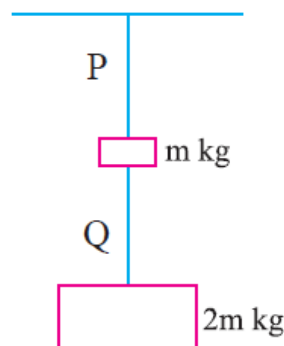
4. Assertion: A perfectly elastic body regains its original shape completely after removing deforming force.
Reason: Elasticity is the property of permanent deformation.
5. Assertion: The SI unit of stress is pascal.
Reason: Stress is defined as force per unit area.

Answer:

1	2	3	4	5
A	A	B	C	A

2-MARKS QUESTIONS

- Define Young's modulus of elasticity.
- State Hooke's law. Deduce expression for young's modulus of material of a wire of length 'l', radius of cross-section 'r' loaded with a body of mass M producing an extension Δl in it.
- Give reasons why the springs are made of steel and not of copper.
- A wire of length l area of crossection A and young's modulus Y is stretched by an amount x. What is the work done ?
- Prove that the elastic potential energy per unit volume is equal to stress $\times$ strain.
- Define the term bulk modulus. Give its SI unit. Give the relation between bulk modulus and compressibility.
- Define shear modulus. With the help of a diagram explain how shear modulus can be calculated.
- Which is more elastic steel or rubber. Explain.
- Two wires P and Q of same diameter are loaded as shown in the figure. The length of wire P is L m and its young's modulus is Y N/m² while length of wire Q is twice that of P and its material has young's modulus half that of P. Compute the ratio of their elongation.



Hint...

$$\begin{aligned}\Delta l_p &= \frac{3mg}{A} \times \frac{L}{Y} \\ \Delta l_Q &= \frac{2mg}{A} \times \frac{2L}{Y/2} = \frac{8mg}{A} \times \frac{L}{Y} \\ \frac{\Delta l_p}{\Delta l_q} &= \frac{3}{8}\end{aligned}$$

10. In case of emergency, a vacuum brake is used to stop the train. How does this brake works ?

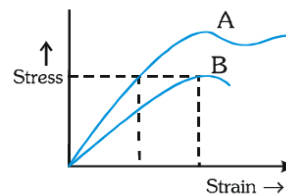
SOLUTION OF 2-MARKS QUESTIONS

- Young's modulus is defined as the ratio of longitudinal stress to longitudinal strain within the elastic limit of a material. It measures the resistance of a solid to a change in its length.
- Derivation: Let a wire have length "l" and radius "r". Area of cross-section $A = \pi r^2$.
When a mass M is hung, the stretching force $F = Mg$.
Longitudinal Stress = $F / A = Mg / \pi r^2$
Longitudinal Strain = $\Delta l / l$ Young's Modulus (Y) = Stress / Strain
 $Y = (Mg / \pi r^2) / (\Delta l / l)$ $Y = (Mg \times l) / (\pi r^2 \times \Delta l)$
- Springs are made of steel because steel is more elastic than copper. Steel has a higher Young's modulus, meaning it can withstand greater deforming forces and returns to its original shape more accurately. A steel spring will have a higher spring constant and will not undergo permanent deformation as easily as a copper one.

4. When a wire is stretched by an amount x , the work done (W) is stored as elastic potential energy.
 Average Force = $(0 + F)/2 = F/2$ Since $F = YAx/l$,
 Work Done = Average Force $\times$ Displacement = $(\frac{1}{2})(YAx/l) \times x$ $W = \frac{1}{2}(YAx^2) / l$
5. Work Done (W) = $\frac{1}{2} \times$ Force $\times$ Extension
 Energy Density (u) = $W / \text{Volume} = (\frac{1}{2} \times F \times \Delta l) / (A \times l)$
 $u = \frac{1}{2} \times (F/A) \times (\Delta l/l)$ $u = \frac{1}{2} \times \text{stress} \times \text{strain}$
8. Steel is more elastic than rubber. In physics, elasticity is defined by the resistance to deformation. For the same amount of stress, steel produces much less strain than rubber. Therefore, Young's modulus for steel is much higher than that for rubber ($Y_{\text{steel}} > Y_{\text{rubber}}$).

3-MARKS QUESTIONS

- Define modulus of elasticity. Write its dimensional formula and S.I. unit. Explain its types.
- Stress and pressure are both force per unit area. Then in what respect does stress differ from pressure? Also explain the types of stress.
- The stress versus strain graphs for two materials A and B are shown. Explain (i) Which material has greater Young's modulus ?
 (ii) Which material is more ductile ?
 (iii) Which material is more brittle ?
 (iv) Which of the two is the stronger material ?



SOLUTIONS OF 3-MARKS QUESTIONS

- Answer: **Modulus of Elasticity**
 Recall the relation between **stress** and **strain**.
 Modulus of elasticity = **ratio of stress to strain**.
 Stress = Force/Area $\rightarrow$ derive dimensions using $F = ma$.
 Types: **Young's modulus** $\rightarrow$ longitudinal stress/strain
Bulk modulus $\rightarrow$ volumetric stress/strain
Shear modulus (Rigidity modulus) $\rightarrow$ shearing stress/strain
- Answer: Both are force per unit area, but think about: Where they act
 Pressure acts normally and equally in all directions in fluids.
 - Stress develops inside a body due to deforming forces.
 - Types of stress:
 - Longitudinal stress**
 - Tensile stress
 - Compressive stress
 - Bulk stress**
 - Shearing stress**
- Answer: **Stress–Strain Graph Questions**
 Use the graph carefully.
 - Greater Young's modulus**
 - More ductile material**
 - More brittle material**
 - Stronger material**
 - Stronger material can withstand greater stress before breaking.
 Compare ultimate/breaking stress values on graph.

CASE BASED QUESTIONS

- I. Engineers use steel cables in suspension bridges because steel can withstand large forces without permanent deformation. When heavy vehicles pass over the bridge, the cables stretch slightly and regain their original shape after the load is removed.

- The property by virtue of which steel regains its original shape is called:
(a) Elasticity (b) Plasticity (c) Rigidity (d) Viscosity
- The restoring force per unit area developed inside the cable is called:
(a) Strain (b) Pressure (c) Stress (d) Thrust
- Steel is preferred for bridges because it has:
(a) Low density (b) High elasticity (c) Low strength (d) High plasticity
- Hooke's law is valid only within:
(a) Breaking point (b) Elastic limit (c) Plastic region (d) Melting point

OR

The SI unit of Young's modulus is:

- (a) Pascal (b) Newton (c) Joule (d) Watt

- II. A student stretches a rubber band and a steel wire using equal forces. The rubber band stretches much more, while the steel wire shows very little extension.

- Which material is more elastic?
(a) Rubber (b) Steel (c) Plastic (d) Wood
- The ratio of change in length to original length is called:
(a) Stress (b) Pressure (c) Strain (d) Elasticity
- The material which undergoes permanent deformation is said to be:
(a) Elastic (b) Rigid (c) Plastic (d) Perfect
- The SI unit of stress is:
(a) N (b) N/m (c) N/m² (d) J

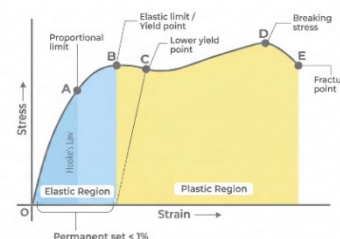
OR

Stress is equal to:

- (a) Force $\times$ Area (b) Force/Area (c) Area/Force (d) Mass $\times$ Acceleration

5-MARKS QUESTIONS

- Draw stress and strain curve. Explain the various region or points that are necessary to understand this graph (i) Proportional Limit (ii) Elastic Limit (iii) Yield Point (iv) Ultimate Stress Point (v) Fracture or Breaking Point.
- State Hooke's law. What does it define? Is this only define for solid? If yes why? In a wire of length L , the increase in its length is l . If the length is reduced to half, What will be the increase in its length.
- A construction engineer is selecting a beam to support a heavy bridge. They have a choice between a solid rectangular bar and an I-shaped girder made of the same material and weight.



- Using the expression for the depression (δ) of a beam loaded at the center, $\delta = (Wl^3)/(4bd^3Y)$, explain why the depth (d) is more critical than the breadth (b) in reducing sag.
 - Why is the I-shape preferred over a solid rectangular bar of the same cross-sectional area? Discuss in terms of weight and strength.
 - A steel girder of length 5 m, breadth 10 cm, and depth 20 cm is loaded with 2000 kg at the center. Calculate the depression produced. ($Y_{\text{steel}} = 2 \times 10^{11} \text{ N/m}^2$, $g = 10 \text{ m/s}^2$).
4. A solid sphere is taken to the bottom of the Mariana Trench, where the pressure is $1.1 \times 10^8 \text{ Pa}$.

$$B = -V(\Delta P/\Delta V).$$

- (b) If the Bulk Modulus of the material is 1.6×10^{11} Pa and the initial radius is 0.5 m, calculate the fractional change in volume.
- (c) What is the relationship between Bulk Modulus and Compressibility? Which state of matter has the highest compressibility?
5. A rectangular block of gelatin is riveted to a table. A horizontal force is applied to the top face.
- (a) Describe the difference between Shearing Stress and Tensile Stress.
- (b) A metal cube of side 10 cm is subjected to a shearing force of 10^6 N. If the top face is displaced by 0.05 cm relative to the bottom, calculate the Shear Modulus (η) of the metal.
- (c) Why is the Shear Modulus typically much smaller than the Young's Modulus for most materials?

SOLUTIONS OF 5-MARKS QUESTIONS

1. The Diagram and explanation

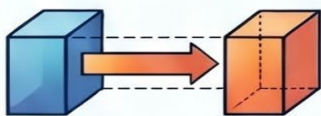
2. Hint- $l \propto L$ i.e. if length is reduced to half then increase in length will be $\frac{l}{2}$.
3. Part (a): In the formula $\delta \propto 1/(bd^3)$, the depression is inversely proportional to the cube of the depth but only linearly proportional to the breadth. Doubling the depth reduces sag by 8 times, while doubling breadth only reduces it by 2 times. Thus, increasing depth is much more efficient for structural stability.
- Part (b): An I-beam provides a large surface area for the load to be distributed on the flanges, while the vertical web resists shear. It provides a high Moment of Inertia (resistance to bending) while saving material and reducing the beam's own weight, preventing buckling.
- Part (c): $W = mg = 2000 \times 10 = 2 \times 10^4$ N; $l = 5$ m, $b = 0.1$ m, $d = 0.2$ m.
 Calcul: $\delta = (2 \times 10^4 \times 5^3) / (4 \times 0.1 \times (0.2)^3 \times 2 \times 10^{11}) = 25 \times 10^5 / 6.4 \times 10^8 \approx 3.9 \text{ mm}$.
4. Part (a): Bulk Modulus is the ratio of Hydraulic Stress to Volumetric Strain. The negative sign indicates that as pressure increases ($+\Delta P$), the volume decreases ($-\Delta V$), ensuring the constant B is positive.
- Part (b): $\Delta V/V = P/B = (1.1 \times 10^8) / (1.6 \times 10^{11}) = 6.875 \times 10^{-4}$.
- Part (c): Compressibility $K = 1/B$. Gases have the highest compressibility because their atoms/molecules have large intermolecular spaces compared to solids and liquids.
5. Part (a): Tensile stress acts perpendicular to the area (causing change in length), while shearing stress acts tangential to the surface (causing change in shape/angle).
- Part (b): Area = $(0.1)^2 = 0.01 \text{ m}^2$. Shearing Stress = $10^6 / 0.01 = 10^8 \text{ N/m}^2$.
 Shear Strain = $\Delta x/L = 0.0005 / 0.1 = 0.005$. $\eta = \text{Stress/Strain} = 10^8 / 0.005 = 2 \times 10^{10} \text{ Pa}$.
- Part (c): It is generally easier to slide layers of atoms over one another (shape change) than it is to pull them apart or compress them (length/volume change). Thus, materials offer less resistance to shear.

MECHANICS OF SOLIDS:

FORMULA SHEET

FUNDAMENTAL ELASTICITY & MODULI

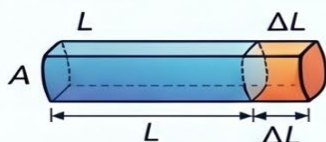
Stress & Strain Equations



$$\text{Stress} = \frac{F}{A}$$

$$\text{Longitudinal Strain} = \frac{\Delta L}{L}$$

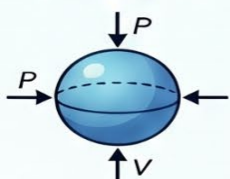
Young's Modulus (Y)



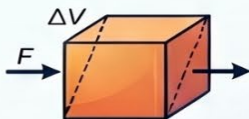
$$Y = \frac{F \times L}{A \times \Delta L}$$

Measures a material's resistance to linear deformation.

Bulk (K) & Rigidity (η) Moduli



$$K = \frac{P}{\Delta V/V}$$



$$\eta = \frac{\text{Shearing Stress}}{\text{Shearing Strain}}$$

SI UNITS & DIMENSIONAL PROPERTIES

Quantity	SI Unit	Dimensional Nature
Stress	Pascal (Pa) / N/m ²	Same as Pressure
Strain	None	Dimensionless
Moduli (Y, K, η)	Pascal (Pa)	Same as Stress

ENERGY, LIMITS, & COMPARISONS

Elastic Potential Energy (U)



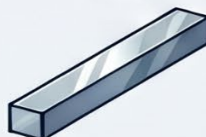
$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

Hooke's Law & The Elastic Limit



This law is valid only when the material is within its elastic limit.

Material Elasticity: Steel vs. Rubber



Steel is more elastic because its Young's modulus is higher than rubber's.



Rubber band because the lower Young's modulus compared to steel.

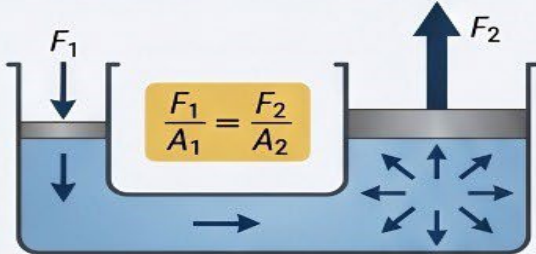
CHAPTER –9 MECHANICAL PROPERTIES OF FLUIDS

FLUID STATICS & SURFACE PHENOMENA

Summarizes mechanical properties of fluids, focusing on behavior at rest and in motion, covering essential laws (Pascal's, Bernoulli's, Stoke's) and critical formulas.

FLUID DYNAMICS & VISCOUS FLOW

Pascal's Law & Hydraulic Systems



Pressure applied to an enclosed fluid is transmitted equally in all directions; used in hydraulic lifts and brakes.

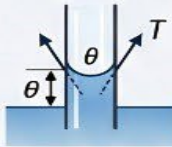
Surface Tension & Capillarity



Surface Tension

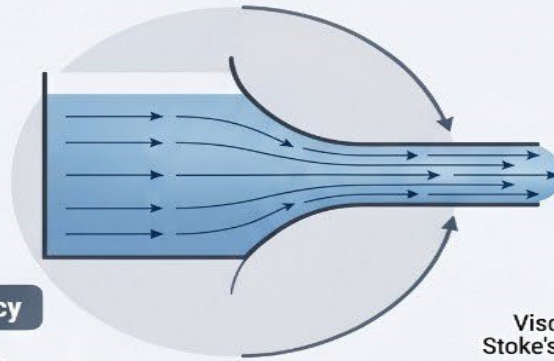
$$T = \frac{F}{l}$$

Liquids minimize surface area via Surface Tension; drives capillary rise and spherical raindrop shapes.

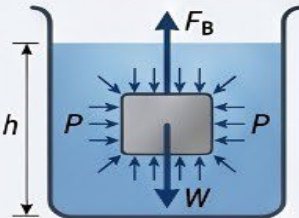


Capillary Rise

$$h = \frac{2T \cos \theta}{\rho r g}$$



Hydrostatic Pressure & Buoyancy



Pressure:

$$P = P_0 + \rho g h$$

Buoyant Force:

$$F_B = \rho V g$$

Pressure increases with depth; Buoyant force equals the weight of the displaced fluid.

COMPARISON: SURFACE PHENOMENA & PRESSURE

Excess Pressure

Drop
(1 surface)



Drop:

$$\frac{2T}{R}$$

Bubble
(2 surfaces)



Bubble:

$$\frac{4T}{R}$$

Angle of Contact

Acute
(Water/Glass)



$\theta < 90^\circ$

Water:

0°–8°

Obtuse
(Mercury/Glass)



$\theta > 90^\circ$

Mercury:

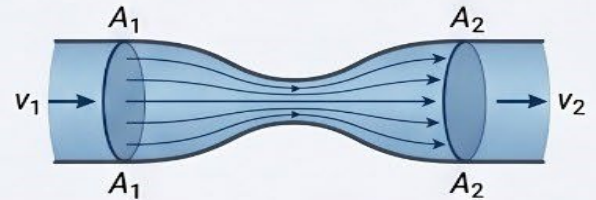
130°–140°

Gauge Pressure

The difference between absolute and atmospheric pressure

$$P - P_0 = \rho g h$$

Conservation of Flow (Continuity & Bernoulli)



Equation of Continuity

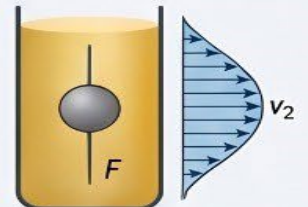
$$A_1 v_1 = A_2 v_2$$

Bernoulli's Theorem

$$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$$

Incompressible fluids follow the Equation of Continuity and Bernoulli's energy conservation.

Viscosity & Stoke's Law

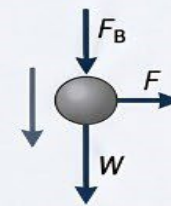


Stoke's Law

$$F = 6\pi\eta r v$$

Viscosity is internal fluid friction; Stoke's Law defines the dragging force on a sphere moving through a fluid.

Terminal Velocity & Reynolds Number



Reynolds Number determines flow type.



Streamline Flow



Turbulent Flow

Objects reach a constant maximum speed (**Terminal Velocity**) when viscous force equals net weight; flow type is determined by the **Reynolds Number**.

MOST COMMON STUDENT MISTAKES

- ✗ **Aircraft Wing Lift:** Buoyant force acts downward. (Corrected principle: Bernoulli's Theorem)
- ✗ **Hydraulic Lift:** Pressure depends only on force (it's F/A). (Corrected principle: Pascal's Law)
- ✗ **Raindrop Shape:** Surface tension and viscosity are the same. (Corrected principle: Surface Tension)

MULTIPLE CHOICE QUESTIONS

1. Pressure at a depth h inside a liquid is given by:

(a) $P = \rho gh$ (b) $P = \rho g/h$ (c) $P = gh/\rho$ (d) $P = \rho h/g$
2. Hydraulic lift works on:

(a) Archimedes' principle (b) Bernoulli's principle (c) Pascal's law (d) Newton's law
3. In hydraulic brakes, pressure applied at one point is transmitted:

(a) Unequally (b) Only upward (c) Equally in all directions (d) Only downward
4. Viscosity is the property of fluid due to which it opposes:

(a) Compression (b) Relative motion of layers (c) Expansion (d) Pressure
5. A body attains terminal velocity when:

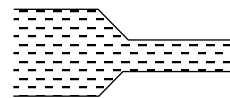
(a) Acceleration is maximum (b) Net force becomes zero
(c) Weight becomes zero (d) Buoyant force vanishes
6. Radius of a soap bubble is increased from R to $2R$ work done in this process in terms of surface tension is

(a) $24\pi S R^2$ (b) $48\pi S R^2$ (c) $12\pi S R^2$ (d) $36\pi S R^2$
7. If pressure at half the depth of a lake is equal to $2/3$ pressure at the bottom of the lake then what is the depth of the lake?

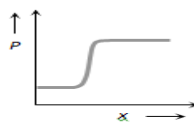
(a) 10 m (b) 20 m (c) 60 m (d) 30 m
8. A cylindrical tank has a hole of 1 cm^2 in its bottom. If the water is allowed to flow into the tank from a tube above it at the rate of $70\text{ cm}^3/\text{sec}$. then the maximum height up to which water can rise in the tank is

(a) 2.5 cm (b) 5 cm (c) 10 cm (d) 0.25 cm
9. There is a hole in the bottom of tank having water. If total pressure at bottom is 3 atm ($1\text{ atm} = 105\text{ N/m}^2$) then the velocity of water flowing from hole is

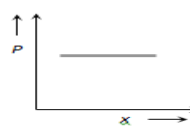
(a) $\sqrt{400}\text{ m/s}$ (b) $\sqrt{600}\text{ m/s}$ (c) $\sqrt{60}\text{ m/s}$ (d) None of these
10. Water flows through a frictionless duct with a cross-section varying as shown in fig. Pressure p at points along the axis is represented by



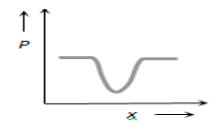
(a)



(b)



(c)



(d)

11. In streamline flow, every particle passing through a point follows:

(a) Different path (b) Circular path (c) Same path (d) Zig-zag path
12. Flow becomes turbulent when velocity exceeds:

(a) Escape velocity (b) Average velocity (c) Critical velocity (d) Terminal velocity
13. Reynolds number is used to determine:

(a) Surface tension (b) Nature of flow (c) Pressure energy (d) Capillary rise

14. Bernoulli's theorem is based on conservation of:

- (a) Charge (b) Momentum (c) Energy (d) Mass

15. The equation $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$ represents:

- (a) Stokes' law (b) Bernoulli's theorem (c) Pascal's law (d) Hooke's law

16. Aeroplanes fly due to:

- (a) Buoyancy (b) Dynamic lift (c) Surface tension (d) Viscosity

17. Surface energy per unit area is numerically equal to:

- (a) Pressure (b) Density (c) Surface tension (d) Viscosity

18. Water flows faster through a narrow pipe because:

- (a) Pressure increases (b) Area decreases
(c) Velocity increases due to continuity equation (d) Density increases

19. After terminal velocity is reached, the acceleration of a body falling through a fluid is

- (a) equal to g (b) zero (c) less than g (d) greater than g

20. A wooden stick 2m long is floating on the surface of water. The surface tension of water 0.07 N/m. By putting soap solution on one side of the sticks the surface tension is reduced to 0.06 N/m. The net force on the stick will be

- (a) 0.07 N (b) 0.06 N (c) 0.01 N (d) 0.02 N

ANSWERS

Q No.	OPTION	Q. No	OPTION	Q No.	OPTION	Q. No	OPTION
1	a	6	a	11	c	16	b
2	c	7	b	12	c	17	c
3	c	8	a	13	b	18	c
4	b	9	b	14	c	19	b
5	b	10	a	15	b	20	d

HINTS OF MULTIPLE-CHOICE QUESTIONS

- Pressure in a liquid depends directly on density, gravity, and depth. Recall the hydrostatic pressure formula.
- Think about the law stating that pressure applied to an enclosed fluid is transmitted equally in all directions.
- In hydraulic systems, liquids transmit pressure uniformly throughout the fluid.
- Viscosity is related to internal friction between adjacent layers of a moving fluid.
- Terminal velocity is reached when all forces acting on the body balance each other.
- A soap bubble has two surfaces. Work done equals increase in surface energy.
- Use pressure relation:

$$P = P_0 + \rho gh \quad \text{Compare pressure at half depth and full depth.}$$

- At maximum height, inflow rate equals outflow rate. Use Torricelli's theorem:

$$v = \sqrt{2gh}$$

- Excess pressure at the hole causes water to flow out. Use:
Convert pressure into equivalent water column height first.
- In a narrower section of a pipe, fluid speed increases and pressure decreases according to Bernoulli's principle.

11. In streamline flow, fluid particles crossing the same point move along identical trajectories.
12. Turbulence starts when flow speed crosses a certain limiting value called critical velocity.
13. Reynolds number helps classify flow as streamline or turbulent.
14. Bernoulli's theorem comes from conservation of mechanical energy in fluid motion.
15. Identify the equation combining pressure energy, kinetic energy, and potential energy of a fluid.
16. Air moving faster above the wing creates lower pressure, producing lift.
17. Surface tension can also be defined as surface energy per unit area.
18. Apply the continuity equation: $Av = \text{constant}$ Smaller area means higher speed.
19. At terminal velocity, net force becomes zero. What does that imply about acceleration?
20. Net force due to surface tension depends on difference in surface tensions on both sides: $F = (T_1 - T_2)L$ where L is the length of the stick.

ASSERTION-REASON TYPE QUESTIONS-

1.
 - (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
 - (b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
 - (c) If assertion is true but reason is false.
 - (d) If the assertion and reason both are false.

Assertion : The concept of surface tension is held only for liquids.
Reason : Surface tension does not hold for gases.
2. **Assertion :** At critical temperature, surface tension of a liquid becomes zero.
Reason : At this temperature, intermolecular forces for liquids and gases become equal. Liquid can expand without any restriction.
3. **Assertion :** A large soap bubble expands while a small bubble shrinks, when they are connected to each other by a capillary tube.
Reason : The excess pressure inside bubble (or drop) is inversely proportional to the radius.
4. **Assertion :** It is better to wash the clothes in cold soap solution.
Reason : The surface tension of cold solution is less than the surface tension of hot solution.
5. **Assertion :** Tiny drops of liquid resist deforming forces better than bigger drops.
Reason : Excess pressure inside a drop is directly proportional to surface tension.

SOLUTION OF ASSERTION-REASON TYPE QUESTIONS-

1. **(b)** We know that the intermolecular distance between the gas molecules is large as compared to that of liquid. Due to it the forces of cohesion in the gas molecules are very small and these are quite large for liquids. Therefore, the concept of surface tension is applicable to liquid but not to gases.
2. **(a)** Zero surface tension means no opposition to expansion.
3. **(a)** Since the excess pressure due to surface tension is inversely proportional to its radius, it follows that smaller the bubble, greater is the excess pressure. Thus, when the larger and the smaller bubbles are put in communication, air starts passing from the smaller into the large bubble because excess pressure inside the former is greater than inside the latter. As a result, the smaller bubble shrinks and the larger one swells.
4. **(d)** The soap solution, has less surface tension as compared to ordinary water and its surface tension decreases further on heating. The hot soap solution can, therefore spread over large surface area and also it has more wetting power. It is on account of this property that hot soap solution can penetrate and clean the clothes better than the ordinary water.
5. **(b)** When a drop of liquid is poured on a glass plate, the shape of the drop is governed by two forces, the force of gravity. For very small drops, the potential energy due to gravity is insignificant compared to

that due to surface tension. Hence, in this case the shape of the drop is determined by surface tension alone and drop becomes spherical.

2-MARKS QUESTIONS

1. A truck carrying heavy loads has broader tyres than a small car. Explain why broad tyres are preferred for heavy vehicles.
2. During a storm, the roof of a weak house may get blown away. Explain this using Bernoulli's principle
3. A student notices that honey flows much slower than water from a bottle.
What property of liquids causes this difference? Explain.
4. A parachutist opens the parachute before reaching the ground.
Why does this help in safe landing?
5. Water comes out with greater speed from the lower hole of a water tank than from the upper hole. Give reason.
6. Small insects can walk on the surface of water without sinking.
Explain this phenomenon.
7. Modern cars and airplanes are given streamlined shapes. Explain the advantage of such shapes.
8. A spinning cricket ball swings in air during a match. Explain the reason briefly.
9. Hydraulic brakes are used in vehicles to stop them easily. Name the law involved and state its role.
10. Soap is added while washing clothes with water. Why does soap improve cleaning action?

SOLUTIONS OF 2-MARKS QUESTIONS

1. Broad tyres increase the area of contact with the ground. This reduces pressure on the road and prevents sinking.
2. Fast-moving air above the roof lowers pressure outside.
Higher pressure inside pushes the roof upward.
3. Honey has greater viscosity than water.
Therefore, it resists flow more strongly.
4. Opening the parachute increases air resistance.
This reduces the speed of the parachutist.
5. Pressure in a liquid increases with depth.
Hence water flows faster from the lower hole.
6. Surface tension acts like a stretched film on water surface.
It supports the weight of small insects.
7. Streamlined shapes reduce fluid friction or air resistance. This helps vehicles move faster and save fuel.
8. The spinning ball creates unequal air speeds around it. This produces pressure difference and causes swinging.
9. Hydraulic brakes work on Pascal's law. Pressure applied to brake fluid is transmitted equally to all wheels.
10. Soap reduces the surface tension of water. This allows water to spread easily and remove dirt better.

3-MARKS QUESTIONS

1. A car driver presses the brake pedal and the braking force is transmitted equally to all four wheels through brake oil. State Pascal's law for fluids. Using this law, explain the principle and working of hydraulic brakes with the help of a neat labelled diagram.
2. The terminal velocity of a tiny droplet is v . N number of such identical droplets combine together forming a bigger drop. Find the terminal velocity of the bigger drop.

3. Eight spherical rain drops of the same mass and radius are falling down with a terminal speed of 6 cm s^{-1} . If they coalesce to form one big drop, what will be the terminal speed of bigger drop? (Neglect the buoyancy of the air)
4. Derive the expression for terminal velocity of a rain drop falling in a vessel filled with fluid.
5. A water drop of diameter 2mm is split up into 10^9 identical water drops. Calculate the work done in this process. (The surface tension of water is $7.3 \times 10^{-2} \text{ Nm}^{-1}$)
6. Define Surface Energy and derive the expression of relation between surface energy and surface tension.
7. Prove that excess pressure inside a soap bubble is inversely proportional to radius of bubble.
8. Write the different type of flows of a fluid Hence define critical velocity.
9. A bucket of water is suspended from a spring balance. Does the reading of balance change (a) when a piece of stone suspended from a string is immersed in the water without touching the bucket? (b) when a piece of iron or cork is put in the water in the bucket?
10. Explain the correct reason behind these phenomena's (i) Special design of wings of an aeroplane. (ii) A person standing near the railway track is pulled toward the train, when fast moving train passes close to him.
11. Explain why the spinning ball deviates from its usual curved path.
12. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water the quantities of water flowing out per second from both the holes are the same. Then what is the value of R .
13. Derive by the method of dimensions, an expression for the volume of a liquid flowing out per second through a narrow tube. Assume that the rate of flow of liquid depends upon
 - i) Coefficient of viscosity of the liquid
 - ii) Radius of the tube
 - iii) Pressure gradient along the tube.
14. What amount of energy is released or absorbed when two mercury droplets of radii 0.1 cm and 0.2 cm collapse into one single drop? Given that the surface tension of mercury $T = 435.5 \times 10^{-3} \text{ Nm}^{-1}$.
15. Explain the reason behind it
 - (i) Blowing off of roofs of houses during a storm
 - (ii) How sprayer works?

SOLUTIONS OF 3-MARKS QUESTIONS

1. Pressure applied to an enclosed fluid is transmitted equally and undiminished in all directions throughout the fluid. Working with diagram.

3. Terminal velocity of bigger drop $= N^{2/3} v$

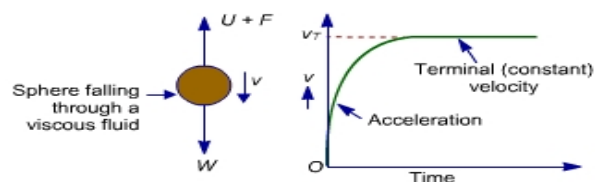
We have $8 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3 \Rightarrow R = 2r$ and we know that terminal velocity $v_T \propto (\text{Radius})^2$

We have $\frac{v_T}{v_r} = \frac{R^2}{r^2}$ or $V_T = v_T \frac{(2r)^2}{r^2} \quad V_T =$

0.24 m/s

4. **Terminal velocity**- when a body is dropped in a liquid, it, accelerates first due to gravity. In liquid it experience weight (force) 'W' along downward, buoyancy force 'U' along upward. Since downward force is greater so

the body moves along downward. As body in motion the layers of air in contact with body also set in motion due to this an upward viscous force 'F' act on the body. As velocity of the solid body increases



Value of 'F' increases. But after same time the downward force 'W' become exactly equal to the total upward force (U+F). Now there is no net force acting on the body so from Newton's first law the velocity becomes constant (and maximum). The constant maximum velocity with which the body moves in a fluid is called terminal velocity.

When the spherical body of radius 'r' moving with constant terminal velocity v_t .

$$\text{Then } U + F = W \quad \text{--- (1)}$$

U = weight of the liquid displaced = mass of the liquid displaced $\times g = \sigma V g$

W = weight of the solid = mass of the solid sphere $\times g = \rho V g$

From Stoke's law $F = 6 \pi \eta r v_t$ (Where ρ is density of ball, σ is density of liquid.)

$$\sigma V g + 6 \pi \eta r v_t = \rho V g \Rightarrow 6 \pi \eta r v_t = \sigma V g - \rho V g \Rightarrow 6 \pi \eta r v_t = V g (\sigma - \rho)$$

$$6 \pi \eta r v_t = \frac{4}{3} \pi r^3 g (\sigma - \rho) \text{ Hence } v_t = \frac{2 r^2 (\rho - \sigma) g}{9 \eta} \text{ From this formula: -}$$

$$1) v_t \propto r^2. \quad 2) v_t \propto 1/\eta \quad 3) v_t \propto (\sigma - \rho).$$

i) If $\sigma > \rho$ then r. h. s of equation is positive, then direction of terminal velocity is downward. ii) If $\sigma < \rho$ then r. h. s of equation is negative. Then direction of sphere is upward i.e. rising up. For this reason, air bubbles in a liquid are seen to move in upward direction.

5. Let a water drop of radius R be split up into 10^9 identical water drops each of radius $(R = D/2 = 2/2 = 1 \text{ mm} = 1 \times 10^{-3} \text{ m})$, No. of droplets $n = 10^9$; $T = 7.3 \times 10^{-2} \text{ N/m}$

$$W = 4\pi R^2 T [n^{1/3} - 1] = 4\pi (10^{-3})^2 \times 7.3 \times 10^{-2} [(10^9)^{1/3} - 1] = 9.17 \times 10^{-4} \text{ J}$$

6. SURFACE ENERGY: Let us consider a rectangular frame of wire PQRS, where arm RS can slide on the arm PR and QS. If this frame is dipped in a soap solution, then a soap film is produced in the frame PQRS. Due to surface tension, the soap film exerts a force on the frame.

Let 'l' be the length of the arm RS, then the force acting on the arm RS towards the film is given by $F = T \times 2l$ (1)

Here the length is taken twice due to the two-surface area of soap film. Let the arm RS be displaced to a new position 'AB' through a distance 'x'. Hence work done is $W = \text{Force} \times \text{Displacement}$ or $W = (T \times 2l) \times x$ (2)

$W = T \times (2l \times x)$ Work $E = T \times \Delta A$ (3) Which is required expression for surface energy Where $\Delta A = 2(l \times x) =$ increase in the area of the film on both sides $T = W / \Delta A$ or $T = E / \Delta A$

"surface tension of a liquid is numerically equal to the surface energy per unit area.

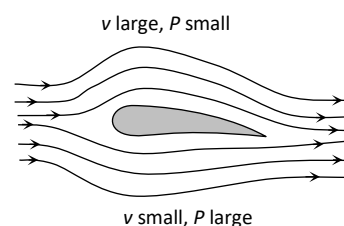
9. Yes, the reading of the balance will increase but the increase in weight will be equal to the loss in weight of the stone ($V\sigma g$) and not the weight of stone ($V\rho g$) [$> V\sigma g$ as $\rho > \sigma$].

(b) Yes, the reading of the balance will increase but the increase in weight will be equal to the weight of iron or cork piece.

10. (i) Wings of an aeroplane are made such that the upper surface is made wing in as compared to the lower surface of the wing is made concave. Due to

their shape of wing, the air currents at the top have a large velocity then at the bottom accordingly to Bernoulli their the pressure above the surface of the.

This difference of pressure is helpful in giving a vertical lift to the plane. But if upper surface is more covered then the airflow does not remain steady and turbulence is set in.



(ii) A person standing near the railway track is pulled toward the train, when fast moving train passes close to him. The fast train large velocity decreases the pressure

between the person and the train while pressure on the other side of man is large which pushes the person towards the track.

11. Curved path of **spinning ball** when a spinning ball is thrown with a large speed, then the layer of the air moving above the ball gets retarded and the layer below the ball gets accelerated due to spin motion because the layer above the ball moves in opposite direction and lower layer moves in the direction of the spinning ball so; the velocity of the ball decreases and hence according to the Bernoulli theorem pressure increases and due to increase in velocity below, pressure decreases .due to difference of pressure on the two sides of the ball , a path of the ball becomes curved.

12. **Ans:** Velocity of efflux when the hole is at depth h , $v = \sqrt{2gh}$

Rate of flow of water from square hole $Q_1 = a_1 v_1 = L^2 \sqrt{2gy}$

Rate of flow of water from circular hole $Q_2 = a_2 v_2 = \pi R^2 \sqrt{2g(4y)}$

and according to problem $Q_1 = Q_2 \Rightarrow L^2 \sqrt{2gy} = \pi R^2 \sqrt{2g(4y)} \Rightarrow R = \frac{L}{\sqrt{2\pi}}$

CASE BASED QUESTIONS

- III. A hydraulic lift is used in service stations to lift heavy vehicles. A small force applied on a small piston produces a much larger force on a bigger piston through the liquid.

1. Hydraulic lift works on the basis of:

- (a) Bernoulli's principle (b) Pascal's law
(c) Archimedes' principle (d) Hooke's law

2. Pressure applied to a confined liquid is transmitted:

- (a) Only upward (b) Only downward (c) Equally in all directions (d) Only sideways

3. The SI unit of pressure is:

- (a) Pascal (b) Joule (c) Newton (d) Watt

4. Hydraulic brakes are commonly used in:

- (a) Aeroplanes (b) Bicycles (c) Vehicles (d) Ships

OR

Pressure is defined as:

- (a) Force $\times$ Area (b) Force/Area
(c) Area/Force (d) Mass/Volume

- IV. A ship made of iron floats on water, while a small iron nail sinks. The ship is designed such that it displaces a large volume of water.

1. The upward force exerted by water on the ship is called:

- (a) Viscous force (b) Surface tension (c) Buoyant force (d) Drag

2. Archimedes' principle relates buoyant force to:

- (a) Weight of displaced fluid (b) Mass of object (c) Density of object (d) Pressure

3. The ship floats because its average density is:

- (a) Greater than water (b) Equal to water (c) Less than water (d) Zero

4. An iron nail sinks in water because:

- (a) Its density is less than water (b) Its density is greater than water
(c) It has no buoyant force (d) Water has no density

OR

The SI unit of density is:

- (a) kg m^{-3} (b) N m^{-2} (c) Pascal (d) Joule

5-MARKS QUESTIONS

- Define different units of pressure and also find the height of atmospheric pressure.
 - Pressure decreases as one ascends the atmosphere. If the density of air is ρ , what is the change in pressure dp over a differential height dh ?
 - A fish is swimming 10 m below the surface of water in a lake. Estimate the pressure on it..
- State and prove Bernoulli's theorem.
 - Explain any two applications of Bernoulli's theorem.
- Explain capillary phenomenon and derive an expression for height of fluid in a capillary tube raised due to capillary action.
- Explain the torricelli's theorem. How can we calculate the range and time taken by the liquid flowing out from a hole. Also tell at what condition the range of the liquid coming from the hole becomes maximum.
- A glass capillary tube of internal radius 0.5 mm is dipped vertically into a beaker of water.
 - Define Surface Tension in terms of surface energy.
 - Derive the "Ascent Formula" for the height (h) in a capillary tube.
 - Calculate the height of the water column ($T = 0.073 \text{ N/m}$, $g = 9.8 \text{ m/s}^2$).
 - What happens if the tube length is less than calculated height h ?

SOLUTIONS OF 5-MARKS QUESTIONS

- Units of pressure are: $1 \text{ atm} = \text{Pressure exerted by } 0.76 \text{ m of mercury column} = h\rho g$
 $= 0.76 \times 13.6 \times 10^3 \times 9.8 = 1.013 \times 10^5 \text{ N/m}^2$

Height of the atmosphere: The atmospheric pressure, $P_a = h\rho g$

$$\text{or } h = P_a / \rho g = 1.013 \times 10^5 / 1.3 \times 9.8 = 7951 \text{ m} \approx 8 \text{ km}$$

(b) Consider a part (packet) of atmosphere of thickness dh , As the pressure at a point in fluid is equal in all directions. So the pressure on upper layer is p acting downward and on lower layer is $(p+dp)$ acting upward. Force due to pressure is balanced by Buoyant force by air $(p+dp)A - pA = -V\rho g$

$$p.A + dp.A - p.A = -A(dh)\rho g$$

$$dp.A = -\rho g(dh)A$$

$$dp/p = -\rho g(dh)$$

Negative shows that pressure decreases as height increases.

(c) $P_a \approx 10^5 \text{ Pa}$ (1 atm), $g = 10 \text{ ms}^{-2}$ Density of water $\rho = 10^3 \text{ kgm}^{-3}$)

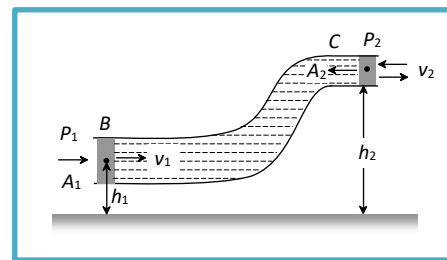
Absolute Pressure $P = P_a + h \rho g$ Absolute Pressure $P = 10^5 + 10 \times 10^3 \times 10$ Absolute Pressure
 $P = 2 \times 10^5 \approx 2 \text{ atm}$

- Mathematically for unit volume of liquid flowing through a pipe.
constant

$$P + \rho gh + \frac{1}{2} \rho v^2 =$$

To prove

If P_1 and P_2 are the pressures at the two ends of the tube respectively, work done in pushing the volume V of incompressible fluid from point B to C through the tube will be $W = P_1 V - P_2 V = (P_1 - P_2)V$ (i)



This work is used by the fluid in two ways.

(i) In changing the potential energy of mass m (in the volume V) from mgh_1 to mgh_2 ,

i.e., $\Delta U = mg(h_2 - h_1)$ (ii)

(ii) In changing the kinetic energy from $\frac{1}{2}mv_1^2$ to $\frac{1}{2}mv_2^2$, i.e., $\Delta K = \frac{1}{2}m(v_2^2 - v_1^2)$ (iii)

Now as the fluid is non-viscous, by conservation of mechanical energy

$$W = \Delta U + \Delta K \quad \text{i.e., } (P_1 - P_2)V = mg(h_2 - h_1) + \frac{1}{2}m(v_2^2 - v_1^2)$$

or $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

This equation is the so-called Bernoulli's equation and represents conservation of mechanical energy in case of moving fluids.

3. A very fine bore is known as the capillarity (hair like).

The property of rise and depression of a liquid inside a capillary tube is called capillarity.

In general the liquid that wet the glass rises inside the tube while those don't wet the glass show a depression

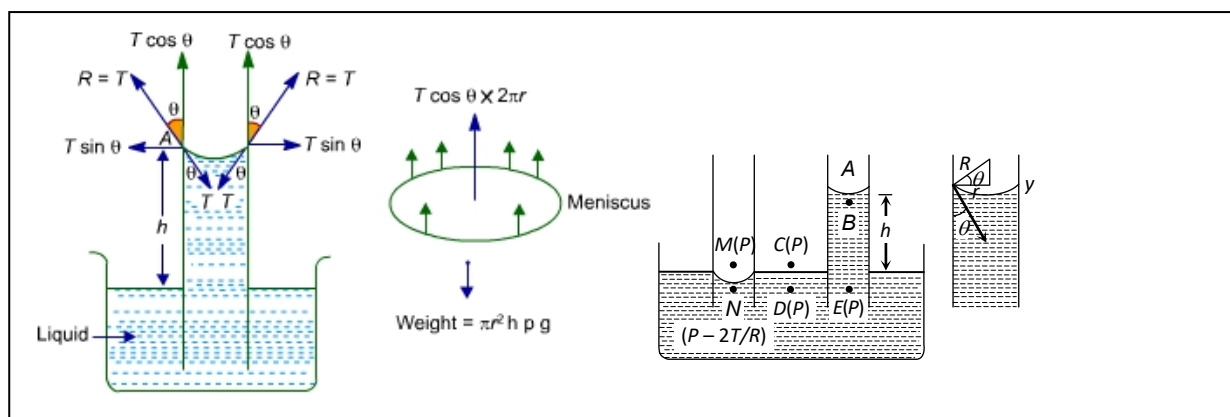
Ascent Formula: Consider a capillary tube dipped in a dish containing water.

Pressure at points C and D just above and below the plane surface of liquid in the vessel is also P (atmospheric pressure). The points N and D are in the same horizontal plane in the liquid but the pressure at these points is different. In order to maintain the equilibrium, the liquid level rises in the capillary tube upto height h .

If T is S.T of water, then on water outside be $P = \frac{2T}{R}$ - - - - - (1)

As the pressure on the liquid side is less, the liquid rises in the capillary. The height to which the liquid rises is such that the excess pressure $2T/R$ is equal to the pressure exerted by liquid column in the capillary tube. If 'r' is the radius of the capillary tube, 'h' is height of liquid rises, 'd' is the density of the liquid and θ is angle of contact then, pressure exerted by the liquid column in capillary tube is $P_{\text{liq}} = h d g$ (2)

For equilibrium, Pressure due to liquid column = pressure difference due to surface tension (excess of pressure)



comparing eq1 and eq2. $h \rho g = \frac{2 T}{R}$ or $T = \frac{h R \rho g}{2}$ -(2)

In Triangle OAB $\cos \theta = \frac{r}{R} \Rightarrow R = \frac{r}{\cos \theta}$ Putting values in eq2

$T = \frac{h r \rho g}{2 \cos \theta}$ When volume of the liquid is taken into account then $T = \frac{(h+r/3) r \rho g}{2 \cos \theta}$

For pure water angle of contact is nearly zero. $\cos 0 = 1$. Then

$$T = \frac{h r \rho g}{2}$$

4. **Torricelli's theorem:- Velocity of Efflux:-Statement-** “The velocity of efflux of a liquid from a hole is equal to the velocity of a body falling freely from height h i.e. $V = \sqrt{2gh}$ ”.

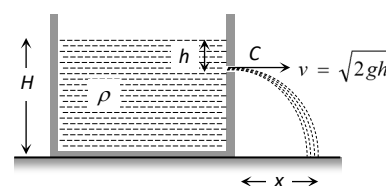
Proof:-

If a liquid is filled in a vessel up to height H and a hole is made at a depth h below the free surface of the liquid as shown in fig. then taking the level of hole as reference level (i.e., zero point of potential energy) and applying Bernoulli's principle to the liquid just inside and outside the hole (assuming the liquid to be at rest inside) we get

$$\therefore (P_0 + h\rho g) + 0 = P_0 + \frac{1}{2}\rho v^2 \text{ or } v = \sqrt{2gh}$$

Since the velocity that an object would acquire in falling from rest through a distance h is v then,

$$v^2 = u^2 + 2as \text{ and } v^2 = 0^2 + 2gh \text{ or } v = \sqrt{2gh}$$



Hence velocity of efflux is same as the velocity that an object would acquire in falling from rest through a distance h

** Velocity with which liquid flowing out from a hole or orifice is called velocity of efflux or velocity of flow.

This result was first given by Torricelli, so this is known as Torricelli's theorem.

- (i) The velocity of efflux is independent of the nature of liquid, quantity of liquid in the vessel and the area of orifice.

- (ii) Time taken by the liquid in falling from hole to ground level $t = \sqrt{\frac{2(H-h)}{g}}$

(iii) The horizontal range covered by the liquid $x = \text{horizontal velocity } (V) \times \text{time } (t)$

$$= \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}$$

(iv) It become maximum when $h=H/2$

5. (a): Surface tension is the surface energy per unit area, representing work done to increase surface area by a unit amount.

(b): $2\pi r T \cos\theta = \pi r^2 h \rho g \Rightarrow h = (2T \cos\theta) / (r \rho g)$.

(c): $h = (2 \times 0.073) / (0.5 \times 10^{-3} \times 1000 \times 9.8) \approx 0.0298 \text{ m}$ or 2.98 cm.

(d): The liquid will rise to the top of the tube, but it will not overflow. The meniscus radius increases to maintain equilibrium.

CHAPTER - 9

Fluid Mechanics: The Essential Formula Guide

A comprehensive visual cheat sheet for the fundamental mathematical laws governing fluid statics and dynamics, including pressure, velocity, and surface phenomena.

FLUID STATICS & DYNAMICS

BERNOULLI'S THEOREM

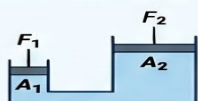
$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$



Represents the conservation of energy for a flowing fluid.

PASCAL'S LAW & HYDRAULIC LIFT

$$\frac{F_2}{F_1} = \frac{A_2}{A_1}$$



Pressure applied to enclosed fluids is transmitted equally in all directions.

THE EQUATION OF CONTINUITY

$$Av = \text{constant}$$



Ensures mass conservation by relating fluid velocity to the cross-sectional area.

EXCESS PRESSURE (ΔP)

Surface Type	Excess Pressure (ΔP)
Liquid Drop	$\Delta P = \frac{2T}{r}$
Soap Bubble	$\Delta P = \frac{4T}{r}$
Air Bubble in Liquid	$\Delta P = \frac{2T}{r}$

VISCOSITY & SURFACE PROPERTIES

TERMINAL VELOCITY

$$v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$$



The constant speed reached by an object falling through a viscous fluid.

STOKES' LAW

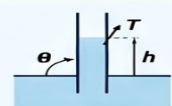
$$F = 6\pi r \eta v$$



Calculates the drag force acting on spheres moving through a viscous fluid.

CAPILLARY RISE

$$h = \frac{2T \cos\theta}{\rho g r}$$



Determines how high a liquid rises or falls in a narrow tube.

SYMBOL KEY

Symbol	Property
ρ (rho)	Fluid Density
η (eta)	Coefficient of Viscosity
T	Surface Tension

CHAPTER-10



THERMAL PROPERTIES OF MATTER



QUICK REVISION CHART

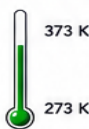
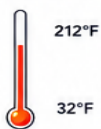
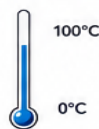
1. HEAT & TEMPERATURE

- Heat is a form of energy that flows from higher temperature to lower temperature.
- Temperature is the degree of hotness or coldness of a body.



2. TEMPERATURE SCALES

Celsius (°C) Fahrenheit (°F) Kelvin (K)



3. CONVERSION FORMULAS

$$F = \frac{9}{5}C + 32$$

$$K = C + 273$$

$C = ^\circ\text{C}$, $F = ^\circ\text{F}$, $K = \text{Kelvin}$

4. HEAT CAPACITY

- Heat Capacity (S):

$$S = \frac{\Delta Q}{\Delta T} \quad (\text{J K}^{-1})$$

- Specific Heat Capacity (s):

$$s = \frac{S}{m} \quad (\text{J kg}^{-1} \text{K}^{-1})$$

- Heat required:

$$\Delta Q = ms\Delta T$$

5. THERMAL EXPANSION OF SOLIDS

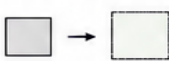
Linear Expansion



$$\frac{\Delta L}{L} = \alpha \Delta T$$

α = Coefficient of linear expansion (K^{-1})

Area Expansion



$$\frac{\Delta A}{A} = \beta \Delta T$$

β = Coefficient of area expansion (K^{-1})

Volume Expansion



$$\frac{\Delta V}{V} = \gamma \Delta T$$

γ = Coefficient of volume expansion (K^{-1})

Relations

$$\beta = 2\alpha$$

$$\gamma = 3\alpha$$

$$\alpha : \beta : \gamma = 1 : 2 : 3$$

Note:

Coefficient of thermal expansion is not strictly constant. It depends on temperature.

6. THERMAL STRESS

When expansion/contraction is prevented, thermal stress develops.

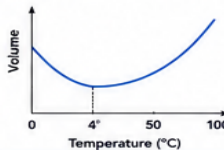
Example: Rod fixed rigidly at both ends $\rightarrow$ develops compressive stress on heating.

Formula:

$$\text{Stress} = Y \alpha \Delta T$$

7. THERMAL PROPERTIES OF MATTER

Water has maximum density at 4°C . It expands when cooled below 4°C .



8. EXPANSION OF GASES

Gases expand more than liquids and solids when heated.



γ (gases) is highly temperature dependent.

9. WATER AS A COOLING AGENT

High specific heat capacity $\rightarrow$ absorbs large heat with small rise in temperature.

Used in radiators of automobiles, engines, cooling systems.



10. CHANGE OF STATE



Melting (Fusion)

Solid $\rightarrow$ Liquid at constant temperature.



Freezing

Liquid $\rightarrow$ Solid at constant temperature.



Vaporization

Liquid $\rightarrow$ Vapour (Evaporation / Boiling).



Boiling Point

Temperature at which liquid changes completely into vapour. Depends on pressure.



Sublimation

Solid $\rightarrow$ Gas without passing through liquid state.



Regelation

Ice melts under pressure, re-freezes when pressure is removed.



Temperature & pressure where solid, liquid & gas coexist in equilibrium. (Water: 273.16 K , $6.11 \times 10^3 \text{ Pa}$)

11. LATENT HEAT

Heat absorbed or released during change of state without change in temperature.

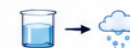
- Latent Heat of Fusion (L_f):

$$Q = mL_f \quad (\text{Solid} \rightleftharpoons \text{Liquid})$$



- Latent Heat of Vaporization (L_v):

$$Q = mL_v \quad (\text{Liquid} \rightleftharpoons \text{Gas})$$



Unit: J kg^{-1}

Where, m = mass of substance

12. HEAT TRANSFER

Conduction

Heat transfer through a stationary medium.

$$H = \frac{KA\Delta T}{L}$$



Heat transfer through bulk motion of fluid (liquid or gas).



Heat transfer through electromagnetic waves. No medium required.

13. RADIATION LAWS

- Wien's Displacement Law

$$\lambda_m T = \text{constant}$$

- Stefan-Boltzmann Law

$$H = A\sigma T^4$$

Where, H = total energy radiated per second (W), A = area (m^2)

IMPORTANT POINTS

- Heat always flows hot $\rightarrow$ cold.
- Water is densest at 4°C .
- Latent heat: changes state, not temperature.
- At boiling point, vapour pressure = atmospheric pressure.
- Good absorbers are good emitters of radiation.

EXAM TIPS



- Write formulas first.
- Use correct units.
- Draw diagrams where useful.
- Read question carefully.
- Check assumptions.
- Practice numericals.

COMMONLY CONFUSED

- Heat vs Temperature
Heat = energy in transit, Temperature = measure.
- Heat Capacity vs Specific Heat
Heat Capacity depends on mass, Specific Heat is mass independent.
- Boiling vs Evaporation
Boiling occurs at fixed temperature, Evaporation at any temperature.

ONE-LINE CONCEPTS

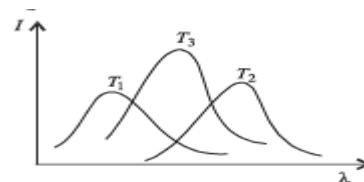
- Heat flows from high to low temperature.
- Thermal expansion increases dimensions of solids.
- Water expands below 4°C .
- Latent heat is hidden heat of change.
- Conduction needs medium, radiation needs none.
- Stefan-Boltzmann: hotter $\rightarrow$ more radiation (T^4).

MULTIPLE CHOICE QUESTIONS

1. The coefficient of linear expansion of a rod is α . The percentage increase in its length when heated through temperature ΔT is: **(NEET 2023)**
 (a) $\alpha\Delta T$ (b) $100\alpha\Delta T$ (c) $\frac{\alpha\Delta T}{100}$ (d) $10\alpha\Delta T$
2. A black body at temperature T radiates energy at maximum wavelength λ . If temperature becomes $2T$, the wavelength of maximum intensity becomes: **(NEET 2022)**
 (a) 2λ (b) λ (c) $\frac{\lambda}{2}$ (d) $\frac{\lambda}{4}$
3. The SI unit of thermal conductivity is: **(NEET 2021)**
 (a) $W m^{-1}K^{-1}$ (b) $J kg^{-1}K^{-1}$ (c) $W m^{-2}K^{-4}$ (d) $J m^{-1}$
4. Water has maximum density at: **(NEET 2020)**
 (a) $0^\circ C$ (b) $4^\circ C$ (c) $100^\circ C$ (d) $-4^\circ C$
5. A leak-proof cylinder floats vertically in water at $0^\circ C$. When water temperature becomes $4^\circ C$, the cylinder rises slightly because: **(JEE Main 2020)**
 (a) Water contracts (b) Water expands
 (c) Density of water increases (d) Density of water decreases
6. A rod is heated and simultaneously compressed so that its length remains unchanged. The coefficient of volume expansion is related to linear expansion coefficient by: **(JEE Main 2019)**
 (a) $\gamma = \alpha$ (b) $\gamma = 2\alpha$ (c) $\gamma = 3\alpha$ (d) $\gamma = 4\alpha$
7. The dimensional formula of Stefan-Boltzmann constant is:
 (a) $[MLT^{-3}K^{-4}]$ (b) $[MT^{-3}K^{-4}]$ (c) $[ML^2T^{-2}]$ (d) $[ML^1T^{-2}]$
8. The heat required to raise the temperature of a body depends upon:
 (a) Mass only (b) Nature of material only
 (c) Mass and specific heat capacity (d) Volume only
9. In solids, heat is mainly transferred by:
 (a) Convection (b) Radiation (c) Conduction (d) Expansion
10. If the temperature of a black body is doubled, the energy radiated per unit time becomes:
 (a) 2 times (b) 4 times (c) 8 times (d) 16 times
11. Two rods A and B of different materials are welded together as shown in figure. Their thermal conductivities are K_1 and K_2 . The thermal conductivity of the composite rod will be 
 (a) $\frac{(2K_1K_2)}{(K_1+K_2)}$ (b) $\frac{3(K_1+K_2)}{2}$ (c) $(K_1 + K_2)/2$ (d) $2(K_1 + K_2)$
12. The coefficient of linear expansion of a crystalline substance in one direction is $2 \times 10^{-4}/0_C$. The coefficient of cubical expansion of crystal is equal to
 (a) $2 \times 10^{-4}/0_C$ (b) $4 \times 10^{-4}/0_C$ (c) $6 \times 10^{-4}/0_C$ (d) $1 \times 10^{-4}/0_C$
13. A cup of hot coffee is left on table after some time coffee cools down. What is the mode of transfer of heat for this cooling?
 (a) conduction (b) convection (c) radiation (d) all of these
14. A block of ice is heated until it melts completely. What happens to the temperature of ice-water mixture during this process?
 (a) It increases continuously (b) it decreases continuously
 (c) it remains constant (d) it increases and then decreases

15. The plots of intensity of radiation versus wavelength of three black bodies at temperatures T_1 , T_2 and T_3 are shown. Then,

- (a) $T_3 > T_2 > T_1$ (b) $T_1 > T_2 > T_3$
 (c) $T_2 > T_3 > T_1$ (d) $T_1 > T_3 > T_2$



16. A bimetallic strip is made of aluminium and steel. On heating, the strip will

- (a) remain straight (b) will bend with aluminium on concave side
 (c) get twisted (d) will bend with steel on concave side.

For questions 17 to 20 , choose the correct option:

(a) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true, but Reason is NOT the correct explanation of Assertion.

(c) Assertion is true, but Reason is false.

(d) Assertion is false, but Reason is true.

17. **ASSERTION:** Specific heat capacity of a body is always lesser than its heat capacity.
REASON: Heat capacity is heat required to raise temperature of unit mass of substance by unity.
18. **ASSERTION:** Thermal coefficient of expansion for volume is approximately $3/2$ times of superficial coefficient of thermal expansion.
REASON: On heating if change in temperature is same then percentage rise in volume is greater than that in surface area.
19. **ASSERTION:** If two different substances of equal mass at different temperature are mixed with each other equilibrium temperature is always different from their mean temperature.
REASON: Two substances have different specific heat capacity.
20. **ASSERTION:** The melting point of ice decreases with increase of pressure.
REASON: Ice contracts on melting.
21. **ASSERTION:** Woollen clothes keep us warm during winter.
REASON: Wool traps air, which is a poor conductor of heat.
22. **ASSERTION:** Shiny surfaces are used inside thermos flasks.
REASON: Shiny surfaces are good absorbers of heat radiation.
23. **ASSERTION:** A metal spoon kept in hot tea becomes hot after some time.
REASON: Metals transfer heat mainly by conduction.
24. **ASSERTION:** Water is used as a coolant in car radiators.
REASON: Water has high specific heat capacity.
25. **ASSERTION:** A black surface cools faster than a polished surface.
REASON: Black surfaces are good emitters of heat radiation.

ANSWER KEY- MCQ

1	b	2	c	3	a	4	b
5	c	6	c	7	b	8	c
9	c	10	d	11	a	12	c
13	d	14	c	15	d	16	d
17	d	18	b	19	a	20	a
21	a	22	c	23	a	24	a
25	a						

PRACTICE QUESTIONS- MCQ

- Water has maximum density at:
(a) 0°C (b) 4°C (c) 100°C (d) -4°C
- The SI unit of thermal conductivity is:
(a) $\text{Jkg}^{-1}\text{K}^{-1}$ (b) $\text{Wm}^{-1}\text{K}^{-1}$ (c) $\text{Wm}^{-2}\text{K}^{-4}$ (d) JK^{-1}
- During melting of ice, the temperature:
(a) Increases continuously (b) Decreases continuously
(c) Remains constant (d) First increases and then decreases
- Assertion (A): Steam burns are more severe than boiling water burns at the same temperature.
Reason (R): Steam contains latent heat of vaporization.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true
- ASSERTION: During boiling, the temperature of water remains constant even when heat is continuously supplied.
REASON: The supplied heat is used to convert liquid into vapour by overcoming intermolecular forces.

2 MARK QUESTIONS

- "Water is a good coolant because of its high specific heat capacity." Is this statement true or false? Explain
- Calculate the heat energy required to raise temperature of 500g water from 20°C to 80°C . (Specific heat capacity of water = $4.186 \text{ J/g}^{\circ}\text{C}$)
- Determine the coefficient of linear expansion of a metal rod given $\Delta L = 0.1\text{m}$, $L_0 = 1\text{m}$, $\Delta T = 25^{\circ}\text{C}$.
- During winter, a lake freezes at the surface while aquatic life survives below it. Explain why.
- Two cooking pans, one black and one shiny, are kept in sunlight. Which one becomes hotter faster and why?
- A person standing near a fire feels warm even though air is a poor conductor of heat. Explain.
- In the construction of the "Iron Pillar of Delhi" or modern steel bridges, why are rollers used at one end of the iron beams instead of fixing both ends rigidly to the concrete pillars?
- Why are burns caused by steam at 100°C much more severe than burns caused by boiling water at the same temperature?
- A railway blacksmith needs to fit a steel rail of length 20 m at a temperature of 25°C . If the maximum temperature expected in the summer is 45°C , calculate the minimum gap that must be left between two such rails so that they do not touch or buckle.
(Given: Coefficient of linear expansion of steel = $1.2 \times 10^{-5} / \text{K}$)
- A student wants to cool 0.5 kg of water at 30°C by adding ice at 0°C . Calculate the mass of ice required to bring the final temperature of the water down to exactly 0°C .
(Given: Specific heat of water = 4200 J/kgK Latent heat of fusion of ice = $3.36 \times 10^5 \text{ J/kg}$)

SOLUTIONS OF 2 MARK QUESTIONS

- True.
* Water has a high specific heat capacity ($4.186 \text{ J/g}^{\circ}\text{C}$), meaning it can absorb and release large amounts of heat energy without a large change in temperature.
* This makes water an effective coolant, as it can absorb heat from surroundings without rising in temperature quickly.
- Given values: Mass of water (m) = 500g = 0.5 kg, Initial temperature (T_1) = 20°C

Final temperature (T_2) = 80°C , Specific heat capacity (s) = $4.186 \text{ J/g}^\circ\text{C} = 4186 \text{ J/kg}^\circ\text{C}$

Calculate temperature difference (ΔT) = $T_2 - T_1 = 80^\circ\text{C} - 20^\circ\text{C} = 60^\circ\text{C}$

Apply formula $Q = ms\Delta T$ $Q = 0.5 \text{ kg} \times 4186 \text{ J/kg}^\circ\text{C} \times 60^\circ\text{C} = Q = 125580 \text{ J}$

3. Formula: $\alpha = \Delta L / (L_0 \Delta T)$ given values: Change in length (ΔL) = 0.1 m

Original length (L_0) = 1 m , Change in temperature (ΔT) = 25°C

using values into formula:

$$\alpha = 0.1 \text{ m} / (1 \text{ m} \times 25^\circ\text{C}) = \alpha = 0.004 \text{ }^\circ\text{C}^{-1} \text{ or } 4 \times 10^{-3} \text{ }^\circ\text{C}^{-1} = \alpha = 4 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$$

4. Water has maximum density at 4°C . The colder water and ice remain on the top while comparatively warmer water stays below, allowing aquatic life to survive.
5. The black pan becomes hotter faster because black surfaces absorb heat radiation more effectively than shiny surfaces.
6. Heat from the fire reaches the person mainly through radiation, which does not require any medium.
7. Rollers are provided to allow for thermal expansion and contraction. When atmospheric temperature increases, the beam expands in length. If both ends were rigid, the resulting thermal stress could buckle the beam or crack the concrete pillars. The rollers provide the necessary space for this linear expansion.
8. Steam at 100°C carries significantly more energy than water at 100°C . This extra energy is the latent heat of vaporization ($22.6 \times 10^5 \text{ J/kg}$). When steam hits the skin, it first condenses into water, releasing this massive amount of latent heat before it even begins to cool down as a liquid.

9. Identify Given Values:

Initial length (L) = 20 m

Change in temperature (ΔT) = $45^\circ\text{C} - 25^\circ\text{C} = 20^\circ\text{C}$

$$\alpha = 1.2 \times 10^{-5} / \text{K} \quad \text{Formula: } \Delta L = L \alpha \Delta T \quad \therefore \Delta L = 20 \times 1.2 \times 10^{-5} \times 20 = 4.8 \text{ mm}$$

The minimum gap required is 4.8 mm .

10. Mass of water (m) = 0.5 kg

- ΔT for water = $30^\circ\text{C} - 0^\circ\text{C} = 30 \text{ K}$
- Specific heat (s_w) = 4200 J/kgK
- Latent heat of ice (L_f) = $3.36 \times 10^5 \text{ J/kg}$

Principle of Calorimetry:

Heat lost by water = Heat gained by ice to melt $ms\Delta T = m L_f$

$$0.5 \times 4200 \times 30 = m \times 3.36 \times 10^5 \quad m = 0.1875 \text{ kg} \quad \text{The mass of ice required is } 187.5 \text{ g}$$

PRACTICE QUESTIONS- 2 MARKS

1. Why are gaps left between railway tracks? Explain.
2. Calculate the heat required to raise the temperature of 0.5 kg of water from 20°C to 60°C . Given: Specific heat capacity of water $s_w = 4200 \text{ Jkg}^{-1}\text{K}^{-1}$
3. Why is the inside of a refrigerator made shiny?
4. A ball and ring apparatus is heated. The metal ball passes easily through the ring before heating but not after heating. Why?
5. Calculate the amount of heat required to raise the temperature of 2 kg of aluminium from 30°C to 80°C ($s = 900 \text{ J kg}^{-1} \text{ K}^{-1}$)

SOLUTION - NUMERICALS OF PRACTICE QUESTION

2. 84000 J
5. 90000 J

3 MARK QUESTIONS

1. Show that the coefficient of area expansion, $(\Delta A/A)/\Delta T$, of a rectangular sheet of the solid is twice its linear expansivity, α_l .

2. Why is the triple point of water taken as a standard fixed point in modern thermometry? What is wrong in taking the melting point of ice and boiling point of water as fixed points?
3. Explain why:
 - (a) A body with large reflectivity is a poor emitter
 - (b) A brass tumbler feels much colder than a wooden tray on a chilly day
 - (c) The earth without its atmosphere would be inhospitably cold
4. A copper vessel of mass 500 g contains 200 g of water at 20°C . Calculate the heat required to raise the temperature of both to 60°C .
($s_w = 4200 \text{ J kg}^{-1} \text{ K}^{-1}$, $s_{\text{Cu}} = 390 \text{ J kg}^{-1} \text{ K}^{-1}$)
5. A brass rod of length 50 cm and diameter 3 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250°C , if the original lengths are at 40°C ? ($\alpha_{\text{brass}} = 2.0 \times 10^{-5}/^{\circ}\text{C}$, $\alpha_{\text{steel}} = 1.2 \times 10^{-5}/^{\circ}\text{C}$)
6. How much heat is required to convert 10 g of ice at -10°C into steam at 100°C ? (Given: $s_{\text{ice}} = 0.5 \text{ cal/g } ^{\circ}\text{C}$, $L_f = 80 \text{ cal/g}$, $L_v = 540 \text{ cal/g}$)
7. Explain the concept of latent heat of fusion and latent heat of vaporization. Using these concepts, calculate the amount of heat required to convert 1 kg of ice at 0°C to water at 0°C .
8. A calorimeter contains 200 g of water at 20°C . A 50 g metal piece at 100°C is placed in the calorimeter. The final temperature of the water and metal is 23°C . Determine the specific heat capacity of the metal.
9.
 - a) Explain why a steel rod and a copper rod of the same dimensions, when heated to the same temperature, will show different rates of expansion.
 - b) Explain why a glass tumbler cracks when a hot liquid is poured into it, while a metal tumbler doesn't.
10. During a summer vacation, Rohan notices that the concrete segments of the highway have small gaps filled with a flexible tar-like material between them. On a particularly hot afternoon, he observes that these gaps seem much smaller than they were in the early morning.
 - (a) Identify the physical phenomenon responsible for this observation and explain the physics behind it.
 - (b) What would happen to the highway if these gaps were not provided during construction?
 - (c) If a concrete slab is 10m long at 20°C , calculate the expansion in its length when the temperature rises to 45°C . (Assume $\alpha_{\text{concrete}} = 1.2 \times 10^{-5}/^{\circ}\text{C}$)

SOLUTIONS OF 3 MARK QUESTIONS

1. Consider a rectangular sheet of the solid material of length a and breadth b (Fig.).

When the temperature increases by ΔT ,

l increases by $\Delta l = \alpha_l \cdot l \Delta T$ and b increases by $\Delta b = \alpha_b \cdot b \Delta T$.

From Fig., the increase in area

$$\Delta A = \Delta A_1 + \Delta A_2 + \Delta A_3 = l \Delta b + b \Delta l + \Delta l \Delta b = l \alpha_b b \Delta T + b \alpha_l l \Delta T + \alpha_l^2 l b \Delta T^2$$

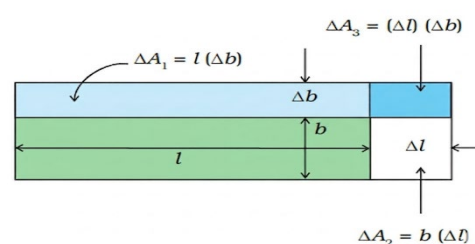
$$= l b \alpha_l \Delta T (2 + \alpha_l \Delta T) = A \alpha_l \Delta T (2 + \alpha_l \Delta T)$$

Since $\alpha_l \approx 10^{-5} \text{ K}^{-1}$, the product $\alpha_l \Delta T$ for fractional temperature is small in comparison with 2 and may be neglected.

$$\therefore \Delta A = A \alpha_l \Delta T \times 2$$

Hence,
$$\frac{\Delta A}{A} \cdot \frac{1}{\Delta T} = 2 \alpha_l$$

2. The triple point of water is used as a standard fixed point because it has a unique and accurately reproducible temperature of 273.16 K, where ice, water and water vapour coexist in equilibrium.



The melting point of ice and boiling point of water are not preferred because they vary with atmospheric pressure and other external conditions. Hence, they are less accurate and less reliable as standard fixed points.

3. (a) A good reflector absorbs very little heat radiation. Since good absorbers are good emitters, a body with large reflectivity becomes a poor emitter of heat radiation.
 (b) Brass is a good conductor of heat and takes heat away from our hand quickly, producing a colder sensation. Wood is a poor conductor, so it does not remove heat rapidly from the hand.
 (c) The atmosphere absorbs and re-radiates heat emitted by the earth, preventing rapid loss of heat into space. Without the atmosphere, most of the earth's heat would escape, making the earth extremely cold.

4. Mass of copper vessel = 0.5 kg , Mass of water = 0.2 kg $\Delta T = 60 - 20 = 40^\circ\text{C}$
 Heat required for water: $Q_1 = 0.2 \times 4200 \times 40 = 33600 \text{ J}$
 Heat required for copper vessel: $Q_2 = 0.5 \times 390 \times 40 = 7800 \text{ J}$
 Total heat required: $Q = Q_1 + Q_2 = 33600 + 7800 = 41400 \text{ J}$
 Total heat required = $4.14 \times 10^4 \text{ J}$

5. Given: $L_1 = L_2 = 50 \text{ cm}$, $\Delta T = 250 - 40 = 210^\circ\text{C}$.
 ○ $\Delta L_{\text{brass}} = L_1 \alpha_{\text{brass}} \Delta T = 50 \times 2.0 \times 10^{-5} \times 210 = 0.21 \text{ cm}$
 ○ $\Delta L_{\text{steel}} = L_1 \alpha_{\text{steel}} \Delta T = 50 \times 1.2 \times 10^{-5} \times 210 = 0.126 \text{ cm}$
 ○ Total Change: $\Delta L = \Delta L_{\text{brass}} + \Delta L_{\text{steel}} = 0.21 + 0.126 = 0.336 \text{ cm}$.

6. Heat to raise ice to 0°C : $Q_1 = ms\Delta T = 10 \times 0.5 \times 10 = 50 \text{ cal}$.
 ➤ Heat to melt ice: $Q_2 = mL_f = 10 \times 80 = 800 \text{ cal}$.
 ➤ Heat to raise water to 100°C : $Q_3 = 10 \times 1 \times 100 = 1000 \text{ cal}$.
 ➤ Heat to convert to steam: $Q_4 = mL_v = 10 \times 540 = 5400 \text{ cal}$.
 ➤ Total Heat: $Q = 50 + 800 + 1000 + 5400 = 7250 \text{ cal}$.

7. Step 1: Explain the concept of latent heat of fusion

Latent heat of fusion is the amount of heat energy required to change the state of a substance from solid to liquid without changing its temperature. For ice, the latent heat of fusion is approximately 334 J/g or $334,000 \text{ J/kg}$.

Step 2: Explain the concept of latent heat of vaporization

Latent heat of vaporization is the amount of heat energy required to change the state of a substance from liquid to gas without changing its temperature. For water, the latent heat of vaporization is approximately 2257 J/g or $2,257,000 \text{ J/kg}$.

Step 3: Calculate the heat required to convert ice to water at 0°C

The heat required to convert 1 kg of ice at 0°C to water at 0°C is given by $Q = m \times L$, where m is the mass and L is the latent heat of fusion. $Q_1 = 1 \text{ kg} \times 334,000 \text{ J/kg} = 334,000 \text{ J}$

8. The heat gained by the water is $Q = m \times c \times \Delta T = 0.2 \text{ kg} \times 4186 \text{ J/kg}^\circ\text{C} \times (23^\circ\text{C} - 20^\circ\text{C}) = 2511.6 \text{ J}$.
 The heat lost by the metal is $Q = m \times c \times \Delta T = 0.05 \text{ kg} \times c \times (100^\circ\text{C} - 23^\circ\text{C}) = 3.85c \text{ J}$.
 Equating the heat gained and lost: $2511.6 \text{ J} = 3.85c \text{ J}$.
 Solving for c : $c = 2511.6 \text{ J} / 3.85 = 652.2 \text{ J/kg}^\circ\text{C}$.

The specific heat capacity of the metal is approximately $652.2 \text{ J/kg}^\circ\text{C}$.

9. a) The steel rod and copper rod will show different rates of expansion because they have different coefficients of thermal expansion. The coefficient of thermal expansion is a material property that determines how much a material expands when heated. Copper has a higher coefficient of thermal expansion than steel, so it will expand more than steel when heated to the same temperature. This means that the copper rod will show a greater rate of expansion than the steel rod.

b) A glass tumbler cracks when a hot liquid is poured into it because glass is a poor conductor of heat and has a relatively high coefficient of thermal expansion. When hot liquid is poured into the glass, the inner surface expands rapidly, while the outer surface remains cooler and doesn't expand as much. This creates stress in the glass, which can cause it to crack.

In contrast, metal is a good conductor of heat and can distribute the heat evenly throughout the tumbler, reducing the stress caused by uneven expansion. As a result, a metal tumbler is less likely to crack when a hot liquid is poured into it.

10. (a) Phenomenon: Thermal Expansion or Linear thermal Expansion.

Explanation: When the temperature increases, the kinetic energy of the molecules in the concrete increases, causing them to vibrate more vigorously and occupy more space. This results in an increase in the length of the concrete slabs.

(b) If gaps were not provided, the expanding concrete slabs would exert immense thermal stress on each other. Since the slabs have no room to expand, this internal pressure would cause the highway to buckle, crack, making it dangerous for vehicles.

(c) Given: Original length (L) = 10 m $\Delta T = 45^\circ\text{C} - 20^\circ\text{C} = 25^\circ\text{C}$

$$\alpha = 1.2 \times 10^{-5}/^\circ\text{C} \quad \Delta L = L \times \alpha \times \Delta T = 10 \times 1.2 \times 10^{-5} \times 25 = 3 \text{ mm}$$

PRACTICE QUESTIONS- 3 MARKS

1. Explain why:
 - (a) A body with large reflectivity is a poor emitter.
 - (b) A brass tumbler feels colder than a wooden tray on a chilly day.
 - (c) The earth without its atmosphere would be extremely cold.
2. A steel rod of length 2 m is heated from 20°C to 120°C . Calculate the increase in length. Given: Coefficient of linear expansion of steel = $1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$.
3. A house owner notices that during summer, rooms with tiled roofs become much hotter in the daytime compared to rooms with roofs painted white. He decides to paint the roof white and also install a false ceiling made of wood below the roof.
Answer the following questions:
 - (a) Why does a white-painted roof keep the room cooler?
 - (b) Why is wood used for making the false ceiling?
 - (c) Name the modes of heat transfer mainly reduced by:
 - (i) White paint
 - (ii) Wooden false ceiling

SOLUTION - NUMERICALS OF PRACTICE QUESTION

2. 2.4 mm

5 MARK QUESTIONS

1. (a) What is thermal expansion? Describe an experiment to demonstrate linear expansion in solids. (1 mark)
 (b) Derive the relationship between change in length (ΔL) of a solid and its coefficient of linear expansion (α), initial length (L_0) and change in temperature (ΔT). (2 marks)
 (c) A metal wire of length 1.5 m expands to 1.502 m when heated from 25°C to 75°C . Calculate its coefficient of linear expansion. (α in $10^{-5}/^\circ\text{C}$) (2 marks)
2. (a) Define thermal conductivity of a material.
 (b) State the factors affecting the rate of conduction of heat through a rod.
 (c) A copper rod of length 2 m and area of cross-section $5 \times 10^{-4} \text{ m}^2$ conducts heat from a hot end at

150°C to a cold end at 50°C. If thermal conductivity of copper is 400 Wm⁻¹K⁻¹, calculate the rate of heat flow through the rod.

3. (a) Define latent heat of fusion and latent heat of vaporization.
 (b) Why does the temperature remain constant during change of state?
 (c) Calculate the heat required to convert 500 g of ice at -20°C into water at 50°C.
 Given: Specific heat capacity of ice = 2100 J kg⁻¹K⁻¹
 Specific heat capacity of water = 4200 J kg⁻¹K⁻¹
 Latent heat of fusion of ice = 3.36 × 10⁵ J kg⁻¹

SOLUTIONS OF 5 MARK QUESTIONS

1. (a) Thermal expansion is the change in dimensions of a substance due to change in temperature. An experiment to demonstrate linear expansion is by heating a metal rod clamped at one end and observing the expansion using a scale.
 (b) Let initial length = L₀, final length = L₀ + ΔL, initial temperature = T₁, final temperature = T₂
 $\Delta T = T_2 - T_1$ $\Delta L = L_0 \times \alpha \Delta T$ or $\alpha = \Delta L / (L_0 \Delta T)$
 (c) Given: ΔL = 1.502 m - 1.5 m = 0.002 m, L₀ = 1.5 m, ΔT = 75°C - 25°C = 50°C
 $\alpha = \Delta L / (L_0 \times \Delta T) = 0.002 / (1.5 \times 50) = 2.67 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$
2. (a) Thermal conductivity is the quantity of heat conducted per second through unit area of a material of unit thickness when the temperature difference between its faces is 1 K.
 (b) Rate of conduction depends on:
 ➤ Area of cross-section
 ➤ Temperature difference
 ➤ Length of conductor
 ➤ Nature of material
 (c) Given: L = 2 m A = 5 × 10⁻⁴ m² ΔT = 150 - 50 = 100°C K = 400 Wm⁻¹K⁻¹
 Using, $\frac{Q}{t} = \frac{KA\Delta T}{L}$ $\frac{Q}{t} = \frac{400 \times 5 \times 10^{-4} \times 100}{2}$ $\frac{Q}{t} = 10 \text{ W}$ Rate of heat flow = 10 W
3. (a) Latent heat of fusion: The amount of heat required to convert unit mass of a solid into liquid at its melting point without change in temperature.
 Latent heat of vaporization: The amount of heat required to convert unit mass of a liquid into vapour at its boiling point without change in temperature.
 (b) During change of state, the supplied heat is used to overcome intermolecular forces between particles rather than increasing kinetic energy. Hence the temperature remains constant.
 (c) Given: m = 500 g
 Step 1: Heat required to raise temperature of ice from -20°C to 0°C
 Using, $Q = mc\Delta T$ $Q_1 = 0.5 \times 2100 \times 20$ $Q_1 = 21000 \text{ J}$
 Step 2: Heat required to melt ice at 0°C
 Using, $Q = mL_f$ $Q_2 = 0.5 \times 3.36 \times 10^5$ $Q_2 = 1.68 \times 10^5 \text{ J}$
 Step 3: Heat required to raise water temperature from 0°C to 50°C
 Using, $Q = mc\Delta T$ $Q_3 = 0.5 \times 4200 \times 50$ $Q_3 = 105000 \text{ J}$
 Total Heat Required
 $Q = Q_1 + Q_2 + Q_3$ $Q = 21000 + 168000 + 105000 = 294000 \text{ J} = 2.94 \times 10^5 \text{ J}$

PRACTICE QUESTIONS- 5 MARKS

1. (a) Define latent heat of fusion and latent heat of vaporization.
 (b) Why does temperature remain constant during change of state?
 (c) Calculate the heat required to convert 500 gm of ice at -20°C into water at 50°C.
 Given: Specific heat capacity of ice = 2100 J kg⁻¹K⁻¹
 Specific heat capacity of water = 4200 J kg⁻¹K⁻¹
 Latent heat of fusion of ice = 3.36 × 10⁵ J kg⁻¹

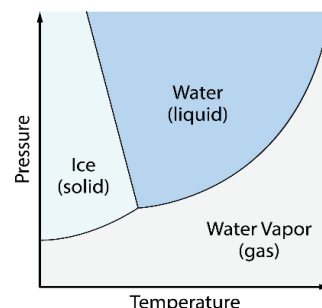
2. (a) State Stefan's law of black body radiation. Explain why a hot object cools faster in the beginning and slower after some time.
 (b) A black body radiates energy at the rate of 800 W at a certain temperature T . If its absolute temperature is doubled, calculate the new rate of energy radiation.

SOLUTION - NUMERICALS OF PRACTICE QUESTION

- 1.(c) 294000 J 2.(c) 12800 J

CASE STUDY QUESTIONS

1. Standards in Thermometry and Anomalous Behavior
 In modern thermometry, the triple point of water is the sole standard fixed point because it occurs at a unique, reproducible temperature and pressure (273.16K at 611.66 Pa). Unlike the traditional ice and steam points used in the Celsius scale, the triple point is independent of external atmospheric fluctuations. This precision is vital for defining the Kelvin scale. Additionally, water exhibits anomalous expansion, where it reaches its maximum density at 4°C. This property is crucial for the survival of aquatic life in cold regions, as the denser 4°C water remains at the bottom of lakes while ice forms on the surface.



Questions:

- 1.1. According to the phase diagram and the text, why is the triple point of water chosen as the standard fixed point?
 (a) It is the temperature at which water boils 1 atm.
 (b) It is a unique state where all three phases (solid, liquid, gas) coexist in equilibrium.
 (c) It is exactly 0°C regardless of pressure.
 (d) It is easier to achieve than the melting point of ice.
- 1.2. At what temperature does water reach its maximum density, ensuring the survival of aquatic life as described?
 (a) 0°C (b) 100°C (c) 4°C (d) 273.16 K
- 1.3. If the temperature of a gas is 300 K, what is its equivalent temperature on the Celsius scale?
 (a) 26.85°C (b) 300°C (c) 573.15°C (d) 27.16°C
- 1.4.A Based on the shift to modern standards, how does the "ice point" change if the atmospheric pressure is significantly increased?
 (a) It increases. (b) It decreases.
 (c) It remains exactly at 273.15 K (d) It becomes equal to the triple point.

OR

- 1.4.B The triple point of water corresponds to a pressure of 611.66 Pa. If the pressure is increased significantly above this value while keeping the temperature at 273.16 K, what is the most likely phase of the water?
 (a) Sublimates directly into water vapor.
 (b) Remains as a mixture of all three phases.
 (c) Transitions completely into the liquid phase.
 (d) Converts into a supercritical fluid.
2. Cooling by Ice and Steam Burns
 Riya went for a picnic with her friends. She observed that ice keeps soft drinks cold for a longer time than chilled water. Later at home, while helping in the kitchen, she learned that burns caused by steam are more severe than burns caused by boiling water at the same temperature. Her physics teacher explained that both situations are related to the concept of latent heat.
- Ice absorbs extra heat while melting and keeps the drink cool for a longer time.
 - Steam releases additional latent heat during condensation, causing severe burns.

- 2.1. Ice cools drinks more effectively because ice:
- (a) Has lower density (b) Absorbs latent heat while melting
(c) Reflects heat (d) Has lower temperature only
- 2.2. The heat required to change unit mass of a substance from solid to liquid without change in temperature is called:
- (a) Specific heat capacity (b) Thermal conductivity
(c) Latent heat of fusion (d) Latent heat of vaporization
- 2.3. Calculate the heat required to melt 100 g of ice at 0°C . Given: $L_f = 3.36 \times 10^5 \text{ J/kg}$
- (a) $1.68 \times 10^4 \text{ J}$ (b) $2.36 \times 10^4 \text{ J}$ (c) $3.36 \times 10^4 \text{ J}$ (d) $4.36 \times 10^4 \text{ J}$
- 2.4. Assertion (A): Steam burns are more severe than boiling water burns at the same temperature.
Reason (R): Steam contains latent heat of vaporization.
- (a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true
- OR
- 2.4. B. During melting of ice, the temperature remains constant because the supplied heat is used to:
- (a) Increase molecular speed (b) Increase density
(c) Overcome intermolecular forces (d) Decrease pressure

SOLUTION OF CASE STUDY QUESTIONS

- 1.1: (b) 1.2: (c) 1.3: (a) 1.4.A: (b) 1.4.B : (c)
2.1: (b) 2.2: (c) 2.3: (c) 2.4.A: (a) 2.4.B : (c)

PRACTICE QUESTIONS- CASE STUDY

1. Riya observed that ice keeps soft drinks cold for a longer time than chilled water. Later, while helping in the kitchen, she learned that steam burns are more severe than burns caused by boiling water at the same temperature. Her teacher explained that these effects are related to latent heat.



Answer the following questions:

- (1) Ice cools drinks more effectively because it:
- (a) Reflects heat (b) Absorbs latent heat while melting
(c) Has lower density (d) Has lower temperature only
- (2) The heat required to change unit mass of solid into liquid without temperature change is called:
- (a) Specific heat capacity (b) Latent heat of vaporization
(c) Thermal conductivity (d) Latent heat of fusion
- (3) Calculate the heat required to melt 100 gm of ice at 0°C . Given: $L_f = 3.36 \times 10^5 \text{ J/kg}$
- (a) $1.68 \times 10^4 \text{ J}$ (b) $2.36 \times 10^4 \text{ J}$ (c) $3.36 \times 10^4 \text{ J}$ (d) $4.36 \times 10^4 \text{ J}$
- (4) During melting of ice, the supplied heat is used to:
- (a) Increase molecular speed (b) Increase temperature
(c) Overcome intermolecular forces (d) Decrease density



ANSWER1: (b) 1.2: (d) 1.3: (c) 1.4. (c)



THERMAL PROPERTIES OF MATTER



TEMPERATURE CONVERSION



$$F = \frac{9}{5} C + 32$$

(F in °F, C in °C)

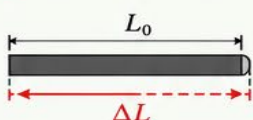


$$K = C + 273$$

(K in Kelvin, C in °C)

THERMAL EXPANSION

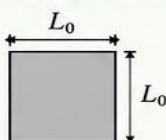
1 LINEAR EXPANSION



$$\Delta L = L_0 \alpha \Delta T$$

L_0 = Original length
 ΔL = Change in length
 α = Coefficient of linear expansion
 ΔT = Change in temperature

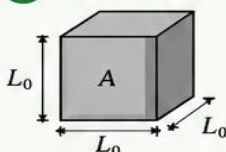
2 AREA EXPANSION



$$\Delta A = A_0 \beta \Delta T$$

A_0 = Original area
 ΔA = Change in area
 β = Coefficient of area expansion
 ΔT = Change in temperature

3 VOLUME EXPANSION



$$\Delta V = V_0 \gamma \Delta T$$

V_0 = Original volume
 ΔV = Change in volume
 γ = Coefficient of volume expansion
 ΔT = Change in temperature

RELATIONSHIP BETWEEN COEFFICIENTS

$$\beta = 2\alpha, \quad \gamma = 3\alpha$$

$$\alpha : \beta : \gamma = 1 : 2 : 3$$

THERMAL STRESS



$$\text{Stress} = Y \alpha \Delta T$$

Y = Young's modulus (modulus of elasticity)
 α = Coefficient of linear expansion
 ΔT = Change in temperature

HEAT CAPACITY & SPECIFIC HEAT



HEAT CAPACITY (S)

ΔQ = Heat added
 ΔT = Change in temperature

$$S = \frac{\Delta Q}{\Delta T}$$



SPECIFIC HEAT CAPACITY (s)

S = Heat capacity
 m = Mass of the substance

$$s = \frac{S}{m}$$



HEAT REQUIRED TO RAISE TEMPERATURE

Quantity of heat required to increase the temperature of m kg substance by ΔT K.

$$\Delta Q = ms\Delta T$$

CHANGE OF STATE



FUSION (SOLID → LIQUID)

Quantity of heat required to convert a solid into a liquid is given by:

$$Q = mL_f$$

L_f = Latent heat of fusion



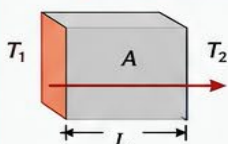
VAPORISATION (LIQUID → VAPOUR)

Quantity of heat required to convert a liquid into vapour is given by:

$$Q = mL_v$$

L_v = Latent heat of vaporisation

HEAT TRANSFER — CONDUCTION



The time rate of heat flow in a material for a given temperature difference is

$$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{L}$$

k = Thermal conductivity
 A = Area of cross-section
 $T_1 - T_2$ = Temperature difference
 Q/t = Rate of heat flow

T = Thickness of separator

THERMAL RADIATION



WIEN'S DISPLACEMENT LAW

$$\lambda_m T = \text{constant}$$

λ_m = Wavelength at maximum intensity
 T = Absolute temperature



STEFAN-BOLTZMANN LAW

$$H = A\sigma T^4$$

H = Total radiant heat energy emitted per second
 A = Area of the surface
 σ = Stefan-Boltzmann constant ($5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$)
 T = Absolute temperature (in K)



These formulas describe how matter responds to temperature changes and how heat is stored, transferred and radiated.

RESOURCE LINKS		
MATERIAL	LINK	Q-R CODE
VOICE NOTE OF THERMAL PROPERTIES OF MATTER	https://notebooklm.google.com/notebook/59b44100-00d4-48c1-a081-1e7259e1d982/artifact/67bd5a43-a14b-4633-9439-cdc73036e99d?utm_source=nlm_web_share&utm_medium=google_oo&utm_campaign=art_share_2&utm_content=&utm_smc=nlm_web_share_google_oo_art_share_2	
INTERACTIVE WORKSHEET	https://docs.google.com/forms/d/e/1FAIpQLSdkOBaQ1q8BrFILhgj2YeutNwu68bGkwbdHuXuo6u12lgDAjg/viewform?usp=sharing&ouid=106416019160863191832	
VIDEO BRIEFING OF THERMAL PROPERTIES OF MATTER	https://notebooklm.google.com/notebook/59b44100-00d4-48c1-a081-1e7259e1d982/artifact/6ba213fa-3df6-46ca-9c97-c4803ed6dcc1?utm_source=nlm_web_share&utm_medium=google_oo&utm_campaign=art_share_2&utm_content=&utm_smc=nlm_web_share_google_oo_art_share_2	

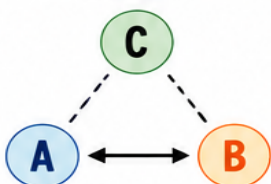
THERMODYNAMICS

ENERGY. TRANSFORMATION. DIRECTION.

1 THE ZEROETH LAW & THERMAL EQUILIBRIUM

The Law:

If two systems (A and B) are in thermal equilibrium with a third system (C), then A and B are in thermal equilibrium with each other.



Significance:

This law justifies the use of thermometers and provides a formal definition of temperature.

2 INTERNAL ENERGY, HEAT, AND WORK

Internal Energy (U):



The sum of kinetic and potential energies of the molecules. It is a state function (depends only on the current state, not how you got there).

Heat (Q):



Energy transfer due to temperature difference.

Work (W):



Energy transfer due to displacement (usually a piston moving in a cylinder).

$$W = P \Delta V$$

3 FIRST LAW OF THERMODYNAMICS (FLOT)

This is essentially the Law of Conservation of Energy applied to thermal systems.

$$Q = \Delta U + \Delta W$$

Interpretation:

The heat energy supplied to a system is used in two ways:

ΔU changing the internal energy of the system

W doing external work.



Energy is conserved, but it can change forms!

4 THERMODYNAMIC PROCESSES

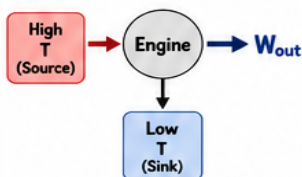
Understanding how a gas changes from one state to another is the core of the chapter:

Process	Condition	Key Feature
 Isothermal	Constant Temperature ($\Delta T = 0$)	Happens slowly ($\Delta U = 0$)
 Adiabatic	No heat exchange ($\Delta Q = 0$)	Happens suddenly. $PV^\gamma = \text{Constant}$
 Isobaric	Constant Pressure ($\Delta P = 0$)	Work done is maximum for a given volume change
 Isochoric	Constant Volume ($\Delta V = 0$)	No work is done ($W = 0$) All heat goes to ΔU

5 HEAT ENGINES AND REFRIGERATORS

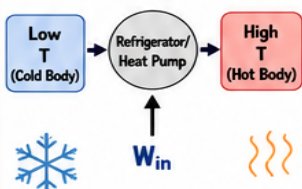
Heat Engine:

Converts heat into work. It takes heat from a source (high T), does work, and rejects the rest to a sink (low T).



Refrigerators/Heat Pumps:

The reverse of an engine. It uses external work to move heat from a cold body to a hot body.



6 SECOND LAW OF THERMODYNAMICS

This law sets the direction of physical processes and proves that 100% efficiency is impossible.

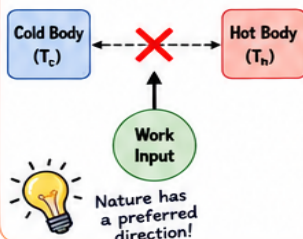
Kelvin-Planck Statement:

No engine can convert all heat from a reservoir into work without some rejection.



Clausius Statement:

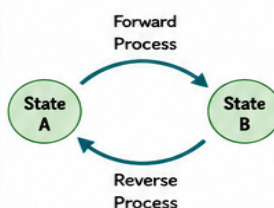
Heat cannot flow spontaneously from a cold body to a hot body without external work.



7 REVERSIBLE AND IRREVERSIBLE PROCESSES

Reversible:

A process that can be retraced such that the system and surroundings return to their original states (idealized).



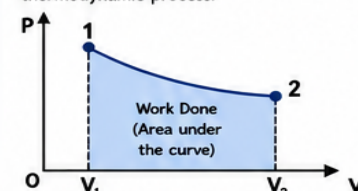
Irreversible:

All natural processes are irreversible due to friction and turbulence.



8 PV INDICATOR DIAGRAM

A pressure-volume (PV) diagram, also known as an indicator diagram, is a graph that visually represents changes in pressure and volume during a thermodynamic process.



Key Features and Applications:

- Work Done:** The area under the curve in a PV diagram represents the work done during a thermodynamic process. 
- Efficiency:** PV diagrams can be used to analyze the efficiency of engines and other thermodynamic systems. 
- Understanding Thermodynamic Processes:** They provide a visual representation of how pressure and volume change during various processes like expansion, compression, and heat transfer. 
- Visualizing Changes:** The slope of the line on a PV diagram can indicate the nature of the process. For example, a steeper slope indicates a larger change in pressure for a given change in volume. 

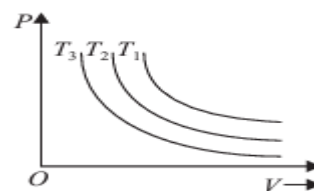
CORE IDEA

Thermodynamics is about **energy**, its **transformations**, and the **direction** of natural processes.



MULTIPLE CHOICE QUESTIONS

- In an isothermal expansion of an ideal gas, which of the following statements is correct regarding the internal energy (U)?
 - Internal energy increases
 - Internal energy remains constant
 - Internal energy decreases
 - Internal energy first increases then decreases
- Which of the following is an intensive property?
 - Enthalpy
 - Volume
 - Molar heat capacity
 - Internal energy
- For an ideal gas, the relationship between molar heat capacity at constant pressure (C_P) and constant volume (C_V) is given by:
 - $C_P + C_V = R$
 - $C_V - C_P = R$
 - $C_P - C_V = R$
 - $\frac{C_P}{C_V} = R$
- For an adiabatic process, which of the following is true?
 - $\Delta Q = 0$
 - $\Delta P = 0$
 - $\Delta T = 0$
 - $\Delta W = 0$
- First law of thermodynamics is based on?
 - Conservation of energy
 - Conservation of momentum
 - Conservation of mass
 - Conservation of work
- Internal energy of a system is defined as?
 - The sum of kinetic energies of all molecules of the system
 - The sum of kinetic and potential energies of all molecules of the system
 - The sum of potential energies of the system
 - The average kinetic energy of all molecules
- The isothermal diagram of a gas at three different temperatures T_1 , T_2 and T_3 , is shown in the given figure. Then:
 - $T_1 < T_2 < T_3$
 - $T_1 > T_2 > T_3$
 - $T_1 < T_2 > T_3$
 - $T_1 > T_2 < T_3$
- The relation between the slope of isothermal curve and slope of adiabatic curve
 - slope of adiabatic curve = γ times slope of isothermal curve
 - slope of isothermal curve = γ times slope of adiabatic curve
 - slope of adiabatic curve = $\frac{1}{\gamma}$ times slope of isothermal curve
 - slope of isothermal curve = $\frac{1}{\gamma}$ times slope of adiabatic curve
- In an adiabatic change the specific heat of a gas is:
 - increase with increase in temperature
 - decrease with increase in temperature
 - not depend upon change in temperature



- d. Always zero.
10. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for gas is:
- 4/3
 - 5/3
 - 2
 - 3/2
11. Which of the following is not a thermodynamic coordinate?
- Gas constant (R)
 - Volume (V)
 - Pressure (P)
 - Temperature (T)
12. Consider the process on a system shown in figure. During the process, the work done by the system
- continuously increases
 - first increases then decreases
 - continuously decreases
 - first decreases then increases
13. A sample of ideal gas ($\gamma = 1.4$) is heated at constant pressure. If 100 J of heat is supplied to the gas the work done by the gas is:
- 28.57 J
 - 38.92 J
 - 56.54 J
 - 65.38 J
14. An ideal gas having molar specific heat capacity at constant volume is $\frac{3}{2}R$, the molar specific heat capacities at constant pressure is:
- $\frac{1}{2}R$
 - $\frac{7}{2}R$
 - $\frac{5}{2}R$
 - $\frac{9}{2}R$
15. Which of the following laws of thermodynamics introduces the concept of temperature?
- First Law of Thermodynamics
 - Zeroth Law of Thermodynamics
 - Second Law of Thermodynamics
 - Third Law of Thermodynamics
16. In an adiabatic process, the relation between pressure P and volume V is given by $PV^\gamma = \text{constant}$. What does γ represent?
- The ratio of specific heat at constant pressure to specific heat at constant volume (C_P/C_V)
 - The product of specific heats ($C_P \times C_V$)
 - The difference between C_P and C_V
 - The sum of the specific heats C_P and C_V
17. Which thermodynamic variable is defined by the Second Law of Thermodynamics?
- Enthalpy
 - Entropy
 - Internal Energy
 - Temperature
18. A Carnot engine works between a source at 127°C and a sink at 27°C . What is its efficiency?
- 25 %
 - 75 %
 - 50 %
 - 100%
19. In an isochoric process, the work done by the system is:
- Positive
 - Zero
 - Negative
 - Infinite
20. For a cyclic process, which of the following is always true?
- $W = 0$
 - $Q = 0$
 - $\Delta U = 0$
 - $\Delta V = 0$



SOLUTIONS:

1. Answer: c
Explanation: Since T is constant and U of an ideal gas depends only on T
2. Answer: c
Explanation: Intensive properties do not depend on the quantity of matter
3. Answer: c
Explanation: This is known as Mayer's formula
4. Answer: a
Explanation: No heat enters or leaves the system
5. Answer : a
Explanation: The first law of thermodynamics is based on the conservation of energy. It deals with work done and heat energy supplied or removed from a system. It basically says that energy supplied to a system is conserved
6. Answer : b
Explanation: The internal energy of a system corresponds to the energy possessed by all molecules. Thus it is the sum of kinetic and potential energies of all molecules in the system considered.
7. Answer: a
Explanation: The given diagram shows that the curves move away from the origin at higher temperature
8. Answer: a
Explanation: For isothermal process, $PV = \text{constant}$, Differentiating both side $PdV + VdP = 0$
or $dP/dV = -P/V$... (i)
Again for adiabatic process $PV^\gamma = \text{constant}$
Again differentiating both side $dP/dV = -\gamma P/V$
slope of adiabatic curve $= \gamma \times \text{slope of isothermal curve}$
9. Answer: d
Explanation: In adiabatic process $Q = \text{constant}$, then $\Delta Q = 0$ and specific heat $s=0$
10. Answer: d)
Explanation: As $P \propto T^3$
or $P/T^3 = \text{constant} \Rightarrow PT^{-3} = \text{constant} \dots\dots\dots(1)$
For an adiabatic change, $PV^\gamma = \text{constant}$
For an ideal gas $PV = nRT$
Or, $V = nRT/P$ Thus, $P(nRT/P)^\gamma = \text{constant}$
Or, $PT^{\gamma/(1-\gamma)} = \text{constant} \dots\dots\dots(2)$
Comparing $\gamma/(1-\gamma) = -3 \Rightarrow \gamma = 3/2$
11. Answer: a
12. Answer: a
13. Answer: a
14. Answer: c
15. Answer: b
16. Answer: a
17. Answer: b
18. Answer: a
19. Answer: b
20. Answer: c

ASSERTION AND REASONING TYPE QUESTIONS

1. **Assertion:** The specific heat of a gas in an adiabatic process is zero.
Reason: In an adiabatic process, there is no exchange of heat between the system and the surroundings ($dq = 0$).
2. **Assertion:** Internal energy of an ideal gas depends only on its temperature.
Reason: In an ideal gas, there are no intermolecular forces of attraction, so the potential energy is zero.
3. **Assertion:** It is impossible to convert all the heat absorbed from a hot reservoir into work.
Reason: The Second Law of Thermodynamics states that heat cannot flow from a colder body to a hotter body without external work.
4. **Assertion:** Air becomes cooler when it expands suddenly from a punctured tyre.
Reason: Sudden expansion is an adiabatic process where the work is done at the expense of internal energy.
5. **Assertion:** An isothermal process must be carried out very slowly.
Reason: For the temperature to remain constant there must be sufficient time for heat exchange between the system and surroundings to maintain thermal equilibrium.
6. **Assertion:** In isothermal process whole of the heat energy supplied to the body is converted into internal energy.
Reason: According to the first law of thermodynamics $\Delta Q = \Delta U$.

SOLUTIONS

1. Answer: a (Both Assertion and Reason are true and Reason is the correct explanation of Assertion)
2. Answer: a (Both Assertion and Reason are true and Reason is the correct explanation of Assertion)
3. Answer: (b) [Both are true statements of the 2nd Law, but the Reason (Clausius statement) does not directly explain the Assertion (Kelvin-Planck statement).]
4. Answer: a (Both Assertion and Reason are true and Reason is the correct explanation of Assertion)
5. Answer: a (Both Assertion and Reason are true and Reason is the correct explanation of Assertion)
6. Answer d (Both A and R is false. Explanation: if $T = \text{constant}$ change in internal energy does not take place and $\Delta Q = \Delta U + W$)

CASE STUDY BASED QUESTIONS

THE PHYSICS OF PHASE CHANGE: A student performs an experiment by taking 1 kg of ice at -10°C and heating it at a constant rate until it becomes steam at 110°C . The student records the temperature at regular intervals and plots a Heating Curve. During the process, the student notices two distinct regions where the temperature remains constant despite a continuous supply of heat. They also note that the volume of the substance changes significantly during the transition from liquid water to steam, performing work against the constant atmospheric pressure.

1. During the transition from ice at 0°C to water at 0°C , the heat supplied is used to:
 - a. Increase the average kinetic energy of the molecules
 - b. Increase the potential energy by breaking intermolecular bonds
 - c. Decrease the temperature of the surroundings
 - d. Increase both kinetic and potential energy equally
2. In the region of the heating curve where water is boiling into steam (100°C), the specific heat of the water is mathematically:
 - a. Infinite
 - b. Zero
 - c. $4186 \text{ J Kg}^{-1}\text{K}^{-1}$
 - d. Negative

3. When 1 gram of water at 100°C turns into 1 gram of steam at 100°C, the internal energy of the system increases. This is because:
 - a. The temperature has increased.
 - b. The molecules are moving slower in steam.
 - c. Heat is absorbed and the volume expands, doing work against atmosphere.
 - d. Potential energy remains constant but kinetic energy increases.
4. According to the First Law of Thermodynamics ($\Delta U + \Delta W$), during the boiling of water at constant pressure:
 - a. $\Delta Q = \Delta U$ because no work is done.
 - b. $\Delta U = \Delta Q - P(V_{\text{steam}} - V_{\text{water}})$.
 - c. $\Delta Q = \Delta W$ because internal energy doesn't change during phase change
 - d. ΔU is always zero for any phase change

SOLUTIONS

1. Answer: b (Explanation: During a phase change, the temperature remains constant, meaning the average kinetic energy does not change. The heat (Latent Heat) is used entirely to overcome the attractive forces between molecules, thereby increasing the potential energy)
2. Answer: a (Explanation: Specific heat is defined as $c = \Delta Q / (m\Delta T)$. During boiling, heat ΔQ is being added, but the change in temperature ΔT is zero. Dividing by zero gives a mathematical value of infinity.)
3. Answer: c (Explanation: Internal energy increases because heat is added to the system ($\Delta Q > 0$). While some of this energy is spent as work (ΔV) to push back the atmosphere as the volume increases, the majority goes into increasing the molecular potential energy.)
4. Answer: b (Explanation: This is the standard application of the First Law. The change in internal energy is the total heat added minus the work done by the system during expansion ($W = P\Delta V$). Since steam occupies much more volume than liquid water, work is a significant factor.)

QUIZ

1 MARK QUESTIONS



1. A gas is suddenly compressed in a container with perfectly insulating walls. Does the temperature of the gas increase, decrease, or remain constant? Justify.
2. A person decides to cool a closed, thermally insulated room by leaving the refrigerator door open. Quantitatively, why will the room temperature actually rise?
3. On a P-V diagram, two curves represent an isothermal and an adiabatic expansion of the same gas from the same initial state. Which curve will have a steeper slope?
4. Explain why the molar specific heat at constant pressure (C_p) is always greater than the molar specific heat at constant volume (C_v) for an ideal gas.
5. After completing one full cycle, a heat engine returns to its initial state. What is the net change in the internal energy of the working substance?
6. You are given two different paths on a P-V diagram to take a gas from State A to State B. Will the work done by the gas be the same in both cases? Why or why not?
7. If 100 J of heat is supplied to a gas kept in a rigid, non-expanding container, how much work is done by the gas, and where does the supplied energy go?
8. Is it possible to create a heat engine that converts the entire heat absorbed from a source into work without rejecting any heat to a sink? Name the law that governs this.

- Between the variables Pressure (P), Volume (V), Temperature (T), Heat (Q), and Work (W), identify which are "path functions."
- For a diatomic gas like Oxygen, what is the value of the adiabatic index ($\gamma = C_P/C_V$) at room temperature?

HINTS / SOLUTIONS

- Temperature increases $\rightarrow$ Sudden compression + insulating walls $\Rightarrow$ Adiabatic process ($Q = 0$), so work done increases internal energy.
- Room temperature rises $\rightarrow$ Refrigerator releases more heat than it absorbs because electrical work is also converted into heat.
- Adiabatic curve is steeper $\rightarrow$ In adiabatic expansion, temperature also decreases, causing pressure to fall faster.
- $C_P > C_V \rightarrow$ At constant pressure, heat is used for both raising internal energy and doing expansion work.
- Net change in internal energy = 0 $\rightarrow$ Internal energy is a state function; cyclic process returns to initial state.
- No, work done may differ $\rightarrow$ Work depends on the path taken on the P–V diagram (area under curve).
- Work done = 0 $\rightarrow$ Rigid container $\Rightarrow \Delta V = 0 \Rightarrow$ no work. Entire 100 J goes into increasing internal energy.
- No, impossible $\rightarrow$ Governed by the Second Law of Thermodynamics (Kelvin–Planck statement).
- Path functions: Heat (Q) and Work (W) $\rightarrow$ depend on the process path followed.
- γ for diatomic gas $\approx 1.4 \rightarrow$ For gases like Oxygen at room temperature.

2 MARK QUESTIONS

- An ideal gas is compressed to half its initial volume. If this is done (a) isothermally and (b) adiabatically, in which case will the final pressure be higher? Provide a reason based on the governing equations for both processes.
- When heat is supplied to a gas at constant volume, the temperature rises quickly. However, when the same amount of heat is supplied at constant pressure, the temperature rise is slower. Explain this observation in terms of the work done by the gas.
- A system is taken from State A to State B via two different paths: Path 1 is a direct straight line, and Path 2 is a curved path on a P–V diagram. (a) Is the change in internal energy the same for both paths? (b) Is the work done the same? Justify your answers.
- When air is pumped into a bicycle tire rapidly, the pump barrel becomes quite hot. Is this heat primarily due to friction, or is there a thermodynamic explanation involving the air inside? Explain using the First Law.
- A scientist claims to have developed a heat engine that operates between 600 K and 300 K with an efficiency of 55%. By calculating the maximum possible efficiency (Carnot efficiency), determine if this claim is valid.
- Define an isochoric process. If 200 J of heat is added to a system during an isochoric process, what is the change in internal energy?
- When a high-pressure bicycle tire suddenly bursts, the air escaping feels noticeably cold to the touch. Since there was no "refrigerator" involved, explain the drop in temperature using the First Law of Thermodynamics and the nature of the expansion.
- An ideal gas is contained in one half of an insulated rigid container, while the other half is a vacuum. If the partition is suddenly removed, the gas occupies the entire volume.

- a. Calculate the work done by the gas.
 - b. What is the change in the internal energy of the gas? Justify your answer.
9. Two containers of equal volume contain 1 mole of Helium (monatomic) and 1 mole of Nitrogen (diatomic) respectively, both at the same temperature T .
- a. Which gas has higher internal energy?
 - b. If the same amount of heat Q is supplied to both at constant volume, which gas will show a greater rise in temperature?
10. Can the efficiency of a Carnot engine be increased more effectively by decreasing the sink temperature (T_2) by 10 K or by increasing the source temperature (T_1) by 10 K? Show the mathematical reasoning behind your choice.

HINTS / SOLUTIONS

1. Final pressure is higher in adiabatic compression
 Isothermal: $PV = \text{constant}$, Adiabatic: $PV^\gamma = \text{constant}$
 Since $\gamma > 1$, pressure rises more rapidly in adiabatic process.
2. Temperature rise is slower at constant pressure
 At constant volume: no work done ($W=0$), all heat increases internal energy.
 At constant pressure: part of heat is used for expansion work ($W=P\Delta V$).
3. (a) Change in internal energy is same $\rightarrow$ depends only on initial and final states (state function).
 (b) Work done is different $\rightarrow$ depends on path followed on P-V diagram.
4. Main reason: adiabatic compression of air
 Rapid compression $\Rightarrow$ little/no heat exchange ($Q \approx 0$).
 Work done on air increases internal energy $\rightarrow$ temperature rises.
 Friction may contribute slightly.
5. Check Carnot efficiency $\eta = 1 - \frac{T_2}{T_1} = 1 - (300/600) = 0.5 = 50\%$.
 Claimed efficiency = 55% > maximum possible 50%. Therefore claim is invalid.
6. Isochoric process: Constant volume process ($\Delta V=0$). Work done $W=0$
 From First Law: $Q=\Delta U$. Hence $\Delta U=200$ J.
7. Air feels cold due to sudden adiabatic expansion. Gas does work while expanding rapidly. No heat enters from surroundings ($Q \approx 0$). Internal energy decreases $\rightarrow$ temperature drops.
8. (a) Work done = 0. Expansion into vacuum $\Rightarrow$ no opposing pressure.
 (b) Change in internal energy = 0. Container insulated $\Rightarrow Q=0$. Since $W=0$, from First Law: $\Delta U=0$.
9. (a) Nitrogen has higher internal energy. Diatomic gases have more degrees of freedom.
 (b) Helium shows greater temperature rise. Monatomic gas has lower C_v , so same heat causes larger ΔT .
10. Efficiency, $\eta = 1 - \frac{T_2}{T_1}$. Decreasing sink temperature by 10 K is more effective in increasing efficiency.

3 MARK QUESTIONS

1. A sample of an ideal gas ($\gamma = 1.4$) at an initial pressure of 1.0×10^5 Pa and temperature of 27°C is compressed suddenly to one-eighth of its original volume. Calculate the final pressure of the gas. (Ans: 1.84×10^6 Pa Using $P_1 V_1^\gamma = P_2 V_2^\gamma$)
2. An ideal gas undergoes a cyclic process represented on a P-V diagram. The process starts at point A (P_1, V_1), moves to B (P_1, V_2) isobarically, then to C (P_2, V_2) isochorically, and finally returns to A via a straight

line path. Data: $P_1 = 300 \text{ kPa}$, $P_2 = 100 \text{ kPa}$, $V_1 = 2 \text{ m}^3$, $V_2 = 5 \text{ m}^3$. Calculate the net work done by the gas during one complete cycle. (Ans: 300 kJ)

3. A thermodynamic system is taken from an initial state A to another state B. Along path ACB, 80 J of heat flows into the system and the system does 30 J of work. Along path ADB, the work done by the system is 10 J. Calculate the heat flow into the system along path ADB. (Ans: 60 J)
4. Two moles of an ideal monatomic gas ($C_V = \frac{3}{2}R$) are heated from 300 K to 600 K at constant pressure. Calculate (a) the heat supplied to the gas, (b) the change in internal energy, and (c) the work done by the gas. (Use $R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$). (Ans: a. 12465 J, b. 7479 J, c. 4986 J)
5. An ideal monatomic gas occupies a volume of 4.0 L at a pressure of $2.0 \times 10^5 \text{ Pa}$ and a temperature of 300 K. The gas is first heated at constant volume until its pressure doubles, and then it is expanded at constant pressure until its volume doubles. Calculate the total heat absorbed by the gas during this two-step process. (Use $R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$ and $C_V = \frac{3}{2}R$). (Ans: 3600 J)

HINTS / SOLUTIONS

1. $P_1 V_1^\gamma = P_2 V_2^\gamma$. $\gamma = 1.4$, $P_1 = 1 \times 10^5 \text{ Pa}$, $V_2 = V_1 / 8$. On solving, $P_2 = 1.84 \times 10^6 \text{ Pa}$.
2. $P_1 - P_2 = 2 \times 10^5 \text{ Pa}$. $V_2 - V_1 = 3 \text{ m}^3$. $W = \frac{1}{2}(P_1 - P_2)(V_2 - V_1) = 300 \text{ kJ}$.
3. Using First Law of Thermodynamics, $Q = \Delta U + W$
 $80 = \Delta U + 30$. $\Delta U = 50 \text{ J}$. $Q = \Delta U + W$, $Q = 50 + 10 = 60 \text{ J}$.
4. $n=2$, $\Delta T=300 \text{ K}$. $C_V = \frac{3}{2}R$, $C_P = C_V + R = \frac{5}{2}R$
 - a. $Q = nC_P\Delta T = 12465 \text{ J}$
 - b. $\Delta U = nC_V\Delta T = 7479 \text{ J}$
 - c. $W = Q - \Delta U = 4986 \text{ J}$
5. At Constant volume, Pressure doubles $\Rightarrow$ temperature doubles.
 $T_1 = 300 \text{ K}$, $T_2 = 600 \text{ K}$. Heat added $= Q_1 = nC_V\Delta T = 1200 \text{ J}$
 At constant Pressure, Volume doubles $\Rightarrow$ temperature doubles again. $600\text{K} \rightarrow 1200\text{K}$
 $Q_2 = nC_P\Delta T = 2400 \text{ J}$. Total $= Q_1 + Q_2 = 3600 \text{ J}$

5 MARK QUESTIONS

1.
 - a. Distinguish between Isothermal and Adiabatic processes.
 - b. A weather balloon containing Helium ($\gamma = 1.67$) at an initial temperature of 27°C and pressure of 1.0 atm rises to an altitude where the pressure is only 0.5 atm. Assuming the expansion is rapid enough to be adiabatic: Determine the final temperature of the gas in the balloon (in Celsius). (Ans: -45.75°C)
2.
 - a. State the First Law of Thermodynamics and explain its physical significance as a restatement of the Law of Conservation of Energy.
 - b. Show that for an ideal gas, the change in internal energy during an isothermal process is zero.
3.
 - a. Define molar specific heat at constant pressure (C_p) and constant volume (C_v).
 - b. Derive Mayer's Relation ($C_p - C_v = R$) for an ideal gas, clearly explaining why C_p is greater than C_v .
 - c. A certain gas has a ratio of specific heats $\gamma = 1.4$. Identify the atomicity of the gas and calculate its degrees of freedom.

HINTS / SOLUTIONS

1. (a)

Isothermal Process	Adiabatic Process
Temperature remains constant ($\Delta T=0$)	No heat exchange ($Q=0$)
Heat exchange occurs with surroundings	System is thermally insulated
For ideal gas: $\Delta U=0$	Internal energy changes
Equation: $PV=\text{constant}$	Equation: $PV^\gamma=\text{constant}$
Happens slowly	Usually happens rapidly

(b) $\gamma = 1.67$, $T_1=27^\circ\text{C} = 300\text{K}$

$P_1 = 1 \text{ atm}$, $P_2 = 0.5 \text{ atm}$

$$T_1 P_1^{\frac{1-\gamma}{\gamma}} = T_2 P_2^{\frac{1-\gamma}{\gamma}} \quad T_2 = 227.25 \text{ K} = -45.75^\circ\text{C}$$

2. (a) Statement: The heat supplied to a system is used: to increase internal energy, and to do external work.
Mathematically: $Q = \Delta U + W$.

Physical Significance: It is the Law of Conservation of Energy applied to thermodynamic systems. Energy cannot be created or destroyed. Heat supplied transforms into internal energy and mechanical work.

(b) Change in internal energy during isothermal process

For an ideal gas: Internal energy depends only on temperature.

In an isothermal process: $\Delta T=0$ Therefore: $\Delta U=0$

Hence, during isothermal expansion/compression of an ideal gas, internal energy remains constant.

3. (a) Specific heat at constant volume (C_V): Heat required to raise temperature of 1 mole of gas by 1K at constant volume.

Specific heat at constant pressure (C_P): Heat required to raise temperature of 1 mole of gas by 1K at constant pressure.

(b) Derive Mayer's Relation

At constant volume: $Q_V = C_V \Delta T$

At constant pressure: $Q_P = C_P \Delta T$

From First Law: $Q = \Delta U + W$

For constant pressure: $Q_P = Q_V + P \Delta V$

Using ideal gas equation: $P \Delta V = R \Delta T$

So: $C_P \Delta T = C_V \Delta T + R \Delta T$

Dividing by ΔT : $C_P - C_V = R$

Why $C_P > C_V$?

At constant pressure, gas expands and does work.

Extra heat is needed for expansion work. Therefore: $C_P > C_V$

(c) $\gamma = C_P / C_V = 1.4$. For gases, $\gamma = (f+2) / f$, therefore $f = 5$.

Degrees of freedom = 5. Gas is diatomic.

FOR PRACTICE

ITEM	LINK	QR CODE
<u>QUIZ</u>	https://wayground.com/embed/quiz/6a02d80ee22c7cfc3ef04649	
<u>WORK SHEET</u>	https://forms.gle/3BznhukLdjtAx1tf7	
<u>AUDIO BOOK</u>	https://drive.google.com/file/d/1nS6G_2GGcnP0vflMjGYwySNW0mqZ9CZP/view?usp=drive_link	
<u>VIDEO BOOK</u>	https://drive.google.com/file/d/15jAamj8WzGFwGp6wDmMeP5_mZMwi0CTm/view?usp=drive_link	
<u>STUDY MATERIAL</u>	https://drive.google.com/file/d/1tePGsYRrI-Tk8TYh0oUKR1pkFrAc9san/view?usp=sharing	
<u>FORMULA SHEET</u>	https://drive.google.com/file/d/16HNvm08z2GMoVkfEpgW6DKEo3SRDs5Nt/view?usp=sharing	
<u>INFOGRAPHICS</u>	https://drive.google.com/file/d/1uAJ5V9vAfilAsCPyR79_EjMqCPBKXgAq/view?usp=sharing	



Heat
Energy in
transit

THERMODYNAMICS

FORMULA SHEET

CLASS XI PHYSICS



Heat flows
from higher
temperature
to lower
temperature.

QUICK REVISION • HIGH WEIGHTAGE • EXAM READY

1 BASIC DEFINITIONS & RELATIONS

- Temperature (T) in kelvin (K)
 $T(K) = t(^{\circ}C) + 273$



- Pressure (P)

$$P = \frac{F}{A}$$

- Density (ρ)

$$\rho = \frac{m}{V}$$

- Ideal Gas Equation

$$PV = nRT$$

- Universal Gas Constant

$$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$$



2 KINETIC THEORY OF GASES

- Pressure in terms of molecular model

$$P = \frac{1}{3} \rho \bar{c}^2$$

($\bar{c}^2$ = mean square speed)

- Root Mean Square Speed

$$c_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}$$

- Average Speed

$$\bar{c} = \sqrt{\frac{8RT}{\pi M}}$$

- Most Probable Speed

$$c_p = \sqrt{\frac{2RT}{M}}$$



3 FIRST LAW OF THERMODYNAMICS

- Internal Energy (Change in U)

$$dU = \delta Q - \delta W$$

- Work done by the system

$$\delta W = P dV$$

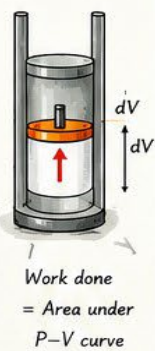
- Heat supplied to the system

$$\delta Q = dU + P dV$$

- For a finite change

$$\Delta U = Q - W$$

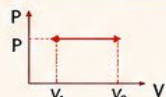
$$(\text{or}) Q = \Delta U + W$$



4 WORK DONE IN THERMODYNAMIC PROCESSES

- Isobaric ($P = \text{constant}$)

$$W = P(V_2 - V_1) = P \Delta V$$



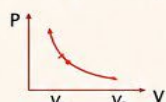
- Isochoric ($V = \text{constant}$)

$$W = 0$$



- Isothermal ($T = \text{constant}$)

$$W = nRT \ln \left(\frac{V_2}{V_1} \right)$$

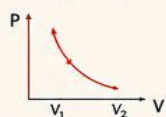


- Adiabatic ($Q = 0$)

$$PV^\gamma = \text{constant}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$= \frac{nR(T_1 - T_2)}{\gamma - 1} \quad \left(\gamma = \frac{C_p}{C_v} \right)$$



5 SPECIFIC HEATS

- At constant volume (C_v)

$$C_v = \left(\frac{\partial U}{\partial T} \right)_v$$

- At constant pressure (C_p)

$$C_p = \left(\frac{\partial H}{\partial T} \right)_p$$

- Relation between C_p and C_v

$$C_p - C_v = R$$

- For an ideal gas

$$C_v = \frac{f}{2} R$$

$$C_p = C_v + R = \frac{f+2}{2} R$$

(f = degrees of freedom)



For diatomic gas at ordinary temperature

$$f = 5 \Rightarrow C_v = \frac{5}{2} R, \quad C_p = \frac{7}{2} R$$

6 HEAT CAPACITY

- Heat Capacity of a body

$$C = \frac{\delta Q}{dT}$$

- Molar Heat Capacity

$$C_m = \frac{C}{n}$$

- Relation between molar heat capacities

$$C_{p,m} - C_{v,m} = R$$

- Heat capacity of an ideal gas

(At constant volume)

$$C_v = \frac{f}{2} nR$$

(At constant pressure)

$$C_p = \frac{f+2}{2} nR$$



7 THERMODYNAMIC PROCESSES – KEY RELATIONS

Process	Condition	Key Relations
• Isothermal ($T = \text{constant}$)	$PV = \text{constant}$	$\Delta U = 0$ $Q = W$
• Isochoric ($V = \text{constant}$)	$\frac{P}{T} = \text{constant}$	$W = 0$ $Q = \Delta U = nC_v \Delta T$
• Isobaric ($P = \text{constant}$)	$\frac{V}{T} = \text{constant}$	$W = P \Delta V$ $Q = nC_p \Delta T$
• Adiabatic ($Q = 0$)	$PV^\gamma = \text{constant}$ $TV^{\gamma-1} = \text{constant}$	$Q = 0$ $\Delta U = -W$
• Cyclic Process	$\Delta U = 0$	$Q = W$

EXAM TIPS

- ✓ Learn derivations of key formulas
- ✓ Practice numericals daily
- ✓ Understand the physical meaning of each formula
- ✓ Draw neat diagrams for processes
- ✓ Revise this sheet before the exam

NOTE

"Thermodynamics is not about what we cannot see, it is about what we can measure and understand."

IMPORTANT DEFINITIONS

- **System:** The part of the universe under study.
- **Surroundings:** Everything outside the system.
- **Boundary:** The surface separating system and surroundings.
- **State:** Condition of the system described by macroscopic physical properties.
- **Equilibrium:** State in which macroscopic properties do not change with time.

STAY CALM
STUDY SMART
SCORE HIGH!

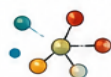


Symbols:

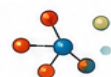
P – Pressure V – Volume T – Temperature
 n – Number of moles R – Gas constant
 U – Internal Energy W – Work Q – Heat
 C_v – Specific heat at constant volume
 C_p – Specific heat at constant pressure
 γ – Adiabatic index f – Degrees of freedom

"Heat is not a substance, it is a mode of energy transfer." – Clausius

CHAPTER 12



KINETIC THEORY OF GASES



1. BEHAVIOUR OF GASES

- Gases at low pressures and high temperatures show ideal gas behavior.
- Work is done when a gas is compressed or expanded. During compression, external force decreases the volume of the gas and increases its internal energy.
- The kinetic theory of gases explains the behaviour of gases based on the motion of molecules.



IDEAL GAS EQUATION

$$PV = \mu RT$$

where μ = number of moles
 R = universal gas constant
 $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$

IN TERMS OF BOLTZMANN CONSTANT

$$PV = k_B n T \quad \text{or} \quad P = k_B n T$$

where $n = \frac{N}{V}$

$$k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

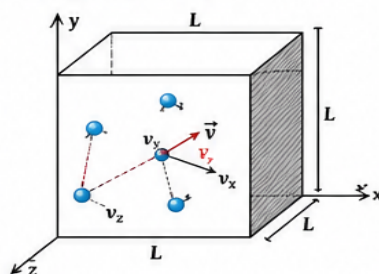
2. POSTULATES OF KINETIC THEORY

- The molecules of a gas are point masses, the size of a molecule being negligible compared to the distance between them.
- There is no force of attraction or repulsion between molecules.
- The molecules are in a state of random motion, moving with all possible velocities in all possible directions.
- During their motion, they collide with one another and also with the walls of the container. These collisions are elastic.
- Between successive collisions, the molecules move in straight lines with uniform velocity. The distance travelled between two successive collisions is called free path. Average distance between the successive collisions is called mean free path.
- Time for a collision is negligibly small compared to the time taken to traverse mean free path.
- The mean KE of the molecule is a constant at a given temperature and is proportional to absolute temperature.

3. CONCEPT OF PRESSURE

The pressure exerted by a gas may be defined as the total momentum imparted to unit area of the walls of the container per second due to molecular impacts (collisions).

Consider an ideal gas (N molecules each of mass m) enclosed in a cubical box of side L .



RESULT

$$P = \frac{1}{3} \rho v_{rms}^2 \quad \text{where } \rho = \frac{M}{V} \text{ (density)}$$

$$\text{Also, } P = \frac{1}{3} m n \overline{v^2} \quad \text{where } n = \frac{N}{V}$$

$$\text{and } v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3RT}{M}}$$

4. RMS SPEED OF GAS MOLECULES

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

where R = universal gas constant
 T = absolute temperature
 M = molar mass of the gas



SPEED ~ km/s

- v_{rms} is of the order of km/s.
- v_{rms} is $\sqrt{3}$ times that of speed of sound in gas.
- v_{rms} does not depend on the pressure of gas (if temperature remains constant).

$$P \propto \rho \quad (\text{Boyle's law})$$

If pressure is increased n times, density increases by n times but v_{rms} remains constant.



A planet or satellite will have atmosphere only and only if $v_{rms} < v_e$ (escape velocity)

5. KINETIC ENERGY OF IDEAL GAS

Quantity of gas	Kinetic energy
1 Molecule (mass m)	$KE = \frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T$
1 mole (M gram) gas	$KE = \frac{3}{2} RT$
1 gram gas	$KE = \frac{3}{2} \frac{R}{M} T$

m = mass of one molecule, M = molar mass in grams,
 N = Avogadro's number

6. DEGREES OF FREEDOM (f)

The total number of independent modes (ways) in which a system can possess energy is called the degree of freedom (f) or it refers to the possible independent motions, systems can have.

General expression: $f = 3N - K$
 $(N = \text{Number of independent particles,}$
 $K = \text{Number of independent restrictions (constraints)})$

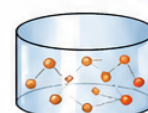
Type of Gas	Description	Degrees of Freedom (f)
1. Monoatomic gas	Can move in any direction in space. (Translational)	$f = 3$
2. Diatomic gas	3 translational + 2 rotational (rotation about 2 axes perpendicular to bond)	$f = 5$
3. Polyatomic gas	3 translational + 3 rotational (rotation about 3 axes through centre of mass)	$f = 6$
4. Non-linear polyatomic gas	3 translational + 3 rotational + 3 vibrational	$f \geq 9$

7. LAW OF EQUIPARTITION OF ENERGY

Statement: At thermal equilibrium, energy is equally distributed among all the degrees of freedom.

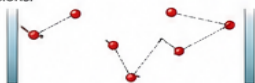
$$E = \frac{f}{2} k_B T$$

where E = average energy of the molecule
 f = degrees of freedom
 k_B = Boltzmann's constant
 T = absolute temperature



8. MEAN FREE PATH (λ)

Mean free path is defined as the average distance travelled by a molecule between two successive collisions.



$$\lambda = \frac{1}{\sqrt{2} n d^2}$$

where d = diameter of the molecule
 n = number of molecules per unit volume

9. SPECIFIC HEAT CAPACITIES OF GASES

From equipartition of energy:

$$C_V = \frac{f}{2} R$$

$$C_P = C_V + R = \left(\frac{f}{2} + 1\right) R$$

$$\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$$

Type of Gas	f	C_V	C_P	γ
Monoatomic	3	$\frac{3}{2} R$	$\frac{5}{2} R$	$\frac{5}{3}$
Diatomic	5	$\frac{5}{2} R$	$\frac{7}{2} R$	$\frac{7}{5}$
Polyatomic	6	$3R$	$4R$	$\frac{4}{3}$
Non-linear polyatomic (≥ 9)	$f \geq 9$	$\frac{f}{2} R$	$\left(\frac{f}{2} + 1\right) R$	$1 + \frac{2}{f}$

LEGEND / CONSTANTS

$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
 $k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$
 N = Avogadro's number
 n = number density = $\frac{N}{V}$
 ρ = density = $\frac{M}{V}$

SYMBOLS

P = Pressure
 V = Volume
 T = Absolute temperature (K)
 m = Mass of one molecule
 M = Molar mass (g mol^{-1})
 v_{rms} = Root mean square speed
 d = Diameter of molecule
 λ = Mean free path
 f = Degrees of freedom



IDEAL GAS EQUATION SUMMARY

$$PV = \mu RT$$

$$\text{or}$$

$$PV = k_B n T$$

$$\text{or}$$

$$P = k_B n T$$

KEY TAKEAWAY

- Pressure arises due to molecular impacts.
- Temperature is a measure of average kinetic energy.
- Higher temperature $\rightarrow$ higher v_{rms} .
- Energy is equally shared among degrees of freedom.
- Kinetic theory successfully explains macroscopic properties of gases from microscopic behavior of molecules.



MULTIPLE CHOICE QUESTIONS (MCQ)

- Suppose ideal gas equation follows $vp^3 = \text{constant}$. Initial temperature and volume of the gas are T and V respectively. If gas expand to $27V$ then its temperature will be come
 - T
 - $9T$
 - $27T$
 - $T/9$
- At what temperature, the mean kinetic energy of O_2 will be the same for H_2 molecules at $-73^\circ C$
 - $127^\circ C$
 - $527^\circ C$
 - $-73^\circ C$
 - $-173^\circ C$
- The temperature of a given mass is increased from $27^\circ C$ to $327^\circ C$. The rms velocity the molecules increase
 - $\sqrt{2}$ times
 - 2 times
 - $2\sqrt{2}$ times
 - 4 times
- The ratio of rms speeds of two gases at same temperature is:
 - $\frac{M_1}{M_2}$
 - $\sqrt{\frac{M_1}{M_2}}$
 - $\sqrt{\frac{M_2}{M_1}}$
 - $\frac{M_2}{M_1}$
- A container has two chambers of volumes $V_1 = 2L$ and $V_2 = 3L$, containing gases at pressures $P_1 = 1\text{atm}$ and $P_2 = 2\text{atm}$ respectively. After removing the partition, the equilibrium pressure becomes:
 - 1.4 atm
 - 1.8 atm
 - 1.3 atm
 - 1.6 atm
- A gas mixture contains 2 moles of O_2 and 4 moles of Ar at temperature T . Neglect vibrational modes. Total internal energy is:
 - $4RT$
 - $15RT$
 - $9RT$
 - $11RT$
- The dimensional formula of the gas constant R is:
 - $[ML^2T^{-2}K^{-1}mol^{-1}]$
 - $[MLT^{-2}]$
 - $[ML^2T^{-1}]$
 - $[M^0L^0T^0]$
- A gas is filled in a container at pressure P . If the mass of molecules is halved and their rms speed is doubled. The resultant pressure would be
 - $2P$
 - $4P$
 - $P/4$
 - $P/2$
- At very high temperature, vibrational modes become active. Degrees of freedom of a diatomic gas become:
 - 3
 - 5
 - 6
 - 7
- The mean free path of molecules of a gas (radius r) is inversely proportional to:
 - r^3
 - r^2
 - r
 - $\sqrt{r}$
- The pressure P for a gas is plotted against its absolute temperature T for two different volumes V_1 and V_2 where $V_1 > V_2$. If P is plotted on y – axis and T on x – axis, then
 - the curve for V_1 has greater slope than that for V_2
 - the curve for V_2 has greater slope than that for V_1
 - both curves have same slope
 - the curves intersect at some point other than $T = 0$
- Four molecules of a gas have speeds 1, 2, 3 and 4 km s^{-1} . The value of the root mean square speed of the gas molecule is
 - $\frac{1\sqrt{15}}{2} \text{ kms}^{-1}$
 - $\frac{1}{2}\sqrt{10} \text{ kms}^{-1}$
 - 2.5 kms^{-1}
 - $\sqrt{\frac{15}{2}} \text{ kms}^{-1}$
- For a gas if ratio of specific heats at constant pressure and volume is γ then value of degrees of freedom is
 - $\frac{3\gamma - 1}{2\gamma - 1}$
 - $\frac{2}{\gamma - 1}$
 - $\frac{9}{2} (\gamma - 1)$
 - $\frac{25}{2} (\gamma - 1)$
- If one mole of monoatomic gas $\gamma = \frac{5}{3}$ is mixed with one mole of diatomic gas $\gamma = \frac{7}{5}$, the value of γ for the mixture is
 - 1.40
 - 1.50
 - 1.53
 - 3.07

15. A monoatomic gas molecule has
- Three degrees of freedom
 - Four degrees of freedom
 - Five degrees of freedom
 - Six degrees of freedom
16. A gas behaves as an ideal gas at
- Low pressure and high temperature
 - low pressure and low temperature
 - High pressure and low temperature
 - high pressure and high temperature
17. A partition divides a container having insulated walls into two compartments I and II. the same gas fills the two compartments. The ratio of the number of molecules in compartments I and II is
- | | |
|-----------|-------------|
| P, V, T | $2P, 2V, T$ |
| I | II |
- 1 : 6
 - 6 : 1
 - 4 : 1
 - 1 : 4
18. . In kinetic theory of gases ,it is assumed that:
- The collisions are not perfectly elastic.
 - The molecular collisions change the density of the gas.
 - The molecules don't collide with each other on the wall.
 - Between two collisions the molecules travel with uniform velocity.
19. The pressure and temperature of an ideal gas in a closed vessel are 720 kPa and 40°C respectively. If $\frac{1}{4}$ th of the gas is released from the vessel and the temperature of the remaining gas is raised to 353°C , the final pressure of the gas is
- 1440 kPa
 - 1080 kPa
 - 720 kPa
 - 540 kPa
20. The mean kinetic energy of molecules of all gases is same at:
- Same pressure
 - Same volume
 - Same temperature
 - Same density

HINTS/SOLUTIONS

1. b 2. C 3. b 4. c 5. d, 6. d 7. a 8. a 9. d 10. b 11. b 12.d
13. b 14. b 15. a 16.a 17. d 18. d 19. b 20. C

1. b, For the gas process: $vp^3 = \text{constant}$, Using the ideal gas law $PV=nRT$, $P = nRT/v$

$$V \cdot \left(\frac{nRT}{V}\right)^3 = \text{constant}, \text{ so } T^3 \propto V^2, \frac{T_2^3}{T_1^3} = \frac{V_2^2}{V_1^2}, T_2 = 9T$$

2. c The mean kinetic energy of gas molecules depends only on absolute temperature, not on the type of gas.

3. b. $V_{\text{rms}} \propto \sqrt{T}$, Convert temperatures to Kelvin: $T_1 = 27^\circ\text{C} = 300 \text{ K}$, $T_2 = 327^\circ\text{C} = 600 \text{ K}$

4.,the ratio of rms speeds of two gases at same temperature is: $\frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}}$

5. Using conservation relation: $P_1V_1 + P_2V_2 = P(V_1 + V_2)$, $P = \frac{1 \times 2 + 2 \times 3}{2+3} = \frac{8}{5} = 1.6 \text{ atm}$

6.For gases: Diatomic gas: $U = \frac{5}{2}nRT$ Monoatomic gas: $U = \frac{3}{2}nRT$

$$U = \frac{5}{2}(2RT) + \frac{3}{2}(4RT) = 5RT + 6RT = 11RT$$

7.a $[\text{ML}^2\text{T}^{-2}\text{K}^{-1}\text{mol}^{-1}]$

8. a Using, $P = \frac{1}{3}\rho v_{\text{rms}}^2 = \frac{1}{3}mNv_{\text{rms}}^2/V \therefore P \propto mv_{\text{rms}}^2$

9. d Translational = 3 Rotational = 2 Vibrational = 2

Total = 7.

11. $PV=nRt$; $p = nRT/V$, Slope of $P - T$ graph $\propto 1/V$

$$12. \therefore V_{\text{rms}} = \sqrt{\frac{1+4+9+16}{4}} = \sqrt{\frac{30}{4}}$$

$$13. \gamma = 1 + \frac{2}{n}$$

$$14. \frac{n_1+n_2}{\gamma-1} = \frac{n_1}{\gamma_1-1} + \frac{n_2}{\gamma_2-1}, \gamma = \frac{3}{2}$$

15. Three

16. a Ideal Gas Behavior A gas behaves as an ideal gas at low pressure and high temperature. , A gas behaves ideally when its molecules are widely spaced, move freely, and do not interact with each other.

17. (d) $n = \frac{PV}{kT}$ Now, $n' = \frac{(2P)(2V)}{kT} = 4 \frac{PV}{kT} = 4n$ or $\frac{n}{n'} = \frac{1}{4}$.

18. D

19. (b) $P_1 = 720 \text{ kPa}$, $T_1 = 40^\circ\text{C} = 273 + 40 = 313 \text{ K}$ $P \propto mT \Rightarrow \frac{P_2}{P_1} = \frac{m_2}{m_1} \frac{T_2}{T_1} = \frac{3}{4} \times \frac{626}{313} = 1.5 \Rightarrow$

$P_2 = 1.5P_1 = 1.5 \times 720 = 1080 \text{ kPa}$

20. A Using, $P = \frac{1}{3} \rho v_{\text{rms}}^2 = \frac{1}{3} m N v_{\text{rms}}^2 / V \therefore P \propto m v_{\text{rms}}^2$

ASSERTION & REASONING BASED MCQS

For question numbers 1 to 10 , two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false and R is also false

1. **Assertion (A)** : Mean free path of gas molecules varies inversely as density of the gas.

Reason (R) : Mean free path of gas molecules is defined as the average distance travelled by a molecule between two successive collisions.

2. **Assertion (A)** : Specific heat of a gas at constant pressure is greater than its specific heat at constant volume. **Reason (R)** : At constant pressure, some heat is spent in expansion of the gas.

3. **Assertion (A)** : Absolute zero is not the temperature corresponding to zero energy.

Reason (R) : The temperature at which no molecular motion ceases is called absolute zero temperature.

4. **Assertion (A)** : Vibrational energy of diatomic molecule corresponding to each mode of freedom is $k_B T$.

Reason (R) : For every molecule, vibrational degree of freedom is 2

5. **Assertion**: The ratio C_p/C_v is more for helium gas than for hydrogen gas.

Reason: Atomic mass of helium is more than that of hydrogen.

6. **Assertion**: An undamped spring-mass system is simplest free vibration system.

Reason: It has three degrees of freedom

7. **Assertion (A)** : The number of degrees of freedom of a linear triatomic molecule is 7.

Reason (R) : The number of degree of freedom depends on number of particle in the system.

8. **Assertion (A)** : For an ideal gas, the internal energy depends only on temperature.

Reason (R): In an ideal gas, intermolecular forces are absent.

9. **Assertion (A)**: At constant temperature, the pressure of a gas increases when its volume decreases.

Reason (R): Gas molecules collide more frequently with the walls of the container when volume decreases.

10. **Assertion (A)**: The pressure exerted by a gas is due to collisions of molecules with the walls of the container.

Reason (R): Gas molecules are in continuous random motion.

HINTS/ SOLUTIONS (ASSERTION AND REASON)

1.b RMS speed increases with temperature: $v_{\text{rms}} \propto \sqrt{T}$

So, the **Assertion is true**. The **Reason is also true** because it correctly defines mean free path. However, the definition given in the reason does **not explain** why mean free path varies inversely with density.

2. a Mathematically: $C_p = C_v + R$

3. c Assertion (A) is true because even at absolute zero, atoms and molecules possess zero-point energy due to quantum effects. Therefore, total energy is not exactly zero.

Reason (R) is false because molecular motion does not completely cease at absolute zero. Classical kinetic motion becomes minimum, but quantum mechanical zero-point motion still exists.

- 4.b One degree of freedom contributes kinetic energy $\frac{1}{2}k_B T$. Another contributes potential energy $\frac{1}{2}k_B T$. Hence total vibrational energy per vibrational mode is: $k_B T = \frac{1}{2}k_B T + \frac{1}{2}k_B T$. The Reason is also true in the sense that each vibrational mode contributes two quadratic degrees of freedom (one kinetic and one potential). However, the statement “for every molecule, vibrational degree of freedom is 2” is not the correct explanation because the number of vibrational degrees depends on the type of molecule and the number of vibrational modes.
- 5.b Both assertion and reason are true but the reason is not a correct explanation of the assertion. Monoatomic helium has 3 degrees of freedom while diatomic hydrogen has 5 degrees of freedom.
- 6.C Assertion is true and reason is false. An undamped spring-mass system is the simplest vibration system with one degree of freedom.
7. b For a linear triatomic molecule: Translational degrees of freedom = 3 Rotational degrees of freedom = 2 Vibrational degrees of freedom (neglected in many cases at ordinary temperatures) = 2 Hence total active degrees of freedom:
 $f = 3 + 2 + 2 = 7$ The Reason is also true because the total degrees of freedom of a system are related to the number of constituent particles and their possible motions. However, the reason does not specifically explain why a linear triatomic molecule has 7 degrees of freedom.
8. a Since there are no intermolecular forces in an ideal gas, there is no potential energy. Hence internal energy consists only of kinetic energy, which depends only temperature.
9. a at constant temperature, $P \propto \frac{1}{V}$. So, when volume decreases, pressure increases. From kinetic theory: Decreasing the volume reduces the space available for molecular motion. Hence gas molecules strike the walls more frequently. Increased collision frequency increases pressure.
10. a Because gas molecules move continuously and randomly, they frequently collide with the container walls, producing pressure.

CASE BASED QUESTIONS

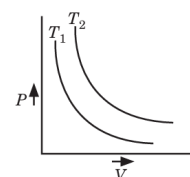
1. Case Study A research team is studying the behaviour of gases in a laboratory cylinder fitted with a movable piston. The cylinder contains an ideal gas. When the piston is pushed inward, the gas molecules collide more frequently with the walls, causing the pressure to increase. The average kinetic energy of the molecules depends only on the absolute temperature of the gas. The scientists also observe that lighter gas molecules move faster than heavier molecules at the same temperature. According to kinetic theory, gas pressure arises due to continuous random motion of molecules.

- According to kinetic theory, the pressure of a gas is due to:
 - Mutual attraction between molecules
 - Collisions of molecules with container walls
 - Weight of molecules
 - Shape of molecules
- If the absolute temperature of an ideal gas doubles, the average kinetic energy of molecules becomes:
 - Half
 - Double
 - Four times
 - Unchanged
- Which of the following gases will have the highest rms speed at the same temperature?
 - Oxygen
 - Nitrogen
 - Hydrogen
 - Carbon dioxide
- Real gases behave nearly ideally under:
 - High pressure and low temperature
 - Low pressure and high temperature
 - High pressure and high temperature
 - Low pressure and low temperature
- The mean kinetic energy of gas molecules depends on:
 - Pressure only
 - Volume only

- What do you mean by mean free path of a gas molecule.
- Mention the different ways of increasing number of molecular collisions per unit time in a gas.
- The ratio of specific heats γ for a diatomic gas (rigid) is:
- How does the mean free path (λ) change with an increase in gas density?

2 MARK QUESTIONS

- State the two main assumptions of the Kinetic Theory of Gases that do not hold true for real gases.
- How does the mean free path of a gas vary with (a) Temperature at constant pressure and (b) Pressure at constant temperature?
- The root mean square (rms) speed of oxygen molecule at certain temperature 'T' is 'V'. If temperature is doubled and oxygen gas dissociates into atomic oxygen what is the speed of atomic oxygen?
- Two vessels of the same volume are filled with the same gas at the same temperature. If the pressure of the gas in these vessels be in the ratio 1 : 2 then state
 - The ratio of the rms speeds of the molecules.
 - The ratio of the number of molecules.
- At what temperature will the RMS speed of oxygen molecules be sufficient so as to escape from the earth? (The escape velocity from Earth is approximately: $v_e = 11.2$ km/s, M for oxygen gas (O_2) is 32×10^{-3} kg/mol)
- A gas molecule has f degrees of freedom. State the expression for its internal energy (U) and molar specific heat at constant volume (C_v).
- Explain why a diatomic molecule has different degrees of freedom at room temperature versus very high temperatures.
- What is specific heat capacity (C_v) of water in terms of degrees of freedom? (assuming water behaves as a solid)
- Isothermal curves for a given mass of gas are shown at two different temperatures T_1 and T_2 . State whether $T_1 > T_2$ or $T_2 > T_1$. Justify your answer.
- Explain why (a) there is no atmosphere on moon. (b) there is fall in temperature with altitude



HINTS/SOLUTIONS

- The volume occupied by the gas molecules themselves is negligible compared to the total volume of the gas. There are no intermolecular forces of attraction or repulsion between the gas molecules.
- For an ideal gas, the **mean free path** λ is the average distance a molecule travels between collisions. It is given by: $\lambda = \frac{kT}{\sqrt{2} \pi d^2 P}$ where: k = Boltzmann constant, T = absolute temperature, d = molecular diameter, P = pressure. From this relation: (a) Variation with Temperature at Constant Pressure At constant P , $\lambda \propto T$. So, the mean free path increases directly with temperature. Higher temperature, molecules move faster and gas expands collisions become less frequent, larger mean free path. (b) Variation with Pressure at Constant Temperature At constant T , $\lambda \propto \frac{1}{P}$. So, the mean free path decreases inversely with pressure.
- $v = \sqrt{\frac{3RT}{M}}$ $c' = \sqrt{\frac{3R(2T)}{M/2}}$ $c' = 2v$
- (i) $v \propto \sqrt{T}$ as the temperature is same rms speeds are same. (ii) $\frac{P_1}{P_2} = \frac{n_1}{n_2} = \frac{1}{2}$
- To escape Earth, the RMS speed of oxygen molecules must equal Earth's escape velocity.

The escape velocity from Earth is approximately: $v_e = 11.2 \text{ km/s} = 1.12 \times 10^4 \text{ m/s}$ For a gas molecule, the RMS speed is: $v_{rms} = \sqrt{\frac{3RT}{M}}$ Where $R = 8.314 \text{ J mol}^{-1}\text{K}^{-1}$ For oxygen gas (O_2) is $32 \times 10^{-3} \text{ kg/mol}$ $1.12 \times 10^4 = \sqrt{\frac{3(8.314)T}{32 \times 10^{-3}}}$ $T \approx 1.6 \times 10^5 \text{ K}$

6. For an ideal gas molecule with f degrees of freedom:

Internal Energy The internal energy of n moles of the gas is $U = \frac{f}{2} nRT$

where: f = degrees of freedom n = number of moles R = universal gas constant

T = absolute temperature, $U = \frac{f}{2} nRT$ The molar specific heat at constant volume is $C_v = \frac{f}{2} R$

$$C_v = \frac{f}{2} R$$

7. A diatomic molecule can store energy in different ways called **degrees of freedom**: 1. Translational motion 2. Rotational motion 3. Vibrational motion The number of active degrees of freedom depends on temperature because energy is quantized. **At Room Temperature** A diatomic molecule mainly has: **3 translational** degrees of freedom **2 rotational** degrees of freedom So, $f = 5$ The vibrational modes are usually **not excited** at room temperature because the thermal energy is too small to overcome the large energy gap between vibrational levels. **At Very High Temperatures** : As temperature increases, molecules gain enough energy to excite vibrational motion. A vibrational mode contributes: 1 kinetic degree and 1 potential degree Thus, vibration adds **2 more degrees of freedom**. So at high temperatures: $f = 7$

8. If water behaves as a **solid**, its molecules mainly possess **vibrational degrees of freedom**. For a solid, each atom contributes: 3 kinetic vibrational degrees, 3 potential vibrational degrees So total degrees of freedom per molecule: $f = 6$

Using the equipartition theorem, the molar specific heat capacity at constant volume is $C_v = \frac{f}{2} R$

Substituting $f = 6$, $C_v = 3R$, $C_v = \frac{f}{2} R$ Thus, for water treated as a solid, $C_v = 3R$

9. Since, $T = PV/nR$, $T \propto PV$ [R = constant] If area under PV is greater for the curve at T_2 than for the curve T_1 . Hence, $T_2 > T_1$

10. The Moon has very small mass, so its gravitational pull is weak. Therefore, the escape velocity on the Moon is low (Escape velocity of the moon is 2.38 km s^{-1}), and gas molecules move fast enough to escape into space. As a result, the Moon cannot retain an atmosphere. As altitude increases, atmospheric pressure decreases. Air expands in the lower-pressure region, and during expansion it cools (adiabatic cooling). Also, the atmosphere is heated by radiation from the Earth's surface, so higher layers receive less heat. Hence, temperature decreases with increase in altitude

3 MARK QUESTIONS

1. Demonstrate your understanding of the kinetic theory of gases by listing its fundamental assumptions and explaining the significance of each.
2. Using assumptions of kinetic theory of gases find the expression for the pressure exerted by an ideal gas.
3. A gas mixture consists of 2.0 moles of oxygen and 4.0 moles of neon at temperature T . Neglecting all vibrational modes, calculate the total internal energy of the system. (Oxygen has two rotational modes.)
4. Derive expression for mean free path of a gas molecule. On which factors does the mean free path depend?
5. Derive an expression showing relation between pressure and kinetic energy using Kinetic theory of gases
6. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at -20°C ? (atomic mass of Ar = 39.9 u, of He = 4.0 u).
7. (a) How much volume does one mole of a gas occupy at NTP? (b) What are the essential characteristics of an ideal gas? (c) What are the conditions under which real gases behave nearly as a perfect gas?

HINTS/SOLUTIONS

3. For an ideal gas mixture, the total internal energy is the sum of the internal energies of each gas component. For a molecule with f degrees of freedom: $U = \frac{f}{2}nRT$ **Oxygen (O_2)** Oxygen is a diatomic gas with: 3 translational modes 2 rotational modes Vibrational modes neglected So, $f = 5$ Given: $n = 2.0\text{mol}$, Internal energy of oxygen: $U_{O_2} = \frac{5}{2}(2.0)RT = 5RT$ **Neon (Ne)** Neon is a monoatomic gas with only: 3 translational modes So, $f = 3$ Given: $n = 4.0\text{mol}$, Internal energy of neon: $U_{Ne} = \frac{3}{2}(4.0)RT = 6RT$ Total Internal Energy $U_{\text{total}} = U_{O_2} + U_{Ne}$, $U_{\text{total}} = 5RT + 6RT = 11RT$

4. $\lambda = \frac{1}{\sqrt{2\pi d^2 n}} = \frac{m}{\sqrt{2\pi d^2 \rho}} = \frac{kT}{\sqrt{2\pi d^2 P}}$ Hence: $\lambda \propto \frac{1}{d^2}$, $\lambda \propto \frac{1}{\rho}$, $\lambda \propto T$, $\lambda \propto \frac{1}{P}$, $\lambda \propto m$ (for fixed pressure and temperature conditions as applicable).

5. Starting from the pressure equation obtained from kinetic theory: $PV = \frac{1}{3}Nm v_{\text{rms}}^2$ Expression for Kinetic Energy - The translational kinetic energy of one molecule is: $\frac{1}{2}mv^2$

For N molecules, total kinetic energy is: $K = \frac{1}{2}Nm v_{\text{rms}}^2$, From
 kinetic theory: $PV = \frac{1}{3}Nm v_{\text{rms}}^2$, $\frac{3}{2}PV = \frac{1}{2}Nm v_{\text{rms}}^2$ But, $\frac{1}{2}Nm v_{\text{rms}}^2 = K$ Therefore,
 $K = \frac{3}{2}PV$ Dividing by V : $P = \frac{2K}{3V}$ Thus, the pressure of a gas is directly related to the translational kinetic energy of its molecules: $K = \frac{3}{2}PV$ Or $P = \frac{2K}{3V}$

6. Let C and C' be the rms velocity of argon and a helium gas atom at temperature T K and T' K respectively. Here, $M = 39.9$; $M' = 4.0$; $T = ?$; $T' = -20 + 273 = 253$ K

$$C = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{39.9}} \quad C' = \sqrt{\frac{3RT'}{M'}} = \sqrt{\frac{3R \times 253}{4}} \quad \text{, Since } C = C' \text{ , } T = 39.9 \times 253 / 4 = 2523.7 \text{ K} \quad 7. a$$

Volume Occupied by One Mole of a Gas at NTP At NTP (Normal Temperature and Pressure):

Temperature = $0^\circ\text{C} = 273 \text{ K}$, Pressure = 1 atm One mole of an ideal gas occupies: 22.4 litres or $22.4 \times 10^{-3} \text{ m}^3$ This is called the **molar volume** of a gas at NTP.

b. Main Characteristics of an Ideal Gas - Volume of molecules is negligible compared to gas volume, No attraction or repulsion between molecules except during collisions. (No inter molecular force) , Total kinetic energy remains conserved (Elastic collision) , Motion is continuous and, in all directions, (Random motion), Obeys ideal gas equation $PV = nRT$ Internal energy depends only on temperature **Real gases approach ideal behavior under:** **1. Low Pressure** - Molecules stay far apart. Intermolecular forces become negligible. **2. High Temperature** - Molecular kinetic energy becomes very large. , Attractive forces become insignificant. Thus, real gases behave nearly as ideal gases at Low pressure and high temperature

Examples: Hydrogen and helium behave most ideally under ordinary conditions

5 MARK QUESTIONS

- State the law of equipartition of energy for a dynamic system. Using this law, determine the expressions for the internal energy and the ratio of specific heats $\gamma = C_p/C_v$ for the following types of ideal gases:
 (a) Monoatomic gas (b) Diatomic gas (c) Triatomic gas (assuming non-linear molecules)
 Also, explain how the degrees of freedom influence the internal energy and heat capacities of each gas
- State kinetic interpretation of temperature, define and derive expression for rms speed of molecules of an ideal gas

HINTS/SOLUTIONS

- The **law of equipartition of energy** states: "For a system in thermal equilibrium, energy is equally distributed among all independent quadratic degrees of freedom, and each degree of freedom contributes an average energy of $\frac{1}{2}kT$ per molecule." For one mole of gas: Energy per degree of freedom = $\frac{1}{2}RT$

If a molecule has f degrees of freedom, then the internal energy of one mole is: $U = \frac{f}{2}RT$ For

n moles: $U = \frac{f}{2}nRT$ From thermodynamics: $C_V = \left(\frac{\partial U}{\partial T}\right)_V$

Hence, $C_V = \frac{f}{2}R$ Also, $C_P = C_V + R$

Thus, $C_P = \left(\frac{f}{2} + 1\right)R$ Therefore, $\gamma = \frac{C_P}{C_V} = \frac{f+2}{f}$ (a)

Monoatomic Gas, Examples: He, Ne, Ar A monoatomic molecule has only: 3 translational degrees of freedom So, $f = 3$ Internal Energy $U = \frac{3}{2}nRT$ Specific Heats $C_V = \frac{3}{2}R$ $C_P = \frac{5}{2}R$ Ratio of

Specific Heats $\gamma = \frac{5}{3}$ (b) **Diatomic Gas** Examples: O_2 , N_2 , H_2 Neglecting vibrational modes: , 3 translational degrees , 2 rotational degrees Thus, $f = 5$ Internal Energy $U = \frac{5}{2}nRT$ Specific Heats $C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$

Ratio of Specific Heats $\gamma = \frac{7}{5}$ (c) **Triatomic Gas (Non-linear Molecules)** Examples: H_2O , NH_3 . For non-linear triatomic molecules: , 3 translational degrees , 3 rotational degrees Hence, $f = 6$

(neglecting vibrational modes) Internal Energy $U = 3nRT$ Specific Heats $C_V = 3R$, $C_P = 4R$

Ratio of Specific Heats $\gamma = \frac{4}{3}$ **Influence of Degrees of Freedom** The degrees of freedom determine

how energy is shared among molecular motions. More degrees of freedom $\Rightarrow$ more energy absorbed by molecules. Therefore: Internal energy increases with f , Specific heat capacities increase with f From:

$U = \frac{f}{2}nRT$ we see: $U \propto f$ Similarly, $C_V = \frac{f}{2}R$, so, $C_V \propto f$ However,

$\gamma = \frac{f+2}{f}$ Thus: As f increases, γ decreases.

2. kinetic interpretation of temperature states: "The absolute temperature of a gas is directly proportional to the average translational kinetic energy of its molecules." Thus, higher temperature means greater molecular motion and greater kinetic energy.

For one molecule of an ideal gas: $\frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT$ where: m = mass of one molecule , $\bar{v}^2$ = mean square speed k = Boltzmann constant T = absolute temperature Hence, Average kinetic energy $\propto T$

Root Mean Square (RMS) Speed The RMS speed of gas molecules is defined as: "The square root of the mean of the squares of the speeds of all molecules." Mathematically, $v_{rms} = \sqrt{\bar{v}^2}$

Derivation of RMS Speed From kinetic theory of gases: $PV = \frac{1}{3}Nm\bar{v}^2$ where: P = pressure , V = volume N = number of molecules m = mass of one molecule Using ideal gas equation: $PV = NkT$, Equating both expressions: $NkT = \frac{1}{3}Nm\bar{v}^2$, $kT = \frac{1}{3}m\bar{v}^2$ Thus, $\bar{v}^2 = \frac{3kT}{m}$

Taking square root: $v_{rms} = \sqrt{\frac{3kT}{m}}$ **Expression in Terms of Molar Mass** Since $m = \frac{M}{N_A}$

and $R = kN_A$ Substituting: $v_{rms} = \sqrt{\frac{3RT}{M}}$ where: R = universal gas constant M = molar

mass of gas The RMS speed of molecules of an ideal gas is: $v_{rms} = \sqrt{\frac{3kT}{m}}$

or $v_{rms} = \sqrt{\frac{3RT}{M}}$

CHAPTER 12

KINETIC THEORY OF GASES

CBSE
CLASS 11
PHYSICS



FORMULA SHEET

★ Big Ideas. Smart Formulas. Top Scores! ★

1 IDEAL GAS EQUATIONS

$$PV = nRT$$

P = Pressure
 V = Volume
 n = Number of moles
 T = Temperature (K)
 $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$

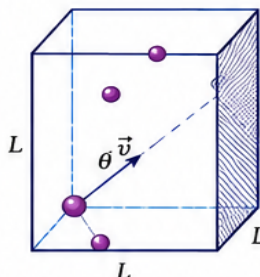
In terms of Boltzmann constant (k_B):

$$PV = Nk_B T$$

$$\text{or } P = nk_B T$$

$$k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

2 PRESSURE OF A GAS



Pressure is due to molecular collisions with the walls.

$$P = \frac{1}{3} \rho v_{rms}^2$$

where ρ = density of gas

v_{rms} = root mean square speed



QUICK RECALL

- ✓ Gas molecules move randomly in all directions.
- ✓ Collisions are perfectly elastic.
- ✓ No intermolecular forces.
- ✓ Pressure $\propto$ number of collisions & their momentum change.
- ✓ Temperature $\propto$ average kinetic energy.

3 RMS SPEED OF GAS MOLECULES

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
 T = Temperature (K)
 M = Molar mass (kg mol^{-1})

FACTS

- ✓ v_{rms} is of the order of km/s.
- ✓ $v_{rms} \approx 3 \times$ (speed of sound in gas)
- ✓ For constant T , v_{rms} is independent of P .
- ✓ Higher $T \rightarrow$ Higher v_{rms} .



4 KINETIC ENERGY OF IDEAL GAS

Quantity of Gas	Kinetic Energy
1 molecule	$\frac{1}{2} m v^2$
N molecules	$\frac{1}{2} N m v_{rms}^2$
1 mole (M g)	$\frac{3}{2} RT$
1 gram	$\frac{3}{2} \frac{R}{M} T$

m = mass of one molecule

M = molar mass (g mol^{-1})

N_A = Avogadro's number ($6.022 \times 10^{23} \text{ mol}^{-1}$)



MOST IMPORTANT

- Ideal Gas Equation: $PV = nRT$
- Pressure Formula: $P = \frac{1}{3} \rho v_{rms}^2$
- rms Speed: $v_{rms} = \sqrt{\frac{3RT}{M}}$
- Kinetic Energy (per mole): $\frac{3}{2} RT$
- Degrees of Freedom (f)
- Equipartition of Energy
- Mean Free Path (λ)
- Specific Heats (C_v, C_p)

5 DEGREES OF FREEDOM (f)

Type of Molecule	Motion	f
Monoatomic	3 Translational	3
Diatomic (Linear)	3 Translational + 2 Rotational	5
Non-linear Triatomic	3 Translational + 3 Rotational	6
Non-linear Polyatomic	3 Translational + 3 Rotational + $(3N-6)$ Vibrational	$3N-3$

General Formula: $f = 3N - K$

N = No. of independent particles,
 K = No. of independent constraints

6 LAW OF EQUIPARTITION OF ENERGY

Energy is equally shared among all degrees of freedom.

$$\text{Energy per degree of freedom} = \frac{1}{2} k_B T$$

$$\text{Total energy per molecule} = \frac{f}{2} k_B T$$

$$\text{Per mole} = \frac{f}{2} RT$$



TRICK More $f \rightarrow$ More internal energy
 $\rightarrow$ Higher specific heat!

7 MEAN FREE PATH (λ)

Average distance travelled by a molecule between two successive collisions.

$$\lambda = \frac{1}{\sqrt{2} \pi d^2 n}$$

d = diameter of molecule

n = number density (No. of molecules per unit volume)

TIP

Higher pressure (or density) $\rightarrow$ More collisions
 $\rightarrow$ Smaller mean free path

8 SPECIFIC HEAT CAPACITIES OF IDEAL GASES

Gas	f	C_v	C_p	$C_p - C_v$
Monoatomic	3	$3R/2$	$5R/2$	R
Diatomic (Linear)	5	$5R/2$	$7R/2$	R
Non-linear Triatomic	6	$3R$	$4R$	R

Remember

$$C_p - C_v = R$$

(For all ideal gases)

Units

$$C_v, C_p \rightarrow \text{J mol}^{-1} \text{ K}^{-1}$$

$$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$$

SHORT TRICKS

Remember
 v_{rms}
Square root of
($3RT/M$)
Easy to recall!

Pressure Trick
 $P \propto \rho$ and $\propto v_{rms}^2$
(Double density
or double rms speed
 $\rightarrow$ Double pressure)

Specific Heat Trick
 $f \uparrow \rightarrow C_v \uparrow \rightarrow C_p \uparrow$
(Always increasing
with degrees of
freedom)

λ Trick
Pressure $\uparrow \rightarrow$
Density $\uparrow \rightarrow$
Mean free path $\lambda \downarrow$

9 PYQ TIPS (CBSE)

- ✓ Derivations: Pressure of gas, RMS speed, Mean free path.
- ✓ Concept Based: Effect of temperature on rms speed, dependence of pressure.
- ✓ Numericals: Ideal gas equation, Kinetic energy, Specific heats.
- ✓ High Weightage Topics: Pressure derivation, degrees of freedom, Equipartition law.

EXAM STRATEGY

- Write formula first, then apply.
- Draw neat diagrams for pressure derivation.
- Mention units.
- Check calculations & significant figures.

“ Understand molecules today,
Score full marks tomorrow! ”



CH – 12 KINETIC THEORY OF GASES

<u>FORMULA SHEET</u>	https://drive.google.com/file/d/1-5CIBZhPUu8Y_WAbts4xCXadRsEn-0MI/view?usp=sharing	
<u>NOTES</u>	https://docs.google.com/document/d/1NSl9ZntnDk_R4rr_qf3yELJ3sGMRJ14yd/edit?usp=sharing&ouid=107220755287004713439&rtpof=true&sd=true	
<u>VIDEO BOOK</u>	https://drive.google.com/file/d/1TxCOEApQ-a7BrFsnZCUco-aSgTC1JZAC/view?usp=sharing	
<u>AUDIO BOOK</u>	https://drive.google.com/file/d/1qo6QKfasZxRSVwjLW_pK29zvrTnxgV7j/view?usp=sharing	
<u>STUDY MATERIAL</u>	https://drive.google.com/file/d/13liRS87fUYSiFS4q43ahhk_sqFYGbbV1_/view?usp=sharing	
<u>QUIZ</u>	https://wayground.com/join?gc=38204465&source=liveDashboard	

CHAPTER-13



1. PERIODIC MOTION – TIME PERIOD, FREQUENCY, DISPLACEMENT AS A FUNCTION OF TIME

A motion is periodic if it repeats itself after regular intervals of time.

Time Period (T)

Time taken to complete one full oscillation.

Unit: second (s)

Frequency (f)

Number of oscillations per second.

$$f = \frac{1}{T}$$

Unit: hertz (Hz)

Displacement as a function of time (for S.H.M.)

$$x(t) = A \sin(\omega t + \phi)$$

where,

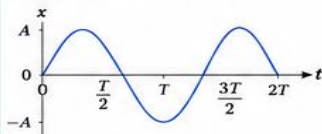
$x(t)$ = displacement at time t

A = amplitude (maximum displacement)

ω = angular frequency = $2\pi f$

ϕ = initial phase

Graph of x vs t



Common Periodic Functions

- Sine function: $y = \sin x$
- Cosine function: $y = \cos x$
- Tangent function: $y = \tan x$

Applications

- Alternating current (AC)
- Sound and music
- Vibrations in mechanical systems
- Electrical circuits (LC, RLC circuits)
- Signals and waveforms in electronics

2. SIMPLE HARMONIC MOTION (S.H.M), UNIFORM CIRCULAR MOTION AND ITS EQUATIONS OF MOTION; PHASE

Simple Harmonic Motion (S.H.M.)

A motion in which the restoring force (or acceleration) is directly proportional to the displacement from the mean position and is directed towards it.

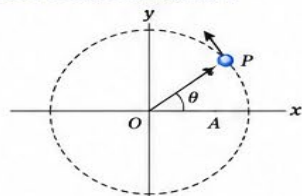
Mathematical form:

$$a = -\omega^2 x \quad (\text{or } F = -kx)$$

where, ω = angular frequency,
 k = force constant.

Uniform Circular Motion (U.C.M.)

A particle moves along a circular path of radius A with constant speed v .



Relation with S.H.M. (Projection on x-axis)

The projection of U.C.M. on any diameter results in S.H.M.

$$x = A \cos \theta$$

Equations of Motion

Displacement: $x = A \cos(\omega t + \phi)$

Velocity: $v = -A\omega \sin(\omega t + \phi)$

Acceleration: $a = -A\omega^2 \cos(\omega t + \phi)$

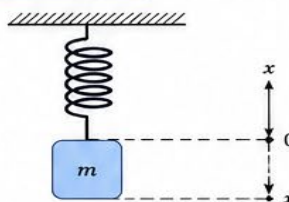
Phase (ϕ)

The phase determines the state of oscillation at any time t .

- Initial phase ϕ is the phase at $t = 0$
- Phase at time t : $(\omega t + \phi)$

3. OSCILLATIONS OF A LOADED SPRING – RESTORING FORCE AND FORCE CONSTANT; ENERGY IN S.H.M. (KINETIC AND POTENTIAL ENERGIES)

Loaded Spring



(a) Restoring Force

When the mass is displaced by x from the equilibrium position, the spring exerts a restoring force towards the mean position.

$$F = -kx$$

where, k = force constant (N m^{-1})

(b) Force Constant (k)

From Hooke's law, $F = -kx$

$$k = \frac{F}{x}$$

Unit: N m^{-1}

(c) Energy in S.H.M.

Total mechanical energy (E) is constant.

Potential Energy (U)

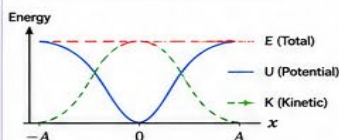
$$U = \frac{1}{2} kx^2$$

Kinetic Energy (K)

$$K = \frac{1}{2} mv^2 = \frac{1}{2} k(A^2 - x^2)$$

Total Energy (E)

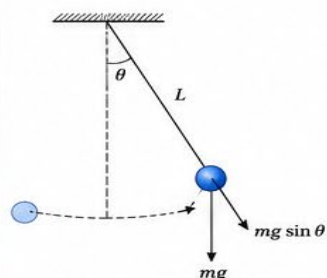
$$E = K + U = \frac{1}{2} kA^2$$



4. SIMPLE PENDULUM – DERIVATION OF EXPRESSION FOR ITS TIME PERIOD

Simple Pendulum

A small bob of mass m suspended by a light, inextensible string of length L .



Derivation (for small angles)

Restoring force (tangential component)

$$F_t = -mg \sin \theta$$

For small angles, $\sin \theta \approx \theta$ (in radians)

$$F_t \approx -mg\theta$$

Linear displacement along arc,

$$x = L\theta$$

$$F_t = -mg \left(\frac{x}{L} \right) = -\frac{mg}{L} x$$

Comparing with $F = -kx$,

$$k = \frac{mg}{L}$$

Time period of a simple pendulum

$$T = 2\pi \sqrt{\frac{L}{g}}$$

where, L = length of pendulum (m)

g = acceleration due to gravity (m s^{-2})

(Valid for small angular displacements)

KEY FORMULAS AT A GLANCE

$$f = \frac{1}{T}, \quad \omega = 2\pi f = \frac{2\pi}{T}$$

$$x(t) = A \sin(\omega t + \phi)$$

$$a = -\omega^2 x, \quad F = -kx$$

$$v = -A\omega \sin(\omega t + \phi)$$

$$a = -A\omega^2 \cos(\omega t + \phi)$$

$$U = \frac{1}{2} kx^2, \quad K = \frac{1}{2} k(A^2 - x^2)$$

$$E = \frac{1}{2} kA^2$$

$$T_{\text{pendulum}} = 2\pi \sqrt{\frac{L}{g}}$$

S.H.M. AT IMPORTANT POSITIONS

Position (x)	v	a	U	K
$x = +A$	0	$-\omega^2 A$	$\frac{1}{2} kA^2$	0
$x = 0$	$A\omega$	0	0	$\frac{1}{2} kA^2$
$x = -A$	0	$+\omega^2 A$	$\frac{1}{2} kA^2$	0
Any x	$\sqrt{\omega^2(A^2 - x^2)}$	$-\omega^2 x$	$\frac{1}{2} kx^2$	$\frac{1}{2} k(A^2 - x^2)$

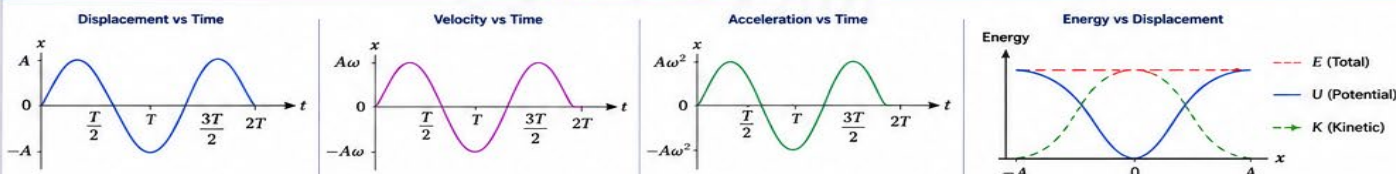
UNIT SUMMARY

- Time (T): second (s)
- Frequency (f): hertz (Hz)
- Angular frequency (ω): rad s^{-1}
- Displacement (x): metre (m)
- Velocity (v): metre per second (m s^{-1})
- Acceleration (a): metre per second squared (m s^{-2})
- Force constant (k): newton per metre (N m^{-1})
- Energy: joule (J)

CONCEPT CHECK

- What is the relation between frequency and time period?
- Why does a restoring force in S.H.M. always point towards the mean position?
- How is S.H.M. related to uniform circular motion?
- Why is $\sin \theta \approx \theta$ used in derivation of the pendulum time period?
- How do kinetic and potential energies vary in S.H.M.?

COMMON GRAPHICAL REPRESENTATIONS



MULTIPLE CHOICE QUESTIONS

- Which of the following examples represents periodic motion but not oscillatory motion?
(a) The motion of a simple pendulum
(b) The rotation of Earth about its axis
(c) A wave traveling along a string
(d) The motion of a mass attached to a spring
- For a particle executing Simple Harmonic Motion (SHM), what is the phase difference between its displacement and velocity?
(a) 0 radians (b) $\pi/4$ radians (c) $\pi/2$ radians (d) π radians
- A particle executes SHM with an amplitude A . At what displacement from the mean position is its kinetic energy equal to its potential energy?
(a) $A/2$ (b) $A/\sqrt{2}$ (c) $A/\sqrt{3}$ (d) $A/2\sqrt{2}$
- The total energy of a particle executing Simple Harmonic Motion is directly proportional to:
(a) The frequency of oscillation (b) The square of the amplitude
(c) The displacement from the mean position (d) The square root of the mass
- If the length of a simple pendulum is increased by 44%, what is the percentage increase in its time period?
(a) 20% (b) 22% (c) 44% (d) 12%
- Which of the following examples is not a simple harmonic motion?
(a) Vertical oscillation of a spring-block system.
(b) Motion of a simple pendulum with large angle.
(c) Motion of a particle moving in a straight line with $a = -2x$.
(d) Oscillations of a liquid in a U-tube.
- The acceleration of a particle executing SHM is given by $a = -16x$, where x is the displacement in meters. What is the angular frequency of the motion?
(a) 16 rad/s (b) 8 rad/s (c) 4 rad/s (d) 2 rad/s
- The nature of the acceleration vs displacement graph for a particle is executing S.H.M.
(a) A straight-line (b) A circle (c) An ellipse (d) A hyperbola
- The change (or changes) observed when the period of a simple pendulum is doubled is
(a) Its length is doubled (b) The mass of the bob is doubled
(c) Its length is made four times
(d) The mass of the bob and the length of the pendulum are doubled
- The period of oscillation of a simple pendulum of constant length at earth's surface is T . Its period inside a mine is
(a) Greater than T (b) Less than T (c) Equal to T (d) Cannot be compared
- The total energy of a particle executing S.H.M. is proportional to
(a) Displacement from equilibrium position (b) Frequency of oscillation
(c) Velocity in equilibrium position (d) Square of amplitude of motion
- When a damping force acts on a vibrating system, what happens to the amplitude of the oscillations over time?
(a) It increases linearly (b) It decreases exponentially
(c) It remains constant (d) It fluctuates randomly
- Resonance occurs when the frequency of the external periodic force is:
(a) Much higher than the natural frequency of the system
(b) Much lower than the natural frequency of the system
(c) Equal to the natural frequency of the system
(d) Completely independent of the system
- A particle executes simple harmonic motion along a straight line with an amplitude A . The potential energy is maximum when the displacement is
(a) $\pm A$ (b) Zero (c) $\pm A/2$ (d) $\pm A/\sqrt{2}$

15. A particle is vibrating in a simple harmonic motion with an amplitude of 4 cm. At what displacement from the equilibrium position, is its energy half potential and half kinetic
 (a) 1 cm (b) $2/\sqrt{2}$ cm (c) 3 cm (d) $2\sqrt{2}$ cm

MULTIPLE CHOICE QUESTION: ANSWER KEY

- (b) Periodic motion repeats at regular intervals. Oscillations require a back-and-forth Movement about a mean position. The rotation of Earth repeats every 24 hours (periodic) but never reverses its direction (not oscillatory). Options A and D represent both periodic and oscillatory motion. Option C represents wave propagation.
- (b) The displacement equation is ($x = A \sin \omega t$). Differentiating gives the velocity equation ($v = A \omega \cos \omega t$) = $A \omega \sin(\omega t + \pi/2)$. The phase angle difference between the sine and cosine functions is $\pi/2$ radians.
- (a) Kinetic Energy (KE) equals Potential Energy (PE). Therefore,
 $\frac{1}{2} k (A^2 - x^2) = \frac{1}{2} k x^2$. Simplifying this gives $A^2 - x^2 = x^2$ $2x^2 = A^2$
 $x = A/\sqrt{2}$.
- (b) The total energy (E) of a simple harmonic oscillator is given by the formula ($E = \frac{1}{2} m \omega^2 A^2$). This shows that the total energy is directly proportional to the square of the amplitude A^2
- (a) The time period is ($T = 2\pi \sqrt{l/g}$), meaning $T \propto \sqrt{l}$ If the new length is ($l' = 1.44l$), the new time period becomes ($T' = \sqrt{1.44}$) , $T = 1.2T$. The increase is ($1.2T - T = 0.2T$), which is equal to a 20% increase.
- (b) Motion of a simple pendulum with large angle.
 SHM requires a small angle ($\sin \theta \approx \theta$) Large angle motions are periodic but not simple harmonic
- (c) The standard equation for acceleration in SHM is ($a = -\omega^2 x$). Comparing this to the given equation ($a = -16x$) shows that ($\omega^2 = 16$). Taking the square root gives an angular frequency = 4 rad/sec .
- (a) The graph of acceleration vs. displacement for a particle in S.H.M. is a straight line with negative slope — indicating that acceleration is always directed opposite to displacement.
- (c) The time period T of a simple pendulum is given by: $T = 2\pi \sqrt{\frac{l}{g}}$ -----(1)
 To make the time period 2T, let the length be changed to l' . Then , $2T = 2\pi \sqrt{\frac{l'}{g}}$ ---(2)
 From (1) and (2), $2 = \sqrt{l'/l}$ (or) $l' = 4l$
- (a) $T \propto 1/\sqrt{g}$. If g increases, T decreases. If g decreases (like on the Moon or inside a mine), T increases.
- (d) The total energy E in S.H.M. is the sum of kinetic and potential energy, and is constant for a given motion: $E = \frac{1}{2} m \omega^2 A^2$; $E \propto A^2$
- (c) Resonance occurs when the driving frequency matches the natural frequency of the oscillator. This state enables maximum energy transfer, causing the system to oscillate with its peak possible amplitude.
- (c) When two perpendicular or out-of-phase harmonic motions combine as ($x = a \sin \omega t + b \cos \omega t$), the resulting amplitude is calculated using the vector sum formula ($A = \sqrt{a^2 + b^2}$). Here $A = 25$.
- (a) The potential energy (P.E.) in S.H.M. at any displacement x is:
 $P.E. = \frac{1}{2} k x^2$. So, P.E. is maximum when x is maximum, i.e., when $x = \pm A$

15. (d) Kinetic energy, $K = \frac{1}{2}m\omega^2(a^2 - x^2)$ Potential energy, $U = \frac{1}{2}m\omega^2x^2$
 Given, $KE = PE$ $\frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{2}m\omega^2x^2$ $x = a/\sqrt{2} = 4/\sqrt{2} = 2\sqrt{2}$

CHALLENGE CORNER

- Which of the following quantities remains constant in simple harmonic motion?
 (a) Displacement (b) Velocity (c) Acceleration (d) Total Energy
- If a simple harmonic oscillator has got a displacement of 0.02 m and acceleration equal to 2.0 m/s² at any time, the angular frequency of the oscillator is equal to
 (a) 10 rad/s (b) 1 rad/s (c) 100 rad/s (d) 1 rad/s
- For a magnet of time period T, magnetic moment is M. If the magnetic moment becomes one fourth of the initial value, then the time period of oscillation becomes
 (a) Half of initial value (b) One fourth of initial value 248
 © Double of initial value (d) Four time initial value
- A particle executes SHM with amplitude **8 cm** and time period **4 s**. The maximum velocity of the particle is:
 (a) 4π cm/s (b) 8π cm/s (c) 6π cm/s (d) 32π cm/s
- A body oscillates between points **A and B**, 20 cm apart. The time taken to move from A to B is **0.5 s**. The time period is:
 (a) 0.5 s (b) 1 s (c) 2 s (d) 4 s

ANSWER KEY OF CHALLENGE CORNER

1. d 2. A 3. C 4. A 5. B

ASSERTION REASON TYPE QUESTIONS

- (a) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are true, but Reason is NOT the correct explanation of Assertion.
 (c) Assertion is true, but Reason is false.
 (d) Assertion is false, but Reason is true.

- Assertion (A):** Every periodic motion is not necessarily a simple harmonic motion.
Reason (R): Simple harmonic motion requires the restoring force to be directly proportional to the displacement.
- Assertion (A):** In Simple Harmonic Motion (SHM), the velocity of the particle is maximum at the mean position.
Reason (R): The acceleration of a particle performing SHM is zero at the mean position.
- Assertion (A):** The total energy of an undamped spring-mass system performing SHM remains constant.
Reason (R): Kinetic energy and potential energy transform into each other continuously during the oscillation.
- Assertion (A):** If a pendulum clock is taken to the moon, it runs slower.
Reason (R): The acceleration due to gravity on the moon is less than that on Earth.
- Assertion (A):** If the amplitude of a simple harmonic oscillator is doubled, its total energy becomes four times.
Reason (R): Total energy of a simple harmonic oscillator is directly proportional to the square of its amplitude.
- Assertion (A):** Liquid in a U-tube executing vertical oscillations performs SHM.
Reason (R): The restoring force on the liquid column is directly proportional to the height of the liquid level displacement.
- Assertion (A):** Resonance occurs when the frequency of the external periodic force matches the natural frequency of the oscillator.
Reason (R): At resonance, the amplitude of oscillation becomes exceptionally small.

8. **Assertion (A):** For a particle executing SHM, the graph between potential energy and displacement is a parabola.
Reason (R): Potential energy in SHM varies linearly with the displacement from the mean position.
9. **Assertion (A):** The time period of a simple pendulum inside a freely falling lift is infinite.
Reason (R): The effective acceleration due to gravity inside a freely falling lift is zero.
10. **Assertion (A):** When two identical springs are connected in parallel, the effective spring constant doubles.
Reason (R): In a parallel combination, both springs experience the same stretching force.

ASSERTION REASON TYPE: ANSWER KEY

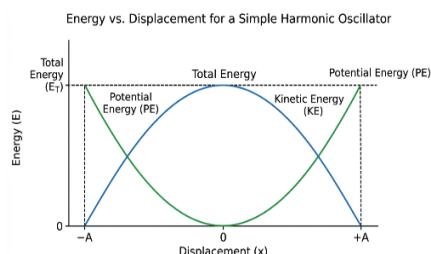
1. (a) 2. (b) 3. (b) 4. (a) 5. (a) 6. (a) 7. (c) 8. (c) 9.(a) 10.(c)

VERY SHORT ANSWER QUESTIONS (2 MARKS)

- Show that for a particle executing Simple Harmonic Motion (SHM), the acceleration is directly proportional to its displacement but directed opposite to it.
- A particle executes SHM with an amplitude (A). At what displacement from the mean position are its kinetic energy and potential energy equal?
- Plot a clean, labelled graph showing the variation of Kinetic Energy (KE) and Potential Energy (PE) of a simple harmonic oscillator as a function of its displacement.
- A linear harmonic oscillator has a total energy of (E). What will be its kinetic energy and potential energy when its displacement is half of its amplitude ($x = A/2$)?
- What is the fundamental difference between damped oscillations and forced oscillations? Mention one physical example of each.
- Explain why the time period of a simple pendulum becomes infinite if it is placed inside a geostationary satellite.
- What is the phase difference between:
 - (a) Displacement and Velocity of a particle in SHM?
 - (b) Velocity and Acceleration of a particle in SHM?
- A simple pendulum has a hollow bob filled with mercury. If a small hole at the bottom lets the mercury drain out slowly, how does the time period change?
- The maximum velocity of a particle executing SHM is (v_0) and its maximum acceleration is (a_0). Find the amplitude of its motion in terms of (v_0) and (a_0).
- How do damping forces affect the amplitude and the time period of a real-world harmonic oscillator over time?

ANSWER KEY

- Correct derivation using ($a = d^2x/dt^2 = -\omega^2 x$)
 Explanation of the negative sign indicating direction towards the mean position.
- Setting $1/2 m \omega^2 (A^2 - x^2) = 1/2 m \omega^2 x^2$, Solving to find x
-



4. Calculating ($PE = 1/2 m \omega^2 (A/2)^2 = E/4$) Calculating ($KE = E - PE = 3E/4$)

5. Definition pointing out energy loss due to resistive forces vs. energy supplied by external periodic force. Correct examples (e.g., oscillating pendulum in water vs. a swing being pushed continuously).
6. Explaining that a satellite is in a state of free fall, making effective gravity ($g_{\text{eff}} = 0$), Substituting into ($T = 2\pi\sqrt{\frac{L}{g}}$).
7. (a) $\pi/2$ radian (b) $\pi/2$ radian
8. Explaining that the centre of gravity first shifts down (increasing l and (T)), then moves back up to the centre of the hollow shell once empty (decreasing (T) to original), Stating the sequence: first increases, then decreases.
9. Using formulas ($v_0 = \omega A$) and ($a_0 = \omega^2 A$)
Eliminating ω by calculating ($A = v_0^2 / a_0$)
10. Damping forces, such as friction or air resistance, continuously dissipate mechanical energy from the system. This causes the amplitude of the oscillations to decrease exponentially over time until the motion stops entirely. Additionally, damping slightly reduces the frequency, which causes the time period of a damped oscillator to be marginally longer than that of an ideal, undamped oscillator.

CHALLENGE CORNER

1. A particle executing Simple Harmonic Motion (SHM) has a velocity of 8 cm when its displacement is 3cm, and a velocity of 6cm/s when its displacement is 4cm. Find the amplitude of its oscillation. 5cm
2. A particle begins executing SHM from its mean equilibrium position at $t = 0$ with a time period T . Find the shortest time taken by the particle for its kinetic energy to become exactly equal to its potential energy. $t/8\text{sec}$
3. The equation of a particle executing SHM is given by ($x=5\sin 2\pi t + \pi/4$) where (x) is in meters and (t) is in seconds. Calculate the initial displacement of the particle at ($t = 0$) and the time when it first crosses its mean equilibrium position. .375sec

ANSWER KEY OF CHALLENGE CORNER

1. $a = 5\text{cm}$ 2. $t = T/8\text{sec}$ 3. $t = .375\text{sec}$

SHORT ANSWER QUESTIONS (3 MARKS)

1. Find the expression for the total energy of a particle executing S.H.M.
2. A body is executing S.H.M of amplitude 1 m. Its velocity while passing through the mean position, is 10ms^{-1} . Find its frequency.
3. The kinetic energy of a particle vibrating in S.H.M. is 4 J when it passes the mean position. If the mass of the body is 2 kg and the amplitude is 1 m, calculate its time period.
4. Two particles execute SHM of the same amplitude and frequency along the same straight line. They pass each other moving in opposite directions whenever their displacement is half of their amplitude. Find the phase difference between them.
5. A bungee jumper oscillates up and down after the initial drop. Explain why this motion is considered Simple Harmonic Motion (SHM). State the condition required for it to remain strictly SHM.
6. A harmonic oscillation is represented by $y = 0.24 \cos (6000 t + 0.44)$, where y and t are in m and s respectively. Deduce: (i) the amplitude, (ii) the frequency and angular frequency, (iii) the period and (iv) the initial phase.
7. At what displacement from the mean position is the kinetic energy of a particle executing SHM exactly three times its potential energy? Express your answer in terms of its amplitude A .
8. A block of mass (0.5 kg) is attached to two identical springs connected in parallel, each having a spring constant of (100 N/m). Calculate the effective spring constant of the system and the frequency of oscillation.

9. A point object executes SHM with a time period of π seconds. If its maximum velocity is (4 m/s), calculate the amplitude of oscillation and the magnitude of its acceleration at a displacement of (1 m) from the mean position.
10. Military troops are strictly ordered to break their uniform marching steps when crossing a suspension bridge. Explain the physical phenomenon behind this instruction and what would happen if they ignored it.

ANSWER KEY

1. Ans. P.E. with a S.H.M. $\frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 x^2$
 kinetic energy with SHM $= \frac{1}{2} mv^2 = \frac{1}{2} m(\omega\sqrt{A^2 - x^2})^2$
 $= \frac{1}{2} m\omega^2(A^2 - x^2)$
 where m is mass, A is amplitude, x is any position and ω is angular frequency
 Total energy $= \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m\omega^2(A^2 - x^2)$
 $\frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 A^2$
2. Here $a = 1$ m, $V_{\max} = 10$ m/s $V_{\max} = a\omega = 2\pi\nu a = \nu = v_{\max}/2\pi a$
 $= 1.592$ Hz
3. In its mean position, the kinetic energy of the particle is maximum.
 Given - $E_{K\max} = \frac{1}{2} m\omega^2 a^2$ $E_{K\max} = 4$ J; $m = 2$ kg; $A = 1$ m
 4 J $= \frac{1}{2} \times 2 \times \omega^2 \times 1$ $\omega = 2$, $T = 2\pi/\omega = \pi$ sec
4. Use the displacement equation: ($x = A \sin \omega t$).
 Given ($x = A/2$), so $A/2 = A \sin \theta$ $\sin \theta = 1/2$.
 The two possible angles are $\theta_1 = \pi/6$ $\theta_2 = 5\pi/6$
 Phase difference: $\theta_2 - \theta_1 = 5\pi/6 - \pi/6 = 4\pi/6$ or 120° .
5. The elastic bungee cord acts like a spring.
 It exerts a restoring force proportional to the displacement from the equilibrium position ($F = -kx$). The motion is strictly SHM only if the displacement remains within the elastic limit of the cord.
6. Ans. Given that, $y = 0.24 \cos(6000t + 0.44)$ While the general expression for displacement is, $y = a \cos(\omega t + \Phi_0)$ Comparing these two expressions,
 (i) Amplitude, $a = 0.24$ m. (ii) Angular frequency $\omega = 6000$ radian sec⁻¹
 Frequency $= \nu = \omega/2\pi = 30000/\pi$ iii) Period $T = 1/\omega = \pi/30000$ iii) Initial phase
 $\Phi_0 = .44$
7. Given Condition: $KE = 3PE$.
 Formulas: $\frac{1}{2} m\omega^2(A^2 - x^2) = 3 \times \frac{1}{2} m\omega^2 x^2$
 Simplification: $(A^2 - x^2) = 3x^2$ $A^2 = 4x^2$ $x^2 = A^2/4$
8. Parallel Formula: $K_p = k_1 + k_2 = 100 + 100 = 200$ N/m.
 Frequency Formula: $\mu = \frac{1}{2} \pi \sqrt{\frac{K_p}{M}}$.
 Calculation: $\mu = \frac{1}{2} \pi \sqrt{\frac{200}{0.5}} = 3.18$ Hz
9. Angular Frequency: $\omega = 2\pi/T$ $\omega = 2\pi/\pi = 2$ rad/s.
 Amplitude: ($V_{\max}$) $= A\omega$ $4 = A \times 2$ $A = 2$ meters.
 Acceleration: $a = \omega^2 x = (2)^2 \times 1 = 4$ m/s².
10. **Phenomenon:** This instruction is based on the phenomenon of **mechanical resonance**. Every physical structure, including a suspension bridge, has its own natural frequency of oscillation.
Mechanism: If soldiers march in unison, their periodic footsteps exert a periodic driving force on the bridge. If the frequency of these steps matches the natural frequency of the bridge, resonance occurs.
Consequence: Resonance causes the amplitude of the bridge's oscillations to increase drastically. This structural stress can exceed design limits, leading to the collapse of the bridge.

CHALLENGE CORNER

1. A particle executing Simple Harmonic Motion (SHM) has an angular frequency $\omega = 4\text{ rad/s}$. When it is at a distance of 3cm from the mean equilibrium position, the magnitude of its velocity is exactly equal to the magnitude of its acceleration.
(a) Determine the amplitude of the oscillation.
(b) Find the maximum velocity of the particle.
2. A point object executes SHM with a time period $T = 12\text{ seconds}$ and an amplitude $A = 10\text{ cm}$. Assuming it starts from its mean position at $t = 0$:
Find the time taken to reach a displacement of $x = 5\text{ cm}$.
Find the time taken to travel from $x = 5\text{ cm}$ to $x = 5\sqrt{3}\text{ cm}$.
2. A body of mass 0.5 kg executes SHM with a frequency of $10/\pi\text{ Hz}$. If its kinetic energy is 0.18 J at a displacement of $x = 4\text{ cm}$ from the mean position, calculate:
The angular frequency of the oscillator.
The spring constant (force constant) of the system.
The amplitude of the oscillation.

ANSWER KEY OF CHALLENGE CORNER

1. $a = 12.37\text{ cm}$ $V_{\text{max}} = 49.48\text{ cm/sec}$
2. $\Delta T = 1\text{ sec}$
3. $a = 5.83\text{ cm}$

CASE BASE STUDY

1. ENERGY DYNAMICS IN A HORIZONTAL SPRING-MASS SYSTEM

An ideal horizontal spring-mass system consists of a block of mass m attached to a spring of force constant k , resting on a frictionless surface. When the block is pulled to a displacement A and released, it executes **Simple Harmonic Motion (SHM)**.

During this motion, the total mechanical energy (E) is conserved, shifting between **Elastic Potential Energy** ($U = \frac{1}{2} kx^2$) and **Kinetic Energy** ($K = \frac{1}{2} mv^2$). The velocity of the block at any displacement x is given by $v = \omega^2(A^2 - x^2)$

1. At which position does the oscillating block possess entirely Kinetic Energy?
(a) At the extreme position ($x = A$) (b) At the mean position ($x = 0$)
(c) At the point where $x = A/2$ (d) At the point where $x = A\sqrt{2}$
2. If the displacement of the block is exactly half of its amplitude ($x = A/2$), the ratio of its Potential Energy to its Total Energy U/E is:
(a) 1:2 (b) 1:4 (c) 3:4 (D) 1:1
3. A block of mass 1 kg is attached to a spring with $k = 100\text{ N/m}$. If the amplitude is 0.1 m , what is the maximum Kinetic Energy of the system?
(a) 0.5 J (b) 1.0 J (c) 0.05 J (d) 5.0 J
4. If the spring is replaced with one having double the spring constant $2k$ while keeping the same amplitude, the total energy of the system will:
(a) Remain unchanged (b) Become half (c) Double (d) Become four times

2. SHOCK ABSORBERS IN AUTOMOBILES

When a car hits a bump, the springs in its suspension system are compressed. Without any control, these springs would continue to vibrate up and down for a long time, making the ride uncomfortable and the car difficult to handle. To prevent this, engineers use **Shock Absorbers** (dampers).

A shock absorber is essentially a piston moving inside a cylinder filled with oil. As the car oscillates, the piston forces the oil through tiny holes. This creates a **viscous damping force** ($F_d = -bv$) that opposes the motion and converts the mechanical energy of the vibration into heat. This ensures the car returns to its equilibrium position quickly.

1. A car's suspension is designed to return to its original position in the shortest possible time without any further oscillation. This state of damping is known as:
(a) Underdamping (b) Overdamping (c) Critical damping (d) Zero damping
2. As the oil in a shock absorber heats up during a long drive on a bumpy road, its viscosity decreases (it becomes "thinner"). How will this affect the damping constant b and the vibrations?

- (a) b increases; vibrations stop faster.
- (b) b decreases; vibrations persist for a longer time.
- (c) b remains constant; no change in vibrations.
- (d) b increases; the car becomes "bouncier."

3. In a damped oscillation, the amplitude of the car's vibration decreases over time. Which of the following equations best represents the decay of amplitude A ?

- (a) $A = A_0 \sin \omega t$ (b) $A = A_0 e^{-bt/2m}$ (c) $A = A_0 + kt$ (d) $A = A_0/kt$

4. Why do military troops "break step" (stop marching in rhythm) when crossing a bridge?

- (d) To increase the damping of the bridge.
- (d) To prevent the frequency of their steps from matching the natural frequency of the bridge (Resonance).
- (d) To reduce the total mass acting on the bridge.
- (d) To decrease the acceleration due to gravity on the bridge.

ANSWER KEY

- 1. 1.b 2.b 3. c 4. c
- 2. 1.c 2.b 3. b 4. b

CHALLENGE CORNER

1. A group of soldiers is marching in step across a long suspension bridge. The rhythmic marching causes the bridge to begin oscillating slightly. The bridge is modelled as a **large mass-spring system**, with a natural frequency of **2 Hz**. Each soldier exerts a force at a frequency of **2 Hz**, in sync with the natural frequency of the bridge. This creates **resonance**.

The oscillations gradually increase in amplitude until the soldiers are ordered to break step and walk randomly to stop the resonance.

1. What kind of oscillation is exhibited by the bridge when soldiers march in step at its natural frequency?

- (a). Free oscillation (b). Damped oscillation (c). Forced oscillation (d). Resonant oscillation

2. What will happen to the amplitude of the bridge's oscillations if resonance continues unchecked?

- (a). It decreases (b). It remains constant
- (c). It increases gradually (d). It becomes zero

3. Why are soldiers asked to "break step" while crossing bridges?

- (a). To increase the time period
- (b). To decrease the time period
- (c). To avoid synchronization with the bridge's natural frequency
- (d). To synchronize with the bridge's natural frequency

4. If the bridge's mass is doubled, what happens to its natural frequency?

- (a). It remains the same (b). It increases (c). It decreases (d). It becomes infinite

2. In an ancient temple, a large pendulum clock has been swinging for decades. The pendulum consists of a metal bob of mass 2 kg suspended by a string 1.5 m long. The pendulum swings with a small amplitude, undergoing simple harmonic motion (SHM). A devotee notices that the pendulum completes one full oscillation in about 2.46 seconds. The air resistance is negligible, and the pendulum is considered ideal.

1. What is the theoretical formula for the time period of this simple pendulum?

- (a). $T = 2\pi\sqrt{g/l}$ (b). $T = 2\pi\sqrt{l/g}$ (c). $T = 2\pi\sqrt{l \cdot g}$ (d). $T = \sqrt{2\pi \cdot g}$

2. Using the observed time period (2.46 s) and the length (1.5 m), estimate the local value of g near the temple.

- (a). A. 9.8 m/s² (b). 10.0 m/s² (c). C. 9.5 m/s² (d). D. 8.5 m/s²

3. What is the nature of the restoring force acting on the pendulum bob during SHM?

- (a). A. Constant force (b). Proportional to mass
 (c). C. Directly proportional to displacement and opposite in direction
 (d).. Always equal to tension
 4. What happens to the time period if the pendulum is taken to a mountain where g is lower?
 (a). A. It increases (b). It decreases (c). C. It remains the same (d). It becomes zero

ANSWER KEY OF CHALLENGE CORNER

1. 1.d 2.c 3. c 4. c
 2. 1.b 2.c 3. c 4. a

LONG ANSWER TYPE QUESTIONS

- The acceleration due to gravity on the surface of moon is 1.7 ms^{-2} . What is the time period of a simple pendulum on the surface of moon if its time-period on the surface of Earth is 3.5 s? (g on the surface of Earth is 9.8 ms^{-2} .)
- Show that for a particle in linear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.
- (a) Define SHM. What are its characteristics? At what distance from the mean position in SHM of amplitude r the energy is half kinetic and half potential? (b) A spring having a force constant K is divided into three equal parts. What would be force constant for each individual parts?

ANSWER KEY

- Here, $g_m = 1.7 \text{ ms}^{-2}$; $g_e = 9.8 \text{ ms}^{-2}$; $T_m = ?$; $T_e = 3.5 \text{ s}^{-1}$
 Since, $T_e = 2\pi\sqrt{\frac{1}{g_e}}$ and $T_m = 2\pi\sqrt{\frac{1}{g_m}}$
 $\therefore \frac{T_m}{T_e} = \sqrt{\frac{g_e}{g_m}} \Rightarrow T_m = T_e \sqrt{\frac{g_e}{g_m}}$
 $= 3.5 \sqrt{\frac{9.8}{1.7}} = 8.4 \text{ s.}$

- Let the particle executing SHM starts oscillating from its mean position. Then displacement equation is

$$x = A \sin \omega t$$

$$\therefore \text{Particle velocity, } v = A\omega \cos \omega t$$

$$\therefore \text{Instantaneous K.E., } K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2 \cos^2 \omega t$$

$$\therefore \text{Average value of K.E. over one complete cycle}$$

$$\begin{aligned} K_{av} &= \frac{1}{T} \int_0^T \frac{1}{2} mA^2 \omega^2 \cos^2 \omega t \, dt = \frac{mA^2 \omega^2}{2T} \int_0^T \cos^2 \omega t \, dt \\ &= \frac{mA^2 \omega^2}{2T} \int_0^T \frac{(1 + \cos 2\omega t)}{2} \, dt \\ &= \frac{mA^2 \omega^2}{4T} \left[t + \frac{\sin 2\omega t}{2\omega} \right]_0^T \\ &= \frac{mA^2 \omega^2}{4T} \left[(T - 0) + \left(\frac{\sin 2\omega T - \sin 0}{2\omega} \right) \right] \\ &= \frac{1}{4} mA^2 \omega^2 \quad \dots(i) \end{aligned}$$

$$\text{Again instantaneous P.E., } U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$$

$$\therefore \text{Average value of P.E. over one complete cycle}$$

$$\begin{aligned} U_{av} &= \frac{1}{T} \int_0^T \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t \, dt = \frac{m\omega^2 A^2}{2T} \int_0^T \sin^2 \omega t \, dt \\ &= \frac{m\omega^2 A^2}{2T} \int_0^T \frac{(1 - \cos 2\omega t)}{2} \, dt \\ &= \frac{m\omega^2 A^2}{4T} \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T \\ &= \frac{m\omega^2 A^2}{4T} \left[(T - 0) - \left(\frac{\sin 2\omega T - \sin 0}{2\omega} \right) \right] \\ &= \frac{1}{4} m\omega^2 A^2 \quad \dots(ii) \end{aligned}$$

Simple comparison of (i) and (ii), shows that

$$K_{av} = U_{av} = \frac{1}{4} m\omega^2 A^2$$

3. (a) Simple harmonic motion is the projection of uniform circular motion on diameter of a circle of reference. Characteristics of SHM: (I) Displacement: The displacement of a particle executing SHM at an instant is defined as the distance of a particle from the mean position at that instant. (II) Amplitude: The maximum displacement on either side of the mean position is called the amplitude of the motion. (III) Velocity: The velocity of the particle executing SHM at any instant, is defined as the time rate of change of its displacement at that instant. (IV) Acceleration. The acceleration of the particle executing SHM at any instant is defined as the time rate of change of its velocity at that instant.

$$\frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 x^2 \text{ kinetic energy with SHM}$$

$$= \frac{1}{2} mv^2 = \frac{1}{2} m(\omega\sqrt{A^2 - x^2})^2$$

where m is mass, A is amplitude, x is any position and ω is angular frequency

$$PE = KE \quad \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} m\omega^2 x^2$$

$$x = \pm A \sqrt{2}$$

b) force constant $K = F/y$ spring is cut into three equal parts therefore
displacement = $y/3$ $k' = F / (y/3) = 3F/y = 3k$

PERIODIC MOTION & OSCILLATIONS FORMULA SHEET

★ MOST IMPORTANT



Quick Revision



High Score



Smart Study



Concept Clarity



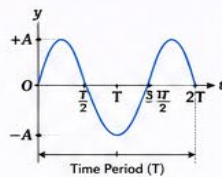
QUICK RECALL

1 PERIODIC MOTION

Time Period & Frequency

Time period (T) =
Time for one complete oscillation

Frequency (f) =
Number of oscillations
per second = $\frac{1}{T}$



Units

$T \rightarrow$ second (s)

$f \rightarrow$ hertz (Hz) = s^{-1}

Displacement as a function of time
(for SHM)

$$x(t) = A \sin(\omega t + \phi)$$

where A = amplitude, ω = angular frequency
 ϕ = initial phase

2 SIMPLE HARMONIC MOTION (S.H.M.)

★ MOST IMPORTANT

Key Features

- Restoring force $\propto$ displacement and opposite in direction.

$$F = -kx$$

Equation of Motion

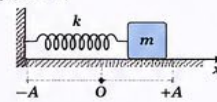
$$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0 \quad \text{or} \quad \frac{d^2x}{dt^2} + \omega^2x = 0$$

Where,

$$\omega = \sqrt{\frac{k}{m}} = \text{angular frequency (rad s}^{-1}\text{)}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$



SHM Conditions

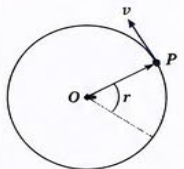
- Force must be directly proportional to displacement.
- Force must act towards mean position.
- System must have constant mass.
- No energy loss (ideal system).

- Periodic motion repeats after equal intervals of time.
- S.H.M. is a special type of periodic motion with restoring force.
- Energy in SHM remains constant.
- Time period (T) = time taken for one complete oscillation.
- Frequency (f) = number of oscillations per second.

SYMBOLS & UNITS

Symbol	Meaning	Unit
x	Displacement	m
A	Amplitude	m
T	Time period	s
f	Frequency	Hz
ω	Angular frequency	rad s ⁻¹
k	Force constant	N m ⁻¹
m	Mass	kg
F	Restoring force	N
E	Mechanical energy	J
g	Acceleration due to gravity	m s ⁻²
l	Length of pendulum	m

3 UNIFORM CIRCULAR MOTION (U.C.M.)



- Linear speed (v) = ωr
- Angular speed (ω) = $\frac{d\theta}{dt}$
- Angular displacement,
 $\theta = \omega t + \theta_0$
- Centripetal acceleration,
 $a_c = \frac{v^2}{r} = \omega^2 r$
- Centripetal force,
 $F_c = ma_c = \frac{mv^2}{r} = m\omega^2 r$

Remember

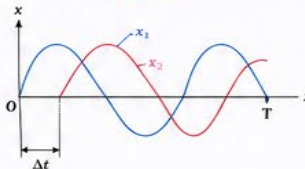
U.C.M. projected on a diameter gives S.H.M.
 $x = r \cos(\omega t + \phi)$ (SHM form)

4 PHASE & PHASE DIFFERENCE

★ MOST IMPORTANT

- Phase (ϕ) decides the position of the oscillator at any instant.
- For $x_1 = A \sin(\omega t + \phi_1)$ and $x_2 = A \sin(\omega t + \phi_2)$

Phase difference,
 $\Delta\phi = \phi_2 - \phi_1$



Relation between phase difference and time lag

$$\Delta\phi = \omega\Delta t = 2\pi\frac{\Delta t}{T}$$

In-phase: $\Delta\phi = 2n\pi$ Out-of-phase: $\Delta\phi = (2n+1)\pi$
($n = 0, 1, 2, \dots$)

ENERGY IN S.H.M.

Total mechanical energy (constant)

$$E = \frac{1}{2}kA^2$$

At any displacement x

- Potential energy, $U = \frac{1}{2}kx^2$
- Kinetic energy, $K = \frac{1}{2}k(A^2 - x^2)$
- Relation: $E = K + U$

At mean position ($x = 0$): $K = E$ (max), $U = 0$

At extreme position ($x = \pm A$): $K = 0$, $U = E$ (max)

COMMON MISTAKES

- Using wrong unit of frequency (Hz not s⁻¹).
- Forgetting negative sign in restoring force.
- Using large angle formula in pendulum.
- Confusing angular frequency (ω) with frequency (f).
- Adding K and U separately ($E = K + U$ only).

MEMORY TRICKS

- " $F = -kx$ " $\rightarrow$ Force Fights Displacement
- " $\omega = \sqrt{\frac{k}{m}}$ " $\rightarrow$ Omega = Spring / Mass
- " $T = 2\pi\sqrt{\frac{l}{g}}$ " $\rightarrow$ 2π times length over gravity

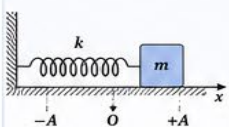
PYQ TIPS (CBSE)

- Derive time period of simple pendulum. (PYQ)
- Find expression for time period of loaded spring. (PYQ)
- Obtain expression for energy in SHM at any position. (PYQ)
- Relate U.C.M. with S.H.M. (PYQ)
- Find phase difference / time lag between two SHM. (PYQ)

APPLICATIONS (REAL LIFE)

- Simple Pendulum $\rightarrow$ clocks, metronome
- Spring Oscillator $\rightarrow$ car suspensions, vibration absorbers
- SHM & Waves $\rightarrow$ sound, musical instruments
- U.C.M. $\rightarrow$ rotating wheels, satellites
- Periodic Functions $\rightarrow$ electricity (AC), wave technology

5 OSCILLATIONS OF A LOADED SPRING



Hooke's Law (Restoring Force)

$$F = -kx$$

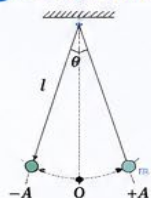
$$k = \frac{F}{x} \quad (\text{N m}^{-1})$$

Equation of Motion

$$m\frac{d^2x}{dt^2} + kx = 0$$

$$\omega = \sqrt{\frac{k}{m}}, \quad T = 2\pi\sqrt{\frac{m}{k}}, \quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

6 SIMPLE PENDULUM



Restoring Torque

$$\tau = -mgl \sin \theta \approx -mgl \theta \quad (\text{for small } \theta \text{ in radian})$$

Equation of Motion

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

Time Period

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where

- l = length of pendulum (m)
- g = acceleration due to gravity (m s⁻²)

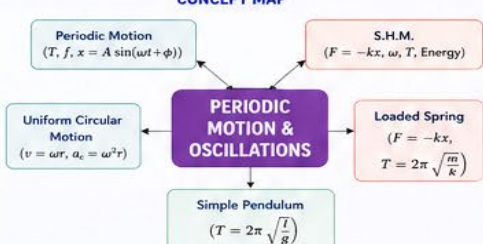
Important

- Valid for small oscillations ($\theta < 5^\circ$ or 0.09 rad)
- Time period is independent of mass and amplitude.

IMPORTANT FORMULA COMPARISON TABLE

Concept	Formula
Time Period (SHM)	$T = 2\pi\sqrt{\frac{m}{k}}$
Frequency (SHM)	$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
Angular Frequency (SHM)	$\omega = \sqrt{\frac{k}{m}} = 2\pi f$
Displacement (SHM)	$x = A \sin(\omega t + \phi)$
Energy (SHM)	$E = \frac{1}{2}kA^2$
Kinetic Energy (SHM)	$E = \frac{1}{2}k(A^2 - x^2)$
Potential Energy (SHM)	$U = \frac{1}{2}kx^2$
Simple Pendulum (T)	$T = 2\pi\sqrt{\frac{l}{g}}$
U.C.M. Speed	$v = \omega r$

CONCEPT MAP



EXAM SUCCESS MANTRA

- Understand Concept
- Memorize Formulas
- Practice PYQs
- Analyze Mistakes
- Revise Regularly

**STAY CONSISTENT,
SCORE EXCELLENT!**

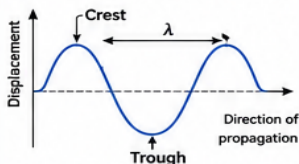


WAVES

1. TRANSVERSE AND LONGITUDINAL WAVES

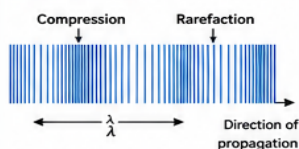
TRANSVERSE WAVES

Particles of the medium vibrate **perpendicular** to the direction of wave propagation.



LONGITUDINAL WAVES

Particles of the medium vibrate **parallel** to the direction of wave propagation.



Examples

Transverse: Waves on a string, electromagnetic waves, ripples on water surface.

Longitudinal: Sound waves in air, waves in slinky.

2. SPEED OF TRAVELLING WAVE

The speed of a wave is the distance travelled by the wave in unit time.

$$v = f\lambda$$

where,

v = speed of the wave (m s^{-1})

f = frequency of the wave (Hz)

λ = wavelength of the wave (m)

Also,

$$v = \frac{\lambda}{T} \quad (T = \text{time period})$$

$$v = \frac{d}{t} \quad (d = \text{distance}, t = \text{time})$$

For waves on a stretched string

$$v = \sqrt{\frac{T}{\mu}}$$

where,

T = tension in the string (N)

μ = mass per unit length (kg m^{-1})

Key Point

The speed of a wave depends on the properties of the medium, not on the frequency or wavelength.

3. DISPLACEMENT RELATION FOR A PROGRESSIVE WAVE

A progressive wave travelling in the $+x$ direction is represented by

$$y(x, t) = A \sin(kx - \omega t + \phi)$$

where,

y = displacement at position x time t

A = amplitude

k = wave number = $\frac{2\pi}{\lambda}$

ω = angular frequency = $2\pi f$

ϕ = initial phase constant

For a wave travelling in the $-x$ direction

$$y(x, t) = A \sin(kx + \omega t + \phi)$$

Relation between wave quantities

Wave number	$k = \frac{2\pi}{\lambda}$
Angular frequency	$\omega = 2\pi f$
Frequency	$f = \frac{1}{T}$
Speed	$v = \omega/k = f\lambda$

Note

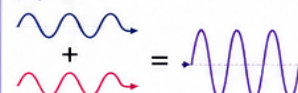
Phase constant (ϕ) determines the initial state of the wave.

4. PRINCIPLE OF SUPERPOSITION OF WAVES

When two or more waves overlap while travelling in the same medium, the resultant displacement at any point is the algebraic (sum) of the displacements produced by each wave individually.

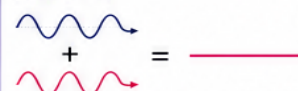
$$y_{\text{net}} = y_1 + y_2 + y_3 + \dots$$

Constructive Interference (in phase)



Resultant amplitude increases.

Destructive Interference (out of phase)



Resultant amplitude decreases (can be zero).



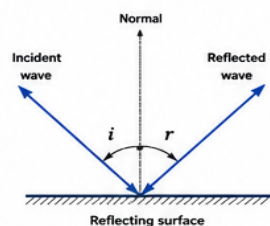
This principle is the basis of many wave phenomena such as standing waves, beats, and interference patterns.

5. REFLECTION OF WAVES

When a wave strikes a boundary between two media, it returns back into the same medium. This phenomenon is called reflection of waves.

Laws of Reflection

- The angle of incidence (i) is equal to the angle of reflection (r).
 $i = r$
- The incident wave, the reflected wave and the normal at the point of incidence lie in the same plane.



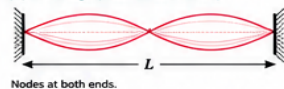
Examples

Echo of sound, reflection of light, reflection of waves on a string.

6. STANDING WAVES IN STRINGS AND ORGAN PIPES

Standing waves are formed due to the superposition of two waves of the same amplitude and frequency travelling in opposite directions.

(A) In Strings (fixed at both ends)

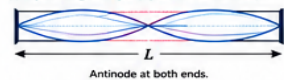


Condition for standing wave

$$L = \frac{n\lambda}{2} \quad (n = 1, 2, 3, \dots)$$

(B) In Organ Pipes

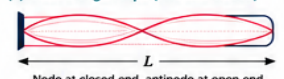
(i) Open Organ Pipe (open at both ends)



Condition

$$L = \frac{n\lambda}{2} \quad (n = 1, 2, 3, \dots)$$

(ii) Closed Organ Pipe (one end closed)



Condition

$$L = \frac{(2n-1)\lambda}{4} \quad (n = 1, 2, 3, \dots)$$

7. FUNDAMENTAL MODE AND HARMONICS

Any system that supports standing waves can vibrate in a number of natural modes.

(A) In Strings and Open Organ Pipes (Open at both ends)

Allowed wavelengths: $\lambda_n = \frac{2L}{n}$

Mode (n)	m	Wavelength (λ_n)	Frequency (f_n)	Name
1	1	$2L$	$\frac{v}{2L}$	Fundamental (First Harmonic)
2	2	L	$\frac{v}{L}$	First Overtone (Second Harmonic)
3	3	$\frac{2L}{3}$	$\frac{3v}{2L}$	Second Overtone (Third Harmonic)
...	...	...	...	...

(B) In Closed Organ Pipes (one end closed)

Allowed wavelengths: $\lambda_n = \frac{4L}{2n-1}$

Mode (n)	m	Wavelength (λ_n)	Frequency (f_n)	Name
1	1	$4L$	$\frac{v}{4L}$	Fundamental (First Harmonic)
2	2	$\frac{4L}{3}$	$\frac{3v}{4L}$	First Overtone (Third Harmonic)
3	3	$\frac{4L}{5}$	$\frac{5v}{4L}$	Second Overtone (Fifth Harmonic)
...	...	...	...	...



Harmonics are integral multiples of the fundamental frequency.

8. BEATS

When two waves of slightly different frequencies interfere, the resultant intensity (or amplitude) at a point varies periodically. This phenomenon is called beats.

Key Points

- Let the frequencies be f_1 and f_2 ($f_1 > f_2$).
- The resultant amplitude varies between maximum and minimum periodically.
- Number of beats per second (beat frequency):

$$f_{\text{beat}} = |f_1 - f_2|$$

Waveform (variation of amplitude)



Examples

Tuning of musical instruments, acoustics of concerts, sound from speakers.

QUICK RECAP



Wave speed

$$v = f\lambda$$

$$v = \frac{\lambda}{T} = \frac{d}{t}$$

(String) $v = \sqrt{\frac{T}{\mu}}$

Wave relation

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f$$

$$f = \frac{1}{T}$$

$$v = f\lambda = \frac{\omega}{k}$$

Progressive wave (+x direction)

$$y(x, t) = A \sin(kx - \omega t + \phi)$$

(-x direction)

$$y(x, t) = A \sin(kx + \omega t + \phi)$$

String (fixed ends)

$$L = \frac{n\lambda}{2}$$

Open pipe (both ends open)

$$L = \frac{n\lambda}{2}$$

Closed pipe (one end closed)

$$L = \frac{(2n-1)\lambda}{4}$$

Beats

$$f_{\text{beat}} = |f_1 - f_2|$$

MULTIPLE CHOICE QUESTIONS

- 1). A boy is swinging in a swing. If he stands, the time period will
(a) First decrease, then increase (b) Decrease (c) Increase (d) Remain same
- 2). A wave pulse travels along a stretched string whose linear mass density decreases continuously from one end to the other. What happens to the frequency of the wave pulse as it propagates?
(a) It increases continuously (b) It decreases continuously
(c) It remains constant (d) It first increases then decreases
- 3). A taut string fixed at both ends vibrates in its fourth harmonic. How many nodes and antinodes are formed inside the string, excluding the boundary endpoints?
(a) 3 nodes, 4 antinodes (b) 4 nodes, 3 antinodes
(c) 5 nodes, 4 antinodes (d) 3 nodes, 3 antinodes
- 4). An open organ pipe is suddenly closed at one end. What happens to the frequency of its third overtone compared to the original fundamental frequency of the open pipe?
(a) It increases by a factor of 3.5 (b) It increases by a factor of 1.75
(c) It decreases by half (d) It remains exactly the same
- 5). Two tuning forks, X and Y, produce 4 beats per second when sounded together. When X is loaded with a little wax, the beat frequency increases to 6 beats per second. If the frequency of Y is 256 Hz, what is the original frequency of X?
(a) 252 Hz (b) 260 Hz (c) 262 Hz (d) 250 Hz
- 6). In a resonance column apparatus, the first resonance occurs at a tube length of 16 cm and the second resonance occurs at 50 cm. What is the end correction of this tube?
(a) 1.0 cm (b) 2.0 cm (c) 0.5 cm (d) 1.5 cm
- 7). A steel wire of length 1 m and mass 5 g is stretched under a tension of 80 N. What is the speed of a transverse wave on this wire?
(a) 100 m/s (b) 126 m/s (c) 400 m/s (d) 160 m/s
- 8). Newton's formula for the speed of sound in a gas was corrected by Laplace because sound propagation is an:
(a) Isothermal process (b) Isobaric process (c) Adiabatic process (d) Isochoric process
- 9). When a sound wave passes from air into water, which of the following properties of the wave does not change?
(a) Wavelength (b) Wave velocity (c) Frequency (d) Amplitude
- 10). A train blowing its whistle moves at a constant speed along a straight track passing a stationary observer on the platform. How does the pitch heard by the observer change as the train passes?
(a) it rises continuously (b) It drops abruptly at the instant the train passes
(c) It remains constant throughout the entire motion
(d) It drops continuously from a high pitch to a low pitch
- 11). The sound intensity level at a point is increased by 20 dB. By what factor does the sound pressure amplitude increase?
(a) 2 (b) 10 (c) 20 (d) 100
- 12). A uniform rope of mass (M) and length (L) hangs vertically from a rigid ceiling. A transverse pulse is generated at the bottom free end. How does the velocity of the pulse change as it travels up the rope?
(a) It remains constant (b) It decreases linearly to zero
(c) It increases proportionally to the square root of the distance from the bottom
(d) It increases linearly with the distance from the bottom
- 13). An ambulance with a siren of frequency 1000 Hz is moving at 30 m/s towards a vertical wall. The speed of sound is 330 m/s. The frequency of the echo heard by the driver is:
(a) 1000 Hz (b) 1100 Hz (c) 1200 Hz (d) 900 Hz
- 14). When two waves of intensities I and 9I interfere, the ratio of maximum to minimum intensity in the interference pattern is:
(a) 4 : 1 (b) 9 : 1 (c) 10 : 8 (d) 16 : 1

15). The fundamental frequency in an open organ pipe is equal to the third harmonic of a closed organ pipe. If the length of the closed organ pipe is 20 cm, the length of the open organ pipe is

- (a) 12.5 cm (b) 13.2 cm (c) 8 cm (d) 16 cm

ANSWER KEY WITH EXPLANATION

1. (b) Time period ($T \propto \sqrt{L}$) decreases.
2. (c) Changing density: Frequency depends only on the source; medium properties do not change it.
3. (a) 4th Harmonic: Has 5 nodes and 4 antinodes. Excluding 2 boundary endpoints leaves 3 nodes, 4 antinodes.
4. (b) Organ pipe shift: Open fundamental ($f_o = v/2L$) Closed 3rd overtone (7th harmonic) ($f_c = 7v/4L = 3.5 f_o$).
5. (a) Tuning forks: Wax reduces X's frequency. Since beats increased (4), original X must be below Y: ($256 - 4 = 252$ Hz).
6. (a) First resonance $L_1 = 16$ cm Second resonance $L_2 = 50$ cm
Difference: $L_2 - L_1 = \lambda/2L$, $50 - 16 = 34$ cm, $\lambda = 68$ cm
For first resonance: $L_1 + e = \lambda/4$ $16 + e = 17$ $e = 1$ cm
7. (c) Linear density: $\mu = mL = 0.005$
Wave speed: $v = \sqrt{\frac{T}{\mu}}$ $v = \sqrt{160000} \approx 126.5$ m/s
8. (c) Laplace corrected Newton because sound propagation in gas is too fast for heat exchange, so process is adiabatic.
9. (c)
10. (b) Approaching $\rightarrow$ higher pitch Receding $\rightarrow$ lower pitch
At exact passing instant $\rightarrow$ sudden drop
11. (b) Sound level relation:
 $\beta = 20 \log(p_2/p_1)$ Given increase = 20 dB $20 = 20 \log(p_2/p_1) = 10$
12. (c) At distance x from bottom: $T = x\mu g$ Wave speed: $v = \sqrt{\frac{T}{\mu}} = \sqrt{xg}$
So velocity increases as square root of distance
13. (c) Double Doppler effect: $f' = f(v + u/v - u)$ Given: $f = 100$ $u = 30$ $v = 330$
 $f' = 1000(360/300) = 1200$
14. (b) Amplitude ratio: $\sqrt{I} : \sqrt{9I} = 1:3$
Maximum: $(1+3)^2 = 16$ Minimum: $(3-1)^2 = 4$ Ratio: $16:4 = 4:1$
15. (b) Closed pipe third harmonic: $f_c = 3v/4L$
Open pipe fundamental: $f_o = v/2L_v$
Given equal: $v/2L_o = 3v/4L_c$, $L_o = 2L_c/3 = (2 \times 20)/3 = 13.3$ cm

CHALLENGE CORNER

- 16). Two cars moving in opposite directions approach each other with speed of 22 m/s and 16.5 m/s respectively. The driver of the first car blows a horn having a frequency 400 Hz. The frequency heard by the driver of the second car is [vel of sound 340 m/s] **(NEET-2017)**
(a) 350 Hz (b) 361 Hz (c) 411 Hz (d) 448 Hz
- 17). A wave of frequency 400 Hz has a velocity of 320 m/s. The distance between the particles differing in phase by 90° is
(a) 20 cm (b) 40 cm (c) 60 cm (d) 80 cm
- 18). A tuning fork vibrating with a sonometer having 20 cm wire produces 5 beats per second. The beat frequency does not change if the length of the wire is changed to 21 cm, the frequency of the tuning fork (in Hertz) must be
(a) 200 (b) 210 (c) 205 (d) 215
- 19). The 3rd overtone of a closed organ pipe is equal to the 2nd harmonic of an open organ pipe. Then the ratio of their lengths is equal to
(a) $7/4$ (b) $3/5$ (c) $3/2$ (d) $7/6$

- 20). Four wires of identical length, diameters and of the same material are stretched on a sonometer wire. If the ratio of their tensions is 1 : 4 : 9 : 16, then the ratio of their fundamental frequencies are
(a) 16 : 9 : 4 : 1 (b) 4 : 3 : 2 : 1 (c) 1 : 4 : 2 : 16 (d) 1 : 2 : 3 : 4

ANSWER KEY OF CHALLENGE CORNER

16. d 17. a 18. c 19. a 20. D

ASSERTION REASON TYPE QUESTIONS

- (a) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true, but Reason is NOT the correct explanation of Assertion.
(c) Assertion is true, but Reason is false.
(d) Assertion is false, but Reason is true.
- Assertion: The propagation of a longitudinal sound wave through a medium creates cyclic variations in local density and pressure.
Reason: Regions of compression exhibit an increase in medium density, whereas regions of rarefaction exhibit a decrease in medium density.
 - Assertion: Solid media can sustain both transverse and longitudinal waves, whereas gaseous media can only transmit longitudinal waves.
Reason: The propagation of transverse mechanical waves strictly requires the medium to possess shear elasticity or rigidity.
 - Assertion: Acoustic waves traveling through an air column serve as a classic example of mechanical longitudinal waves.
Reason: In a longitudinal wave architecture, the individual constituent particles of the medium oscillate perpendicular to the vector of wave energy propagation.
 - Assertion: In a progressive harmonic wave, both the individual particle velocity and the overall wave propagation velocity remain entirely independent of time.
Reason: A material medium must inherently possess both elasticity and inertia to support and sustain mechanical wave propagation.
 - Assertion: Two separate transverse waves propagating along the same uniform string under constant tension cannot travel at different velocities.
Reason: The inertial and elastic characteristics of a specific string are identical for all waves, and the wave propagation velocity depends solely on these intrinsic medium properties.
 - Assertion: When transverse waves are continuously generated at one end of an exceptionally long string, only a progressive wave pattern is visible near the driven end.
Reason: The energy of the wave reflected from the distant fixed boundary is completely dissipated before it can travel back to the source end.
 - Assertion: High-energy acoustic explosions occurring on the surface of neighbouring planets cannot be detected or heard on Earth.
Reason: Mechanical sound waves lack the structural capacity to travel across vast, long-distance spatial intervals.
 - Assertion: Two astronauts standing on the lunar surface can easily communicate verbally with each other by utilizing specialized microphone radio headsets.
Reason: A microphone converts audio pressure signals into electromagnetic transverse waves, which can freely propagate through a vacuum.
 - Assertion: In a standing wave system, a stationary observer hears a significantly louder sound at the displacement nodes compared to the displacement antinodes.
Reason: During the execution of a stationary wave pattern, all vibrating particles across the medium pass through their mean positions in identical phase.
 - Assertion : The change in air pressure affect the speed of sound.
Reason : The speed of sound in a gas is proportional to the square root of pressure.

ANSWER KEY

1. a 2. a 3. c 4. d 5. a 6. a 7. c 8. a 9. b 10. d

VERY SHORT ANSWER QUESTIONS (2 MARKS)

1. Why can the sound of a distant train whistle or outdoor loudspeaker be heard much more clearly and over longer distances during a cold night than on a hot, sunny day?
2. In a progressive wave, energy is transmitted continuously through the medium. How does the energy distribution in a standing (stationary) wave differ from a progressive wave? Specify the energy states at the nodes and antinodes.
3. The frequency of the fundamental note of a closed organ pipe and that of an open organ pipe are the same. What is the ratio of their lengths?
4. How will the fundamental frequency of a closed organ pipe be affected if instead of air it is filled with a gas heavier than air?
5. Sound travels faster on a rainy day than on a dry day. Why?
6. When a longitudinal sound wave traveling through an organ pipe hits a closed, rigid end, it undergoes reflection. What type of phase change happens to the displacement wave, and what happens to the pressure wave at this boundary?
7. A piano tuner strikes a key and a standard 440 Hz tuning fork simultaneously and hears a rhythmic, pulsating sound that rises and falls in loudness 3 times every second. What wave phenomenon is occurring, and how does the tuner use this to fix the piano string?
8. An open organ pipe has a physical length of 50 cm. If the velocity of sound in air is 340 m/s, calculate the fundamental frequency of this pipe. What will the fundamental frequency become if one end of the pipe is tightly closed?

ANSWER KEY

1. Sound speed is directly proportional to temperature $v \propto \sqrt{t}$. On a hot day, the air near the ground is warmer than the air above, causing upward refraction (bending) of sound waves away from the ground.

On a cold night, the ground cools down, making the lower air layers colder than the upper layers (temperature inversion). This causes downward refraction of sound waves back toward the earth, keeping the acoustic energy near the ground and allowing it to travel further.

2. In a standing wave, energy is not transmitted forward; it remains confined within the loops between adjacent nodes. At the nodes, the displacement is always zero, meaning the kinetic energy is zero, and the energy is purely potential (maximum pressure variation). At the antinodes, displacement is maximum, meaning the energy periodically shifts completely into kinetic energy.

3. Ratio is 1:2

4. The fundamental frequency of the closed organ pipe will decrease when filled with a gas heavier than air.

5. Sound travels faster on a rainy day because moist air is less dense than dry air. The speed of sound is inversely proportional to the square root of the medium's density. Therefore, with the lower density of wet air, sound travels faster.

6. The displacement wave experiences a phase change of π radians 180 because the particles at the rigid wall are physically restrained from moving, creating a displacement node. The pressure wave undergoes zero phase change 0° because the compressions reflect as compressions, bouncing off the rigid wall to form a pressure antinode.

7. The phenomenon is beats, caused by the interference of two sound waves of slightly different frequencies. The beat frequency is $\{\Delta f = f_1 - f_2\} = 3\text{Hz}$. The tuner adjusts the tension of the piano string until the pulsating sound slows down and

stops completely (beat frequency becomes (0 Hz), matching the string's frequency perfectly to the 440 Hz fork.

8. For an open pipe: The fundamental frequency is $(f_o = \{v\}/\{2L\})$.

Given: $(L = 50 \text{ cm}) = 0.5 \text{ m}$, $(v = 340 \text{ m/s})$. $(f_o = 340/\{2 * 0.5\} = 340\text{Hz})$.

When one end is closed: The new fundamental frequency is $(f_c = v/4L) = f_o/2$.

$(f_c = 340/(4* 0.5) = 170\text{Hz})$.

SHORT ANSWER QUESTIONS (3 MARKS)

1. Why are longitudinal waves called pressure waves?
2. Define stationary waves and explain why they are named so.
3. State three essential differences between progressive waves and stationary waves.
4. A transverse harmonic wave on a wire is expressed as:

$$y(x, t) = 3 \sin (36t + 0.018x + \pi/4).$$

(i) Is it a stationary wave or a travelling one?

(ii) If it is a travelling wave, give the speed and direction of its propagation.

(iii) Find its frequency and amplitude.

(iv) Give the initial phase at the origin.

(v) Calculate the smallest distance between two adjacent crests in the wave.

[X and y are in cm and t in seconds. Assume the left to the right direction as the positive direction of x]

5. A travelling harmonic wave is given as: $y (x, t) = 2.0 \cos 2\pi (10t - 0.0080x + 0.35)$. What is the phase difference between the oscillatory motion of two points separated by a distance of: (i) 8 m (ii) 1 m (iii) $\lambda/2$ (iv) $6\lambda/4$ [X & y are in cm, & t in secs
6. A 1 m long pipe with a movable piston at one end and an opening at the other will be in resonance with a tuning fork vibrating at 340 Hz, if the length of the pipe is 79.3 cm or 25.5 cm. Calculate the speed of sound in the air. Neglect the edge effects.
7. Steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that the speed of a transverse wave on the wire equals the speed of sound in dry air at $20^\circ\text{C} = 343 \text{ m s}^{-1}$.
8. Two tuning forks give 6 beats/s when sounded simultaneously. The frequency of one of the forks is 480 Hz. When the other fork is loaded with a piece of wax, the beats increase. What is the frequency of the second fork?
9. A guitarist finds that her string sounds flat (i.e., the pitch is lower than expected). Explain how she can correct the pitch without changing the string. If she tightens the string, which physical property of the wave on the string is directly affected?
(a) Amplitude (b) Frequency (c) Wavelength (d) Time period
10. Using the formula for the speed of longitudinal waves in a given medium of density ρ , explain why the speed of sound in air (a) is independent of pressure (b) increases with temperature, and (iii) increases with humidity

ANSWER KEY

1. Longitudinal waves propagate through a medium via alternating compressions and rarefactions. In compressions, the density and pressure of the medium are higher than the average, while in rarefactions, they are lower. Because the wave propagation depends on these fluctuations in pressure and volume, longitudinal waves are referred to as pressure waves.

2. Stationary waves (or standing waves) are formed by the superposition of two identical waves traveling in opposite directions in a bounded medium. They are "stationary" because the waveform does not advance in the medium, and there is no net energy transport through any point in the medium.

3. Propagation: Progressive waves propagate through the medium, while stationary waves do not.

Amplitude: In progressive waves, all particles vibrate with the same amplitude. In stationary waves, amplitude varies (zero at nodes, max at antinodes).

Phase: In progressive waves, phase changes continuously. In stationary waves, all particles in a loop are in phase, but out of phase with adjacent loops.

4. Given, $y(x, t) = 3 \sin(36t + 0.018x + \pi/4)$ (1)

(i) We know the equation of a progressive wave travelling from right to left is:

$$y(x, t) = a \sin(\omega t + kx + \Phi) \quad \dots \dots \dots (2)$$

Comparing equation (1) to equation (2), we see that it represents a wave travelling from right to left and also we get:

$$a = 3 \text{ cm}, \omega = 36 \text{ rad/s}, k = 0.018 \text{ cm} \text{ and } \phi = \pi/4.$$

(ii) Therefore, the speed of propagation, $v = \omega/k = 36/0.018 = 20 \text{ m/s}$.

(iii) Amplitude of the wave, $a = 3 \text{ cm}$ Frequency $\nu = \omega / 2\pi = 36 / 2\pi = 5.7 \text{ Hz}$.

(iv) Initial phase at the origin $= \pi/4$

(v) The smallest distance between two adjacent crests, $\lambda = 2\pi/k = 2\pi/0.018 = 349 \text{ cm}$.

5. Given, The equation for a travelling harmonic wave:

$$y(x, t) = 2.0 \cos 2\pi(10t - 0.0080x + 0.35) = 2.0 \cos(20\pi t - 0.016\pi x + 0.70\pi)$$

Where, Propagation constant, $k = 0.0160\pi$, $a = 2 \text{ cm}$ $\omega = 20\pi \text{ rad/s}$

We know, Phase difference $\Phi = kx = 2\pi/\lambda$

(i) For $x = 8 \text{ m} = 800 \text{ cm}$ $\Phi = 0.016\pi \times 800 = 12.8\pi \text{ rad}$.

(ii) For $x = 1 \text{ m} = 100 \text{ cm}$ $\Phi = 0.016\pi \times 100 = 1.6\pi \text{ rad}$.

(iii) For $x = \lambda/2$ $\Phi = (2\pi/\lambda) \times (\lambda/2) = \pi \text{ rad}$.

(iv) For $x = 6\lambda/4$ $\Phi = (2\pi/\lambda) \times (6\lambda/4) = 3\pi \text{ rad}$.

6. Given, Frequency of the tuning fork, $\nu F = 340 \text{ Hz}$

Length of the pipe, $l = 0.255 \text{ m}$

As the given pipe has a piston at one end, it will behave as a pipe with one end closed and the other end open, as depicted in the figure below:

This kind of system creates odd harmonics. We know that the

fundamental note in a closed pipe is written as $l = \lambda/4$

$$0.255 \times 4 = \lambda = 1.02 \text{ m}$$

Therefore, the speed of sound, $v = \lambda \nu F = 340 \times 1.02 = 346.8 \text{ m/s}$

7. Given, Length of the steel wire, $l = 12 \text{ m}$

Mass of the steel wire, $m = 2.0 \text{ kg}$, Velocity of the transverse wave, $v = 343 \text{ m/s}$

Mass per unit length, $\mu = M/l = 2.10/12 = 0.175 \text{ kg/m}$

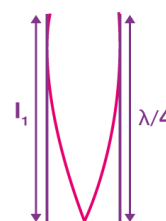
We know, Velocity of the transverse wave, $v =$ Therefore, $T = V^2 \mu = 343^2 \times 0.175$

$= 2.06 \times 10^4 \text{ N}$ Therefore, the tension in the wire is $343^2 \times 0.175 = 2.06 \times 10^4 \text{ N}$.

8. beat frequency is the absolute difference between the frequencies of the two forks.

frequency of the second fork can be either $480 + 6 = 486 \text{ Hz}$ or $480 - 6 = 474 \text{ Hz}$.

Loading the second fork with wax decreases its frequency. If the second fork's



frequency was 486 Hz, loading it with wax would decrease the frequency and the beat frequency would decrease. Since the beat frequency increases, the second fork's frequency must be 474 Hz.

9. To raise the pitch, the guitarist needs to increase the frequency of vibration. She can do this by increasing the tension in the string (by tightening the tuning peg). This increases the wave speed $v = \sqrt{\frac{T}{\mu}}$ and since $f = v/(2L)$, the frequency increases, raising the pitch. (b) Frequency
10. (a) Pressure: The speed of sound in air is independent of pressure because both pressure and density increase or decrease together. Their ratio P/ρ remains constant at a fixed temperature, so the speed doesn't change.
(b) Temperature: The speed of sound increases with temperature. This is because higher temperature means molecules move faster, increasing the wave speed. Mathematically, speed is proportional to the square root of temperature: $v \propto \sqrt{T}$
(c) Humidity: The speed of sound increases with humidity. Water vapor is lighter than dry air, so when air is more humid, its average molecular mass decreases. This makes the air less dense and allows sound to travel faster.

CASE BASE STUDY

1. Acoustic Interference and Tuning Musical Instruments

When two sound sources of **slightly different frequencies** emit continuous waves simultaneously, our ears do not perceive two distinct individual pitches. Instead, we hear a single, fluctuating sound that regularly rises and falls in loudness. This periodic variation in sound intensity is called **beats**.

The phenomenon is a direct consequence of the **superposition principle**. At a given point in space, the sound waves alternately reinforce each other (constructive interference) and cancel each other out (destructive interference).

The number of times the sound intensity reaches a maximum (or minimum) per second is known as the **beat frequency**. This frequency is exactly equal to the absolute difference between the frequencies of the two interfering waves

$f_b = f_1 - f_2$. Piano tuners and musicians use this physical principle extensively to tune instruments by adjusting strings until the audible beats vanish completely.

1. To hear distinct and audible beats, what are the primary physical constraints on the two interfering sound waves?
 - (a). They must have identical frequencies and travel in opposite directions.
 - (b). They must have nearly equal frequencies and travel in the same direction.
 - (c). They must have vastly different amplitudes and travel perpendicular to each other.
 - (d). They must be longitudinal and transverse waves respectively.
2. Looking at the sound envelope of a beat pattern, what mechanical event takes place at the point of minimum loudness (complete destructive interference)?
 - (a). crest of one wave perfectly overlaps with a crest of the second wave.
 - (b). The phase difference between the two waves becomes zero.
 - (c). A crest of one wave perfectly overlaps with a trough of the second wave.
 - (d). The net amplitude of the combined sound wave doubles.
3. Two sound waves are described by equations ($y_1 = A \sin 2\pi f_1 t$) and ($y_2 = A \sin 2\pi f_2 t$). When they superimpose, the mathematical time interval between two consecutive intensity maxima is found to be:
 - (a). $\frac{1}{f_1 + f_2}$
 - (b). $\frac{2}{f_1 + f_2}$
 - (c). $\frac{1}{f_1 - f_2}$
 - (d). $\frac{f_1 - f_2}{1}$
4. A tuning fork of unknown frequency produces 4 beats/second when sounded alongside a standard fork of 256 Hz. When the prongs of the unknown fork are loaded with a small amount of wax, the observed beat frequency decreases to 2 beats/second. What was the initial frequency of the unknown

fork?

- (a). 252Hz (b). 260Hz (c). 258 Hz (d). 254 Hz

STANDING WAVES IN STRETCHED STRINGS (MUSICAL INSTRUMENTS)

When a guitar string of length (L) is plucked, transverse waves travel along the string in opposite directions. These waves reflect off the fixed ends of the string and superimpose on each other. Because the waves have the same amplitude, frequency, and speed but travel in opposite directions, they form a **standing wave** or stationary wave pattern.

The fixed ends cannot move, creating points of zero displacement known as **nodes**. The middle sections experience maximum displacement, creating **antinodes**. The string can vibrate at specific, discrete frequencies called natural frequencies or **harmonics**. The lowest frequency at which a standing wave forms is the **fundamental frequency** or first harmonic. Higher frequencies are integer multiples of this fundamental frequency, forming a harmonic series ($f_n = n \cdot f_1$)

- For a string of length (L) fixed at both ends vibrating in its third harmonic, what is the total number of nodes (including the boundaries) and the distance between any two consecutive nodes?
(a). 3 nodes, distance = $L/3$ (b). 4 nodes, distance = $L/3$
(c). 4 nodes, distance = $L/2$ (d). 5 nodes, distance = $L/4$
- A guitar player tightens a string, increasing its tension by a factor of 4 while maintaining the same string length. How does the fundamental frequency of this string change?
(a). It stays exactly the same (b). It increases by a factor of 2
(c). It increases by a factor of 4 (d). It decreases to half its original value
- Consider two particles located on the same string vibrating in a standing wave pattern. If both particles lie within the exact same loop (between two consecutive nodes), what is the phase difference between their motions?
(a). 0 rad (They are perfectly in phase) (b). $\pi/2$ rad
(c). π rad (They are completely out of phase) (d). 2π rad
- The fundamental frequency of a stretched string fixed at both ends is 200Hz. What is the frequency of its third overtone?
(a). 400 Hz (b). 600 Hz (c). 800 Hz (d). 1000 Hz

3. Organ Pipes and Resonance Columns

Organ pipes use the concept of longitudinal standing waves inside an air column to produce musical notes. These pipes are classified into two types: **open organ pipes** (open at both ends) and **closed organ pipes** (closed at one end and open at the other).

The air at an open end is free to move, creating a displacement **antinode** (or pressure node). The air at a closed end is rigidly blocked, forcing a displacement **node** (or pressure antinode). Because of these distinct structural boundary conditions, open and closed pipes generate completely different configurations of harmonics. This variation changes the sound profile, which is why instruments like flutes (open) sound distinct from instruments like clarinets (closed at one end).

- Which of the following statements correctly details the allowed harmonic frequencies generated inside a closed organ pipe?
(a). It allows all integer multiples of the fundamental frequency (both even and odd).
(b). It allows only even multiples of the fundamental frequency.
(c). It allows only odd multiples of the fundamental frequency.
(d). It does not generate any predictable harmonic series.
- An open organ pipe and a closed organ pipe have the exact same physical length L . What is the mathematical ratio of their fundamental frequencies ($f_{\text{open}} : f_{\text{closed}}$)?
(a). 1 : 1 (b). 1 : 2 (c). 2 : 1 (d). 4 : 1
- If the temperature of the air inside an open organ pipe increases, what happens to the pitch (fundamental frequency) of the sound produced?

- (a). The pitch increases because wave speed increases with temperature.
 (b). The pitch decreases because the air column expands significantly.
 (c). The pitch stays completely constant.
 (d). The pitch drops to zero.
4. A closed organ pipe has a length of 0.25 m. Taking the speed of sound in air to be 340 m/s, calculate the fundamental frequency of this pipe.
 (a). 170 Hz (b). 340 Hz (c). 680 Hz (d). 1360 Hz

ANSWER KEY

- | | | | |
|--------|-----|------|------|
| 1. 1.b | 2.c | 3. c | 4. b |
| 2. 1.b | 2.b | 3. a | 4. c |
| 3. 1.c | 2.c | 3. a | 4. b |

LONG ANSWER TYPE QUESTIONS

- (a) What is the principle of superposition of waves?
 (b) Two progressive waves traveling in opposite directions overlap to form a stationary wave. Derive an expression for a stationary wave and prove that the distance between two consecutive nodes is $\lambda/2$.
 (c) State two differences between a progressive wave and a stationary wave.
- (a) State Newton's formula for the speed of sound in air. (b) Explain why it was incorrect and how Laplace corrected it.
 (b) Discuss the reflection of a wave pulse from (i) a rigid boundary and (ii) an open/free boundary. Include the phase change in each case.
- (a) Define the phenomenon of beats. What is the essential condition for beats to occur?
 (b) Mathematically prove that the number of beats produced per second is equal to the difference in frequencies of the two sounding bodies.
 (c) A tuning fork of frequency (512Hz) produces (4beats/sec) with an unknown fork (A). When fork (A) is filed, the beat frequency decreases to (2beats/sec). What is the original frequency of fork (A)?

ANSWER KEY

- (a) Principle of Superposition: When two or more wave pulses overlap in the same medium, the resulting displacement at any point is the vector sum of the displacements produced by the individual waves.
 Mathematically: $(y = y_1 + y_2 + \dots + y_n)$
 (b) Consider two waves of the same amplitude (a) and frequency (ω) travelling in opposite directions: Wave moving right: $(y_1 = a \sin(kx - \omega t))$ Wave moving left: $(y_2 = a \sin(kx + \omega t))$ According to the superposition principle:
 $(y = y_1 + y_2)$ $(y = a \sin(kx - \omega t) + \sin(kx + \omega t))$
 Using the math identity: $(\sin(A-B) + \sin(A+B) = 2 \sin A \cos B)$
 $(y = [2a \sin(kx)] \cos \omega t)$ In this equation, $(A = 2a \sin(kx))$ represents the amplitude of the stationary wave, which depends on the position (x).
 Distance between two consecutive Nodes:
 At nodes, the amplitude is zero: $\sin(kx) = 0$, $kx = n\pi$ (where $(n = 0, 1, 2, \dots)$)
 Since $(k = 2\pi/\lambda)$, $(2\pi/\lambda) * x = n\pi$, $x = n\lambda/2$
 For $(n=1)$, $(x_1 = \lambda/2)$ For $(n=2)$, $(x_2 = \lambda)$ Distance = $(x_2 - x_1 = \lambda - \lambda/2)$

(c) Differences

Feature	Progressive Wave	Stationary Wave
Energy	Energy is transferred through the medium.	Energy is "trapped" and not transferred.
Amplitude	Every particle has the same amplitude.	Amplitude varies from zero (nodes) to max (antinodes).

2. (a) Newton's Formula: ($v = \sqrt{p/\rho}$). He assumed the process was isothermal (constant temperature).

Correction: Laplace argued that sound travel is an adiabatic process (no heat exchange) because it is very fast.

Corrected Formula: ($v = \sqrt{p\gamma/\rho}$, where γ is the ratio of specific heats (1.4 for air). This correction brought the theoretical value (approx. 332 m/s) close to experimental results. (b) Rigid Boundary: The reflected wave is inverted. The phase changes by 180° because the boundary exerts an equal and opposite reaction force.

Free Boundary: The reflected wave is NOT inverted. There is zero phase change because the end is free to move, acting like an antinode.

3. a) Definition: When two sound waves of nearly equal frequencies traveling in the same direction superimpose, the intensity of the resultant sound waxes (increases) and wanes (decreases) periodically. These periodic variations in intensity are called beats.

Condition: The difference in frequencies between the two sources should be small (typically less than 10Hz) for the human ear to distinguish them.

(b) Consider two sound waves of same amplitude (a) but slightly different frequencies (n_1) and (n_2) (where $n_1 > n_2$). Their displacement equations at a point ($x=0$) are:

$$(y_1 = a \sin(2\pi n_1 t) \quad y_2 = a \sin(2\pi n_2 t))$$

By the Principle of Superposition:

$$(y = y_1 + y_2) \quad (y = a [\sin(2\pi n_1 t) + \sin(2\pi n_2 t)])$$

Using the identity ($\sin C + \sin D = 2 \sin \{C+D\}/2 \cos \{C-D\}/2$):

$$(y = 2a \cos \pi(n_1 - n_2)t \sin \pi(n_1 + n_2)t)$$

The term in the bracket is the Resultant Amplitude (A):

$$(A = 2a \cos \pi(n_1 - n_2)t)$$

For Maxima (Waxing): (A) is maximum when ($\cos t = \pi(n_1 - n_2)t \pm 1$)

$$(\pi(n_1 - n_2)t = k\pi) \quad (\text{where } (k = 0, 1, 2, \dots)) \quad (t = k/(n_1 - n_2))$$

Time intervals for maxima: (0, $1/(n_1 - n_2)$, $2/(n_1 - n_2)$,)

Beat Period (T): Time between two successive maxima = $1/(n_1 - n_2)$

Beat Frequency (b): ($1/T = n_1 - n_2$)

(c) Frequency of known fork ($n = 512$ Hz). Beats = 4.

Possible frequencies for (A): ($512 + 4 = 516$ Hz) or ($512 - 4 = 508$ Hz).

Filing a fork increases its frequency. If (A) was (516 Hz), filing makes it (517, 518) (Beat count would increase). If (A) was (508 Hz), filing makes it (509, 510) (Beat count decreases as it gets closer to (512). The original frequency of fork (A) is (508 Hz).

PERIODIC MOTION & OSCILLATIONS

FORMULA SHEET

MOST IMPORTANT

Quick Revision High Score Smart Study Concept Clarity

QUICK RECALL

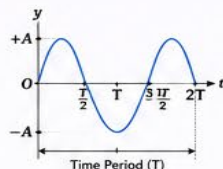
- Periodic motion repeats after equal intervals of time.
- S.H.M. is a special type of periodic motion with restoring force.
- Energy in SHM remains constant.
- Time period (T) = time taken for one complete oscillation.
- Frequency (f) = number of oscillations per second.

1 PERIODIC MOTION

Time Period & Frequency

Time period (T) =
Time for one complete oscillation

Frequency (f) =
Number of oscillations
per second = $\frac{1}{T}$



Units

$T \rightarrow$ second (s)
 $f \rightarrow$ hertz (Hz) = s^{-1}

Displacement as a function of time (for SHM)

$$x(t) = A \sin(\omega t + \phi)$$

where A = amplitude, ω = angular frequency
 ϕ = initial phase

2 SIMPLE HARMONIC MOTION (S.H.M.)

MOST IMPORTANT

Key Features

- Restoring force $\propto$ displacement and opposite in direction.

$$F = -kx$$

Equation of Motion

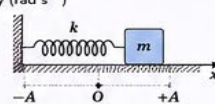
$$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0 \quad \text{or} \quad \frac{d^2x}{dt^2} + \omega^2x = 0$$

Where,

$$\omega = \sqrt{\frac{k}{m}} = \text{angular frequency (rad s}^{-1}\text{)}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$$

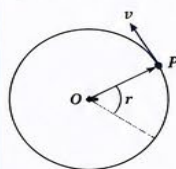
$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$



SHM Conditions

- Force must be directly proportional to displacement.
- Force must act towards mean position.
- System must have constant mass.
- No energy loss (ideal system).

3 UNIFORM CIRCULAR MOTION (U.C.M.)



- Linear speed (v) = ωr
- Angular speed (ω) = $\frac{d\theta}{dt}$
- Angular displacement,
 $\theta = \omega t + \theta_0$
- Centripetal acceleration,
 $a_c = \frac{v^2}{r} = \omega^2 r$
- Centripetal force,
 $F_c = ma_c = \frac{mv^2}{r} = m\omega^2 r$

Remember

U.C.M. projected on a diameter gives S.H.M.
 $x = r \cos(\omega t + \phi)$ (SHM form)

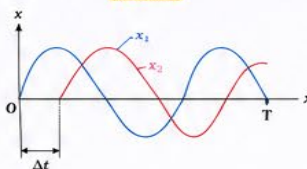
4 PHASE & PHASE DIFFERENCE

MOST IMPORTANT

- Phase (ϕ) decides the position of the oscillator at any instant.

- For $x_1 = A \sin(\omega t + \phi_1)$
and $x_2 = A \sin(\omega t + \phi_2)$

Phase difference,
 $\Delta\phi = \phi_2 - \phi_1$

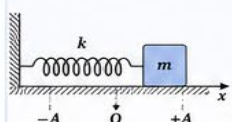


Relation between phase difference and time lag

$$\Delta\phi = \omega \Delta t = 2\pi \frac{\Delta t}{T}$$

In-phase: $\Delta\phi = 2n\pi$ Out-of-phase: $\Delta\phi = (2n+1)\pi$
($n = 0, 1, 2, \dots$)

5 OSCILLATIONS OF A LOADED SPRING



Hooke's Law (Restoring Force)

$$F = -kx$$

$$k = \frac{F}{x} \quad (\text{N m}^{-1})$$

Equation of Motion

$$m \frac{d^2x}{dt^2} + kx = 0$$

$$\omega = \sqrt{\frac{k}{m}}, \quad T = 2\pi \sqrt{\frac{m}{k}}, \quad f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

6 SIMPLE PENDULUM



Restoring Torque

$$\tau = -mgl \sin \theta \approx -mgl \theta \quad (\text{for small } \theta \text{ in radian})$$

Equation of Motion

$$\frac{d^2\theta}{dt^2} + \frac{g}{l} \theta = 0$$

Time Period

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Where
 l = length of pendulum (m)
 g = acceleration due to gravity (m s^{-2})

Important

- Valid for small oscillations ($\theta < 5^\circ$ or 0.09 rad)
- Time period is independent of mass and amplitude.

COMMON MISTAKES

- Using wrong unit of frequency (Hz not s^{-1}).
- Forgetting negative sign in restoring force.
- Using large angle formula in pendulum.
- Confusing angular frequency (ω) with frequency (f).
- Adding K and U separately ($E = K + U$ only).

MEMORY TRICKS

- " $F = -kx$ " $\rightarrow$ Force Fights Displacement
- " $\omega = \sqrt{\frac{k}{m}}$ " $\rightarrow$ Omega = Spring / Mass
- " $T = 2\pi \sqrt{\frac{l}{g}}$ " $\rightarrow$ 2π times length over gravity

PYQ TIPS (CBSE)

- Derive time period of simple pendulum. (PYQ)
- Find expression for time period of loaded spring. (PYQ)
- Obtain expression for energy in SHM at any position. (PYQ)
- Relate U.C.M. with S.H.M. (PYQ)
- Find phase difference / time lag between two SHM. (PYQ)

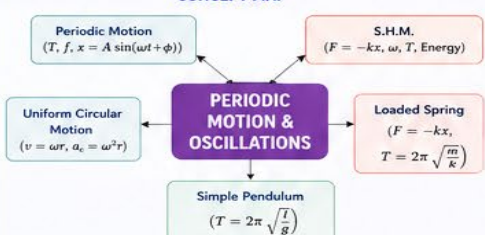
APPLICATIONS (REAL LIFE)

- Simple Pendulum $\rightarrow$ clocks, metronome
- Spring Oscillator $\rightarrow$ car suspensions, vibration absorbers
- SHM & Waves $\rightarrow$ sound, musical instruments
- U.C.M. $\rightarrow$ rotating wheels, satellites
- Periodic Functions $\rightarrow$ electricity (AC), wave technology

IMPORTANT FORMULA COMPARISON TABLE

Concept	Formula
Time Period (SHM)	$T = 2\pi \sqrt{\frac{m}{k}}$
Frequency (SHM)	$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$
Angular Frequency (SHM)	$\omega = \sqrt{\frac{k}{m}} = 2\pi f$
Displacement (SHM)	$x = A \sin(\omega t + \phi)$
Energy (SHM)	$E = \frac{1}{2} k A^2$
Kinetic Energy (SHM)	$E = \frac{1}{2} k (A^2 - x^2)$
Potential Energy (SHM)	$U = \frac{1}{2} k x^2$
Simple Pendulum (T)	$T = 2\pi \sqrt{\frac{l}{g}}$
U.C.M. Speed	$v = \omega r$

CONCEPT MAP



EXAM SUCCESS MANTRA

- Understand Concept
- Memorize Formulas
- Practice PYQs
- Analyze Mistakes
- Revise Regularly

**STAY CONSISTENT,
SCORE EXCELLENT!**



DIGITAL RESOURCES

CHAPTERWISE INFORGRAPHICS, FORMULA SHEET, INTERACTIVE WORKSHEETS AND QUESTION PAPERS	FORMULA SHEET Infographics XI 2026-27 Interactive worksheets class XI Practice QP Class XI
NCERT YOUTUBE OFFICIAL XI PHYSICS	https://www.youtube.com/@NCERTOFFICIAL/search?query=physics%20class%20XI
XI PRACTICAL	https://www.youtube.com/@NCERTOFFICIAL/search?query=physics%20class%20XI%20PRACTICAL



Physics Is Not Just About Formulas And Laws—It Is The Art Of Understanding Nature Through Curiosity, Observation, And Reasoning. This Support Material Has Been Developed With The Sincere Effort To Simplify Concepts, Strengthen Fundamentals, And Inspire Every Learner To Think Scientifically And Confidently